CHAPTER 13

Ahmed Kabil

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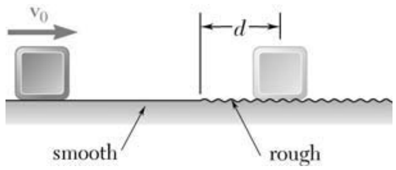
PROBLEM 13.CQ1



Block A is traveling with a speed v_0 on a smooth surface when the surface suddenly becomes rough with a coefficient of friction of μ causing the block to stop after a distance d. If block A were traveling twice as fast, that is, at a speed $2v_0$, how far will it travel on the rough surface before stopping?

(*a*) $d/2$

(*b*) d

(*c*) $\sqrt{2}d$

(*d*) $2d$

(*e*) $4d$

SOLUTION

Answer: (*e*)

PROBLEM 13.1

A 400-kg satellite was placed in a circular orbit 1500 km above the surface of the earth. At this elevation the acceleration of gravity is 6.43 m/s^2. Determine the kinetic energy of the satellite, knowing that its orbital speed is 25.6×10^3 km/h.

SOLUTION

Mass of satellite: $m = 400$ kg

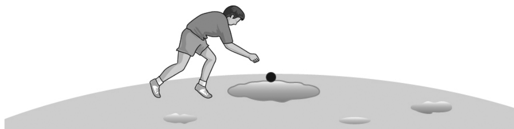
Velocity: $v = 25.6\times10^3 \text{ km/h} = 7.111\times10^3 \text{ m/s}$

Kinetic energy: $T = \dfrac{1}{2}mv^2 = \dfrac{1}{2}(400 \text{ kg})(7.111\times10^3 \text{ m/s})^2$

$T = 10.113\times10^9$ J $\qquad T = 10.11$ GJ ◀

Note: Acceleration of gravity has no effect on the mass of the satellite.

PROBLEM 13.2



A 1-lb stone is dropped down the "bottomless pit" at Carlsbad Caverns and strikes the ground with a speed of 95ft/s. Neglecting air resistance, determine (*a*) the kinetic energy of the stone as it strikes the ground and the height *h* from which it was dropped, (*b*) Solve Part *a* assuming that the same stone is dropped down a hole on the moon. (Acceleration of gravity on the moon = 5.31 ft/s^2.)

SOLUTION

Mass of stone: $m = \dfrac{W\text{ lb}}{g} = \dfrac{1\text{ lb}}{32.2\text{ ft/s}^2} = 0.031056\text{ lb}\cdot\text{s}^2/\text{ft}$

Initial kinetic energy: $T_1 = 0$ (rest)

(*a*) Kinetic energy at ground strike:

$$T_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}(0.031056)(95)^2 = 140.14\text{ ft}\cdot\text{lb}$$

$T_2 = 140.1\text{ ft}\cdot\text{lb}$ ◀

Use work and energy: $T_1 + U_{1\to 2} = T_2$

where $U_{1\to 2} = wh = mgh$

$$0 + mgh = \frac{1}{2}mv_2^2$$

$$h = \frac{v_2^2}{2g} = \frac{(95)^2}{(2)(32.2)}$$

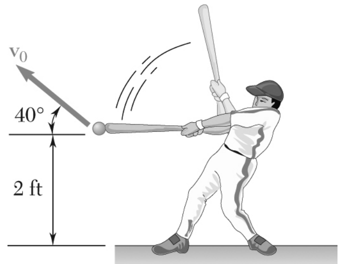
$h = 140.1\text{ ft}$ ◀

(*b*) On the moon: $g = 5.31\text{ ft/s}^2$

T_1 and T_2 will be the same, hence $T_2 = 140.1\text{ ft}\cdot\text{lb}$ ◀

$$h = \frac{v_2^2}{2g} = \frac{(95)^2}{(2)(5.31)}$$

$h = 850\text{ ft}$ ◀



PROBLEM 13.3

A baseball player hits a 5.1-oz baseball with an initial velocity of 140 ft/s at an angle of 40° with the horizontal as shown. Determine (*a*) the kinetic energy of the ball immediately after it is hit, (*b*) the kinetic energy of the ball when it reaches its maximum height, (*c*) the maximum height above the ground reached by the ball.

SOLUTION

Mass of baseball: $$W = (5.1\text{ oz})\left(\frac{1\text{ lb}}{16\text{ oz}}\right) = 0.31875\text{ lb}$$

$$m = \frac{W}{g} = \frac{0.31875\text{ lb}}{32.2\text{ ft/s}^2} = 0.009899\text{ lb}\cdot\text{s}^2/\text{ft}$$

(*a*) Kinetic energy immediately after hit.

$$v = v_0 = 140\text{ ft/s}$$

$$T_1 = \frac{1}{2}mv^2 = \frac{1}{2}(0.009899)(140)^2 \qquad T_1 = 97.0\text{ ft}\cdot\text{lb} \blacktriangleleft$$

(*b*) Kinetic energy at maximum height:

$$v = v_0\cos 40° = 140\cos 40° = 107.246\text{ ft/s}$$

$$T_2 = \frac{1}{2}mv^2 = \frac{1}{2}(0.009899)(107.246)^2 \qquad T_2 = 56.9\text{ ft}\cdot\text{lb} \blacktriangleleft$$

Principle of work and energy: $T_1 + U_{1\to2} = T_2$

$$U_{1\to2} = T_2 - T_1 = -40.082\text{ ft}\cdot\text{lb}$$

Work of weight: $U_{1\to2} = -Wd$

Maximum height above impact point.

$$d = \frac{T_2 - T_1}{-W} = \frac{-40.082\text{ ft}\cdot\text{lb}}{-0.31875\text{ lb}} = 125.7\text{ ft} \qquad 125.7\text{ ft} \blacktriangleleft$$

(*c*) Maximum height above ground:

$$h = 125.7\text{ ft} + 2\text{ ft} \qquad h = 127.7\text{ ft} \blacktriangleleft$$

PROBLEM 13.4

A 500-kg communications satellite is in a circular geosynchronous orbit and completes one revolution about the earth in 23 h and 56 min at an altitude of 35800 km above the surface of the earth. Knowing that the radius of the earth is 6370 km, determine the kinetic energy of the satellite.

SOLUTION

Radius of earth: $R = 6370 \text{ km}$

Radius of orbit: $r = R + h = 6370 + 35800 = 42170 \text{ km} = 42.170 \times 10^6 \text{ m}$

Time one revolution: $t = 23 \text{ h} + 56 \text{ min}$

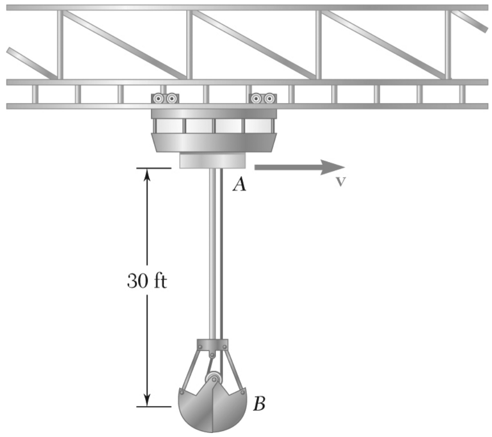
$$t = (23 \text{ h})(3600 \text{ s/h}) + (56 \text{ min})(60 \text{ s/min}) = 86.160 \times 10^3 \text{ s}$$

Speed: $$v = \frac{2\pi r}{t} = \frac{2\pi(42.170 \times 10^6)}{86.160 \times 10^3} = 3075.2 \text{ m/s}$$

Kinetic energy: $$T = \frac{1}{2} m v^2$$

$$T = \frac{1}{2} (500 \text{ kg})(3075.2 \text{ m/s})^2 = 2.3643 \times 10^9 \text{ J}$$

$T = 2.36 \text{ GJ}$ ◀



PROBLEM 13.5

In an ore-mixing operation, a bucket full of ore is suspended from a traveling crane which moves along a stationary bridge. The bucket is to swing no more than 10 ft horizontally when the crane is brought to a sudden stop. Determine the maximum allowable speed *v* of the crane.

SOLUTION

Let position ① be the position with bucket *B* directly below *A*, and position ② be that of maximum swing where $d = 10$ ft. Let *L* be the length *AB*.

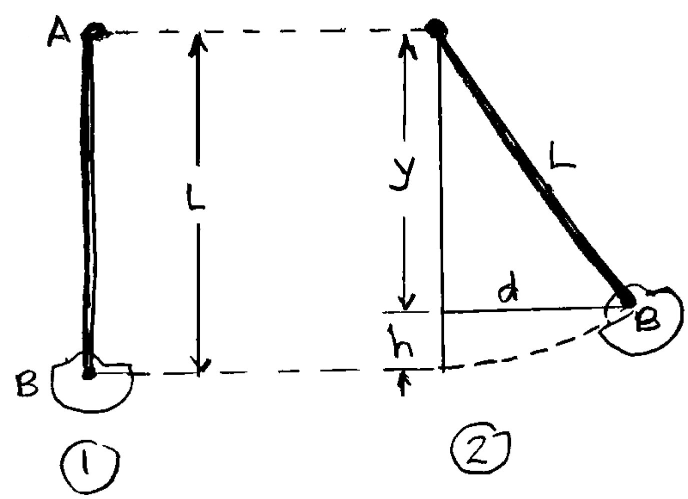
Kinetic energies: $$T_1 = \frac{1}{2}mv^2, \; T_2 = 0$$

Work of the weight: $$U_{1\to 2} = -Wh = -mgh$$

where *h* is the vertical projection of position ② above position ①

From geometry (see figure),

$$\begin{aligned} y &= \sqrt{L^2 - d^2} \\ h &= L - y \\ &= L - \sqrt{L^2 - d^2} \\ &= 30 - \sqrt{(30)^2 - (10)^2} \\ &= 1.7157 \text{ ft} \end{aligned}$$

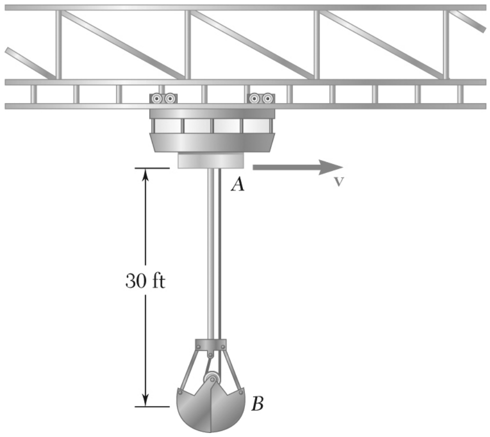


Principle of work and energy: $$T_1 + U_{1\to 2} = T_2$$

$$\frac{1}{2}mv^2 - mgh = 0$$

$$v^2 = 2gh = (2)(32.2 \text{ ft/s}^2)(1.7157 \text{ ft}) = 110.49 \text{ ft}^2/\text{s}^2$$

$v = 10.51$ ft/s ◄



PROBLEM 13.6

In an ore-mixing operation, a bucket full of ore is suspended from a traveling crane which moves along a stationary bridge. The crane is traveling at a speed of 10 ft/s when it is brought to a sudden stop. Determine the maximum horizontal distance through which the bucket will swing.

SOLUTION

Let position ① be the position with bucket B directly below A, and position ② be that of maximum swing where the horizontal distance is d. Let L be the length AB.

Kinetic energies: $$T_1 = \frac{1}{2}mv^2, \; T_2 = 0$$

Work of the weight: $$U_{1\to 2} = -Wh = -mgh$$

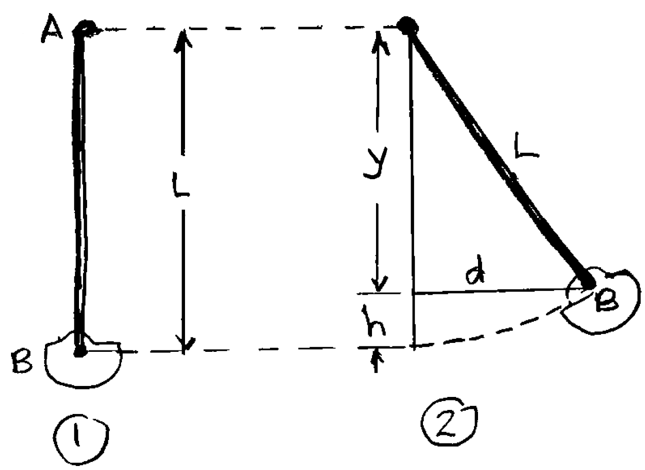
where h is the vertical projection of position ② above position ①.

Principle of work and energy: $$T_1 + U_{1\to 2} = T_2$$

$$\frac{1}{2}mv^2 - mgh = 0$$

$$h = \frac{v^2}{2g} = \frac{(10 \text{ ft/s})^2}{(2)(32.2 \text{ ft/s}^2)} = 1.5528 \text{ ft}$$

From geometry (see figure),



$$\begin{aligned} d &= \sqrt{L^2 - y^2} \\ &= \sqrt{L^2 - (L-h)^2} \\ &= \sqrt{(30)^2 - (30 - 1.5528)^2} \\ &= 9.53 \text{ ft} \end{aligned}$$

$d = 9.53$ ft ◀

PROBLEM 13.7

Determine the maximum theoretical speed that may be achieved over a distance of 110 m by a car starting from rest assuming there is no slipping. The coefficient of static friction between the tires and pavement is 0.75, and 60 percent of the weight of the car is distributed over its front wheels and 40 percent over its rear wheels. Assume (*a*) front-wheel drive, (*b*) rear-wheel drive.

SOLUTION

Let W be the weight and m the mass. $W = mg$

(*a*) *Front wheel drive*: $N = 0.60W = 0.60mg$

$$\mu_s = 0.75$$

Maximum friction force without slipping:

$$F = \mu_s N = (0.75)(0.60W) = 0.45mg$$

$$U_{1\to 2} = Fd = 0.45mgd$$

$$T_1 = 0, \qquad T_2 = \frac{1}{2}mv_2^2$$

Principle of work and energy: $T_1 + U_{1\to 2} = T_2$

$$0 + 0.45mgd = \frac{1}{2}mv_2^2$$

$$v_2^2 = (2)(0.45gd) = (2)(0.45)(9.81\text{ m/s}^2)(110\text{ m}) = 971.19\text{ m}^2/\text{s}^2$$

$v_2 = 31.164$ m/s $\qquad v_2 = 112.2$ km/h ◀

(*b*) *Rear wheel drive*: $N = 0.40W = 0.40mg$

$$\mu_s = 0.75$$

Maximum friction force without slipping:

$$F = \mu_s N = (0.75)(0.40W) = 0.30mg$$

$$U_{1\to 2} = Fd = 0.30mgd$$

$$T_1 = 0, \qquad T_2 = \frac{1}{2}mv_2^2$$

Principle of work and energy: $T_1 + U_{1\to 2} = T_2$

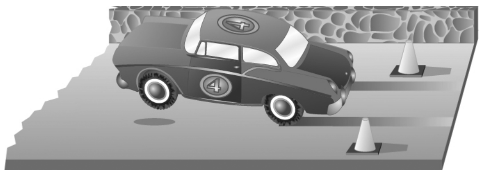
$$0 + 0.30mgd = \frac{1}{2}mv_2^2$$

$$v_2^2 = (2)(0.30)gd = (2)(0.30)(9.81\text{ m/s}^2)(110\text{ m}) = 647.46\text{ m}^2/\text{s}^2$$

$v_2 = 25.445$ m/s $\qquad v_2 = 91.6$ km/h ◀

Note: The car is treated as a particle in this problem. The weight distribution is assumed to be the same for static and dynamic conditions. Compare with sample Problem 16.1 where the vehicle is treated as a rigid body.

PROBLEM 13.8



Skid marks on a drag racetrack indicate that the rear (drive) wheels of a car slip for the first 20 m of the 400-m track. (*a*) Knowing that the coefficient of kinetic friction is 0.60, determine the speed of the car at the end of the first 20-m portion of the track if it starts from rest and the front wheels are just off the ground. (*b*) What is the maximum theoretical speed for the car at the finish line if, after skidding for 20 m, it is driven without the wheels slipping for the remainder of the race? Assume that while the car is rolling without slipping, 60 percent of the weight of the car is on the rear wheels and the coefficient of static friction is 0.75. Ignore air resistance and rolling resistance.

SOLUTION

(*a*) For the first 20 m, the normal force at the real wheels is equal to the weight of the car. Since the wheels are skidding, the friction force is

$$F = \mu_k N = \mu_k W = \mu_k mg$$

Principle of work and energy: $T_1 + U_{1\to 2} = T_2$

$$0 + Fd = \frac{1}{2}mv_2^2$$

$$0 + \mu_k mgd = \frac{1}{2}mv_2^2$$

$$v_2^2 = 2\mu_k gd = (2)(0.6)(9.81\text{ m/s}^2)(20\text{ m}) = 235.44\text{ m}^2/\text{s}^2$$

$$v_2 = 15.34\text{ m/s} \blacktriangleleft$$

(*b*) Assume that for the remainder of the race, sliding is impending and $N = 0.6\ W$

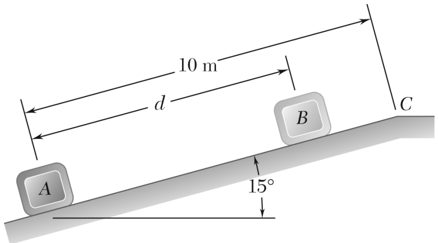
$$F = \mu_s N = \mu_s(0.6W) = (0.75)(0.6mg) = 0.45mg$$

Principle of work and energy: $T_2 + U_{2\to 3} = T_3$

$$\frac{1}{2}mv_2^2 + (0.45mg)d' = \frac{1}{2}mv_3^2$$

$$\begin{aligned} v_3^2 &= v_2^2(2)(0.45)gd' \\ &= 235.44\text{ m}^2/\text{s}^2 + (2)(0.45)(9.81\text{ m/s}^2)(400\text{ m} - 20\text{ m}) \\ &= 3590.5\text{ m}^2/\text{s}^2 \end{aligned}$$

$$v_3 = 59.9\text{ m/s} \blacktriangleleft$$



PROBLEM 13.9

A package is projected up a 15° incline at A with an initial velocity of 8 m/s. Knowing that the coefficient of kinetic friction between the package and the incline is 0.12, determine (*a*) the maximum distance d that the package will move up the incline, (*b*) the velocity of the package as it returns to its original position.

SOLUTION

(*a*) Up the plane from A to B:

$$T_A = \frac{1}{2}mv_A^2 = \frac{1}{2}\frac{W}{g}(8\text{ m/s})^2 = 32\frac{W}{g} \qquad T_B = 0$$

$$U_{A-B} = (-W\ \sin 15° - F)d \qquad F = \mu_k N = 0.12\,\text{N}$$

$$\nwarrow \Sigma F = 0 \quad N - W\cos 15° = 0 \quad N = W\cos 15°$$

$$U_{A-B} = -W(\sin 15° + 0.12\cos 15°)d = -Wd(0.3747)$$

$$T_A + U_{A-B} = T_B: \quad 32\frac{W}{g} - Wd(0.3743) = 0$$

$$d = \frac{32}{(9.81)(0.3747)} \qquad d = 8.71\text{ m} \blacktriangleleft$$

(*b*) Down the plane from B to A: (F reverses direction)

$$T_A = \frac{1}{2}\frac{W}{g}v_A^2 \qquad T_B = 0 \qquad d = 8.71\text{ m/s}$$

$$U_{B-A} = (W\ \sin 15° - F)d$$

$$= W(\sin 15° - 0.12\cos 15°)(8.70\text{ m/s})$$

$$U_{B-A} = 1.245W$$

$$T_B + U_{B-A} = T_A \quad 0 + 1.245W = \frac{1}{2}\frac{W}{g}v_A^2$$

$$v_A^2 = (2)(9.81)(1.245)$$

$$= 24.43$$

$$v_A = 4.94\text{ m/s} \qquad \mathbf{v}_A = 4.94\text{ m/s} \swarrow 15° \blacktriangleleft$$

PROBLEM 13.10

A 1.4 kg model rocket is launched vertically from rest with a constant thrust of 25 N until the rocket reaches an altitude of 15 m and the thrust ends. Neglecting air resistance, determine (*a*) the speed of the rocket when the thrust ends, (*b*) the maximum height reached by the rocket, (*c*) the speed of the rocket when it returns to the ground.

SOLUTION

Weight: $W = mg = (1.4)(9.81) = 13.734 \text{ N}$

(*a*) First stage: $T_1 = 0$

$$U_{1\to 2} = (25 - 13.734)(15) = 169.0 \text{ N}\cdot\text{m}$$

$$T_1 + U_{1\to 2} = T_2$$

$$T_2 = \frac{1}{2}mv^2 = U_{1\to 2} = 169.0 \text{ N}\cdot\text{m}$$

$$v_2 = \sqrt{\frac{2U_{1\to 2}}{m}} = \sqrt{\frac{(2)(169.0)}{1.4}}$$

$v_2 = 15.54 \text{ m/s}$ ◀

(*b*) Unpowered flight to maximum height *h*:

$$T_2 = 169.0 \text{ N}\cdot\text{m} \qquad T_3 = 0$$

$$U_{2\to 3} = -W(h - 15)$$

$$T_2 + U_{2\to 3} = T_3$$
$$W(h - 15) = T_2$$

$$h - 15 = \frac{T_2}{W} = \frac{169.0}{13.734}$$

$h = 27.3 \text{ m}$ ◀

(*c*) Falling from maximum height:

$$T_3 = 0 \qquad T_4 = \frac{1}{2}mv_4^2$$

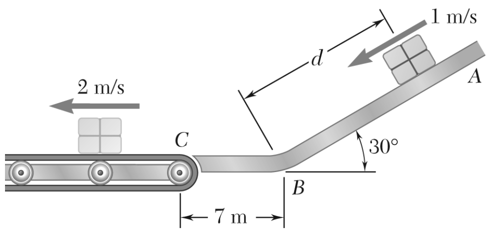
$$U_{3\to 4} = Wh = mgh$$

$$T_3 = U_{3\to 4} = T_4: \qquad 0 + mgh = \frac{1}{2}mv_4^2$$

$$v_4^2 = 2\,gh = (2)(9.81 \text{ m/s}^2)(27.3 \text{ m}) = 535.6 \text{ m}^2/\text{s}^2$$

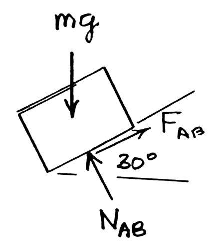
$v_4 = 23.1 \text{ m/s}$ ◀

PROBLEM 13.11



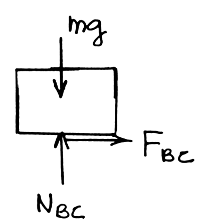
Packages are thrown down an incline at A with a velocity of 1 m/s. The packages slide along the surface ABC to a conveyor belt which moves with a velocity of 2 m/s. Knowing that $\mu_k = 0.25$ between the packages and the surface ABC, determine the distance d if the packages are to arrive at C with a velocity of 2 m/s.

SOLUTION



On incline AB:

$$N_{AB} = mg\cos 30°$$
$$F_{AB} = \mu_k N_{AB} = 0.25 mg \cos 30°$$
$$U_{A\to B} = mgd\sin 30° - F_{AB}\, d$$
$$= mgd(\sin 30° - \mu_k \cos 30°)$$



On level surface BC:

$$N_{BC} = mg \qquad x_{BC} = 7 \text{ m}$$
$$F_{BC} = \mu_k mg$$
$$U_{B\to C} = -\mu_k mg\, x_{BC}$$

At A, $$T_A = \frac{1}{2} m v_A^2 \quad \text{and} \quad v_A = 1 \text{ m/s}$$

At C, $$T_C = \frac{1}{2} m v_C^2 \quad \text{and} \quad v_C = 2 \text{ m/s}$$

Assume that no energy is lost at the corner B.

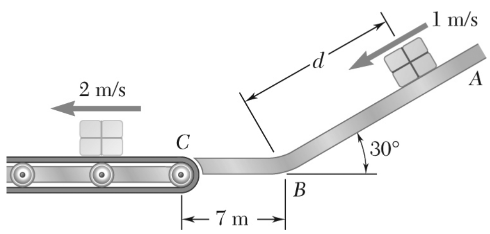
Work and energy. $$T_A + U_{A\to B} + U_{B\to C} = T_C$$

$$\frac{1}{2} m v_A^2 + mgd(\sin 30° - \mu_k \cos 30°) - \mu_k mg\, x_{BC} = \frac{1}{2} m v_0^2$$

Dividing by m and solving for d,

$$d = \frac{\left[v_C^2/2g + \mu_k x_{BC} - v_A^2/2g\right]}{(\sin 30° - \mu_k \cos 30°)}$$
$$= \frac{(2)^2/(2)(9.81) + (0.25)(7) - (1)^2/(2)(9.81)}{\sin 30° - 0.25\cos 30°}$$

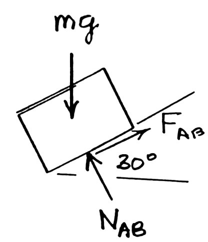
$d = 6.71$ m ◀



PROBLEM 13.12

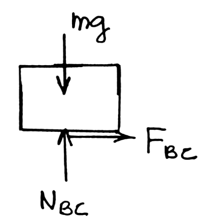
Packages are thrown down an incline at A with a velocity of 1 m/s. The packages slide along the surface ABC to a conveyor belt which moves with a velocity of 2 m/s. Knowing that $d = 7.5\text{ m}$ and $\mu_k = 0.25$ between the packages and all surfaces, determine (*a*) the speed of the package at C, (*b*) the distance a package will slide on the conveyor belt before it comes to rest relative to the belt.

SOLUTION



(*a*) On incline AB:

$$N_{AB} = mg\cos 30°$$
$$F_{AB} = \mu_k N_{AB} = 0.25mg\cos 30°$$
$$U_{A\to B} = mgd\sin 30° - F_{AB}d$$
$$= mgd(\sin 30° - \mu_k\cos 30°)$$



On level surface BC:

$$N_{BC} = mg \quad x_{BC} = 7\text{ m}$$
$$F_{BC} = \mu_k mg$$
$$U_{B\to C} = -\mu_k mg\, x_{BC}$$

At A, $\quad T_A = \dfrac{1}{2}mv_A^2 \quad$ and $\quad v_A = 1\text{ m/s}$

At C, $\quad T_C = \dfrac{1}{2}mv_C^2 \quad$ and $\quad v_C = 2\text{ m/s}$

Assume that no energy is lost at the corner B.

Work and energy. $\quad T_A + U_{A\to B} + U_{B\to C} = T_C$

$$\frac{1}{2}mv_A^2 + mgd(\sin 30° - \mu_k\cos 30°) - \mu_k mg\, x_{BC} = \frac{1}{2}mv_0^2$$

Solving for v_C^2,

$$v_C^2 = v_A^2 + 2gd(\sin 30° - \mu_k\cos 30°) - 2\mu_k g\, x_{BC}$$
$$= (1)^2 + (2)(9.81)(7.5)(\sin 30° - 0.25\cos 30°) - (2)(0.25)(9.81)(7)$$
$$= 8.3811\text{ m}^2/\text{s}^2$$

$v_C = 2.90\text{ m/s}$ ◀

(*b*) Box on belt: Let x_{belt} be the distance moves by a package as it slides on the belt.

$$+\uparrow \Sigma F_y = ma_y \quad N - mg = 0 \quad N = mg$$
$$F_x = \mu_k N = \mu_k mg$$

At the end of sliding, $\quad v = v_{\text{belt}} = 2\text{ m/s}$

PROBLEM 13.12 (Continued)

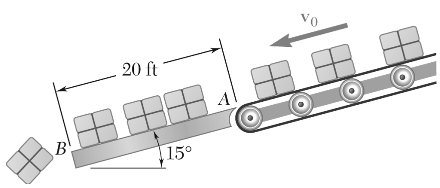
Principle of work and energy:

$$\frac{1}{2}mv_C^2 - \mu_k mg\, x_{\text{belt}} = \frac{1}{2}mv_{\text{belt}}^2$$

$$x_{\text{belt}} = \frac{v_C^2 - v_{\text{belt}}^2}{2\mu_k g}$$

$$= \frac{8.3811-(2)^2}{(2)(0.25)(9.81)}$$

$x_{\text{belt}} = 0.893 \text{ m}$ ◀



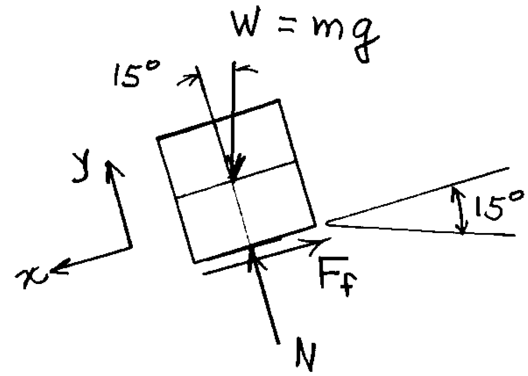
PROBLEM 13.13

Boxes are transported by a conveyor belt with a velocity $\mathbf{v}_0$ to a fixed incline at A where they slide and eventually fall off at B. Knowing that $\mu_k = 0.40$, determine the velocity of the conveyor belt if the boxes leave the incline at B with a velocity of 8 ft/s.

SOLUTION

Forces when box is on AB.

$$\Sigma F_y = 0: \quad N - W\cos 15° = 0$$
$$N = W\cos 15°$$



Box is sliding on AB.

$$F_f = \mu_k N = \mu_k W\cos 15°$$

Distance $\qquad AB = d = 20\text{ ft}$

Work of gravity force: $\qquad (U_{A-B})_g = Wd\sin 15°$

Work of friction force: $\qquad -F_f d = -\mu_k Wd\cos 15°$

Total work $\qquad U_{A\to B} = Wd(\sin 15° - \mu_k \sin 15°)$

Kinetic energy: $\qquad T_A = \dfrac{1}{2}\dfrac{W}{g}v_0^2$

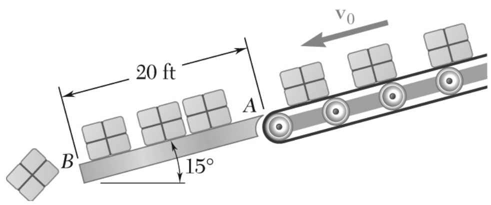
$$T_B = \frac{1}{2}\frac{W}{g}v_B^2$$

Principle of work and energy: $\qquad T_A + U_{A\to B} = T_B$

$$\frac{1}{2}\frac{W}{g}v_0^2 + Wd(\sin 15° - \mu_k\cos 15°) = \frac{1}{2}\frac{W}{g}v_B^2$$

$$v_0^2 = v_B^2 - 2gd(\sin 15° - \mu_k \cos 15°)$$
$$= (8)^2 - (2)(32.2)(20)[\sin 15° - (0.40)(\cos 15°)]$$
$$= 228.29\text{ ft}^2/\text{s}^2$$

$\mathbf{v}_0 = 15.11\text{ ft/s} \nearrow 15°$ ◀



PROBLEM 13.14

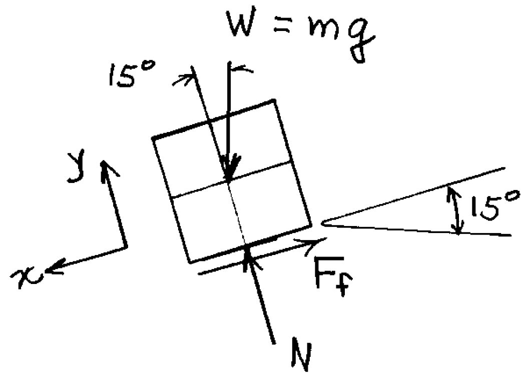
Boxes are transported by a conveyor belt with a velocity $\mathbf{v}_0$ to a fixed incline at A where they slide and eventually fall off at B. Knowing that $\mu_k = 0.40$, determine the velocity of the conveyor belt if the boxes are to have zero velocity at B.

SOLUTION

Forces when box is on AB.

$$\Sigma F_y = 0: \quad N - W\cos 15° = 0$$

$$N = W\cos 15°$$



Box is sliding on AB.

$$F_f = \mu_k N = \mu_k W\cos 15°$$

Distance $\quad AB = d = 20 \text{ ft}$

Work of gravity force: $\quad (U_{A-B})_g = Wd\sin 15°$

Work of friction force: $\quad -F_f d = -\mu_k Wd\cos 15°$

Total work $\quad U_{A-B} = Wd(\sin 15° - \mu_k\cos 15°)$

Kinetic energy:

$$T_A = \frac{1}{2}\frac{W}{g}v_0^2$$

$$T_B = \frac{1}{2}\frac{W}{g}v_B^2$$

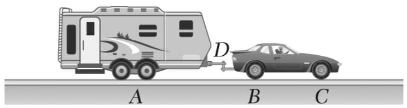
Principle of work and energy: $\quad T_A + U_{A-B} = T_B$

$$\frac{1}{2}\frac{W}{g}v_0^2 + Wd(\sin 15° - \mu_k\cos 15°) = \frac{1}{2}\frac{W}{g}v_B^2$$

$$\begin{aligned} v_0^2 &= v_B^2 - 2gd(\sin 15° - \mu_k\cos 15°) \\ &= 0 - (2)(32.2)(20)[\sin 15° - (0.40)(\cos 15°)] \\ &= 164.29 \text{ ft}^2/\text{s}^2 \end{aligned}$$

$\mathbf{v}_0 = 12.81 \text{ ft/s} \measuredangle 15°$ ◀

PROBLEM 13.15



A 1200-kg trailer is hitched to a 1400-kg car. The car and trailer are traveling at 72 km/h when the driver applies the brakes on both the car and the trailer. Knowing that the braking forces exerted on the car and the trailer are 5000 N and 4000 N, respectively, determine (*a*) the distance traveled by the car and trailer before they come to a stop, (*b*) the horizontal component of the force exerted by the trailer hitch on the car.

SOLUTION

Let position 1 be the initial state at velocity $v_1 = 72 \text{ km/h} = 20 \text{ m/s}$ and position 2 be at the end of braking $(v_2 = 0)$. The braking forces and $F_C = 5000 \text{ N}$ for the car and 4000 N for the trailer.

(*a*) Car and trailer system. (d = braking distance)

$$T_1 = \frac{1}{2}(m_C + m_T)v_1^2 \qquad T_2 = 0$$

$$U_{1\to 2} = -(F_C + F_T)d$$

$$T_1 + U_{1\to 2} = T_2$$

$$\frac{1}{2}(m_C + m_T)v_1^2 - (F_C + F_T)d = 0$$

$$d = \frac{(m_C + m_T)v_1^2}{2(F_C + F_T)} = \frac{(2600)(20)^2}{(2)(9000)} = 57.778$$

$d = 57.8 \text{ m}$ ◀

(*b*) Car considered separately.

Let H be the horizontal pushing force that the trailer exerts on the car through the hitch.

$$T_1 = \frac{1}{2}m_C v_1^2 \qquad T_2 = 0$$

$$U_{1\to 2} = (H - F_C)d$$

$$T_1 + U_{1\to 2} = T_2$$

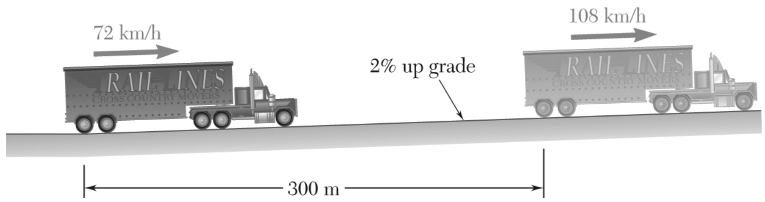
$$\frac{1}{2}m_C v_1^2 + (H - F_C)d = 0$$

$$H = F_C - \frac{m_C v_1^2}{2d} = 5000 - \frac{(1400)(20)^2}{(2)(57.778)}$$

Trailer hitch force on car: $\mathbf{H} = 154 \text{ N} \longrightarrow$ ◀

PROBLEM 13.16

A trailer truck enters a 2 percent uphill grade traveling at 72 km/h and reaches a speed of 108 km/h in 300 m. The cab has a mass of 1800 kg and the trailer 5400 kg. Determine (*a*) the average force at the wheels of the cab, (*b*) the average force in the coupling between the cab and the trailer.



SOLUTION

Initial speed: $v_1 = 72 \text{ km/h} = 20 \text{ m/s}$

Final speed: $v_2 = 108 \text{ km/h} = 30 \text{ m/s}$

Vertical rise: $h = (0.02)(300) = 6.00 \text{ m}$

Distance traveled: $d = 300 \text{ m}$

(*a*) Traction force. Use cab and trailer as a free body.

$m = 1800 + 5400 = 7200 \text{ kg} \qquad W = mg = (7200)(9.81) = 70.632 \times 10^3 \text{ N}$

Work and energy: $T_1 + U_{1\to 2} = T_2 \qquad \frac{1}{2}mv_1^2 - Wh + F_t d = \frac{1}{2}mv_2^2$

$$F_t = \frac{1}{d}\left[\frac{1}{2}mv_1^2 + Wh - mv_1^2\right] = \frac{1}{300}\left[\frac{1}{2}(7200)(30)^2 + (70.632\times 10^3)(6.00) - \frac{1}{2}(7200)(20)^2\right]$$

$= 7.4126 \times 10^3 \text{ N}$ $\qquad F_t = 7.41 \text{ kN}$ ◀

(*b*) Coupling force F_c. Use the trailer alone as a free body.

$m = 5400 \text{ kg} \qquad W = mg = (5400)(9.81) = 52.974 \times 10^3 \text{ N}$

Assume that the tangential force at the trailer wheels is zero.

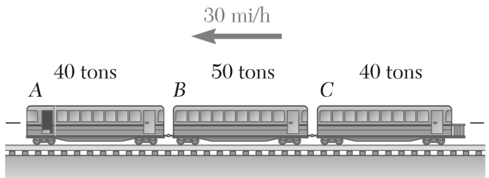
Work and energy: $T_1 + U_{1\to 2} = T_2 \qquad \frac{1}{2}mv_1^2 - Wh + F_c d = \frac{1}{2}mv_2^2$

The plus sign before F_c means that we have assumed that the coupling is in tension.

$$F_c = \frac{1}{d}\left[\frac{1}{2}mv_2^2 + Wh - \frac{1}{2}mv_1^2\right] = \frac{1}{300}\left[\frac{1}{2}(5400)(30)^2 + (52.974\times 10^3)(6.00) - \frac{1}{2}(5400)(20)^2\right]$$

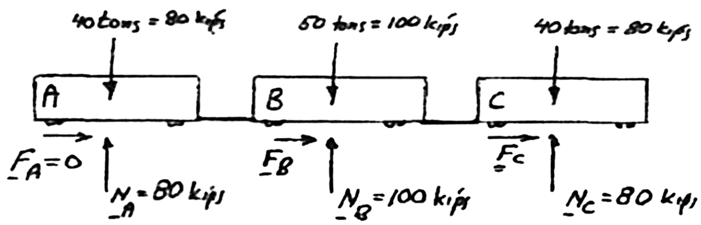
$= 5.5595 \times 10^3 \text{ N}$ $\qquad F_c = 5.56 \text{ kN (tension)}$ ◀

PROBLEM 13.17



The subway train shown is traveling at a speed of 30 mi/h when the brakes are fully applied on the wheels of cars *B* and *C*, causing them to slide on the track, but are not applied on the wheels of car *A*. Knowing that the coefficient of kinetic friction is 0.35 between the wheels and the track, determine (*a*) the distance required to bring the train to a stop, (*b*) the force in each coupling.

SOLUTION



$$\mu_k = 0.35 \quad F_B = (0.35)(100 \text{ kips}) = 35 \text{ kips}$$

$$F_C = (0.35)(80 \text{ kips}) = 28 \text{ kips}$$

$$v_1 = 30 \text{ mi/h} = 44 \text{ ft/s} \longleftarrow \qquad v_2 = 0 \quad T_2 = 0$$

(*a*) Entire train: $T_1 + U_{1-2} = T_2$

$$\frac{1}{2}\frac{(80 \text{ kips} + 100 \text{ kips } + 80 \text{ kips})}{32.2 \text{ ft/s}^2}(44 \text{ ft/s})^2 - (28 \text{ kips} + 35 \text{ kips})\, x = 0$$

$$x = 124.07 \text{ ft}$$

$x = 124.1$ ft ◀

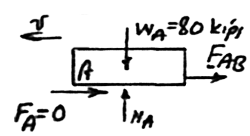
(*b*) Force in each coupling: Recall that $x = 124.07$ ft

Car *A*: Assume F_{AB} to be in tension

$$T_1 + V_{1-2} = T_2$$

$$\frac{1}{2}\frac{80 \text{ kips}}{32.2}(44)^2 - F_{AB}(124.07 \text{ ft}) = 0$$

$$F_{AB} = +19.38 \text{ kips}$$



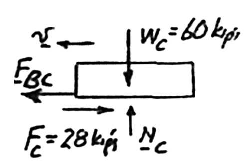
$F_{AB} = 19.38$ kips (tension) ◀

Car *C*: $T_1 + U_{1-2} = T_2$

$$\frac{1}{2}\frac{80 \text{ kips}}{32.2}(44)^2 + (F_{BC} - 28 \text{ kips})(124.07 \text{ ft}) = 0$$

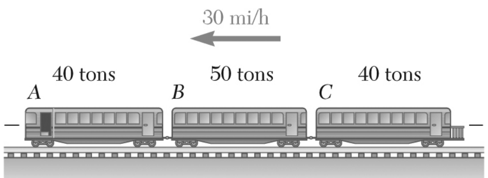
$$F_{BC} - 28 \text{ kips} = -19.38 \text{ kips}$$

$$F_{BC} = +8.62 \text{ kips}$$



$F_{BC} = 8.62$ kips (tension) ◀

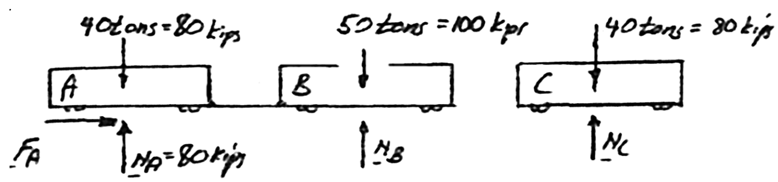
PROBLEM 13.18



The subway train shown is traveling at a speed of 30 mi/h when the brakes are fully applied on the wheels of cars A, causing it to slide on the track, but are not applied on the wheels of cars A or B. Knowing that the coefficient of kinetic friction is 0.35 between the wheels and the track, determine (a) the distance required to bring the train to a stop, (b) the force in each coupling.

SOLUTION

(a) Entire train:



$$F_A = \mu N_A = (0.35)(80\text{ kips}) = 28\text{ kips}$$

$$v_1 = 30\text{ mi/h} = 44\text{ ft/s} \leftarrow \qquad v_2 = 0 \quad T_2 = 0$$

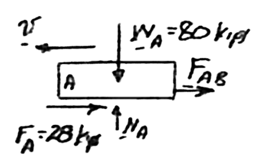
$$T_1 + V_{1-2} = T_2$$

$$\frac{1}{2}\frac{(80\text{ kips} + 100\text{ kips} + 80\text{ kips})}{32.2\text{ ft/s}^2}(44\text{ ft/s})^2 - (28\text{ kips})\,x = 0$$

$$x = 279.1\text{ ft} \qquad x = 279\text{ ft} \blacktriangleleft$$

(b) Force in each coupling:

Car A: Assume F_{AB} to be in tension



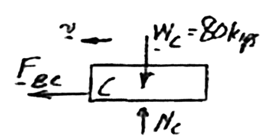
$$T_1 + V_{1-2} = T_2$$

$$\frac{1}{2}\frac{80\text{ kips}}{32.2\text{ ft/s}^2}(44\text{ ft/s})^2 - (28\text{ kips} + F_{AB})(279.1\text{ ft}) = 0$$

$$28\text{ kips} + F_{AB} = +8.62\text{ kips}$$

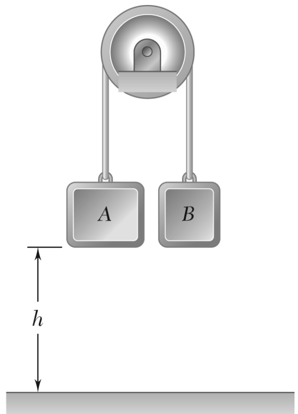
$$F_{AB} = -19.38\text{ kips} \qquad F_{AB} = 19.38\text{ kips (compression)} \blacktriangleleft$$

Car C: $\quad T_1 + V_{1-2} = T_2$



$$\frac{1}{2}\frac{80\text{ kips}}{32.2\text{ ft/s}^2}(44\text{ ft/s})^2 + F_{BC}(279.1\text{ ft}) = 0$$

$$F_{BC} = -8.617\text{ kips} \qquad F_{BC} = 8.62\text{ kips (compression)} \blacktriangleleft$$



PROBLEM 13.19

Blocks A and B weigh 25 lbs and 10 lbs, respectively, and they are both at a height 6 ft above the ground when the system is released from rest. Just before hitting the ground block A is moving at a speed of 9 ft/s. Determine (a) the amount of energy dissipated in friction by the pulley, (b) the tension in each portion of the cord during the motion.

SOLUTION

By constraint of the cable block B moves up a distance h when block A moves down a distance h. ($h = 6$ ft) Their speeds are equal.

Let F_A and F_B be the tensions on the A and B sides, respectively, of the pulley.

Masses:

$$M_A = \frac{W_A}{g} = \frac{25}{32.2} = 0.7764 \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$M_B = \frac{W_B}{g} = \frac{10}{32.2} = 0.31056 \text{ lb}\cdot\text{s}^2/\text{ft}$$

Let position 1 be the initial position with both blocks a distance h above the ground and position 2 be just before block A hits the ground.

Kinetic energies:

$$(T_1)_A = 0, \qquad (T_1)_B = 0$$

$$(T_2)_A = \frac{1}{2}m_A v^2 = \frac{1}{2}(0.7764)(9)^2 = 31.444 \text{ ft}\cdot\text{lb}$$

$$(T_2)_B = \frac{1}{2}m_B v^2 = \frac{1}{2}(0.31056)(9)^2 = 12.578 \text{ ft}\cdot\text{lb}$$

Principle of work and energy:

$$T_1 + U_{1\to 2} = T_2$$

Block A:

$$U_{1\to 2} = (W_A - F_A)h$$

$$0 + (25 - F_A)(6) = 31.444 \qquad F_A = 19.759 \text{ lb}$$

Block B:

$$U_{1\to 2} = (F_B - W_B)h$$

$$0 + (F_B - 10)(6) = 12.578 \qquad F_B = 12.096 \text{ lb}$$

At the pulley F_A moves a distance h down, and F_B moves a distance h up. The work done is

$$U_{1\to 2} = (F_A - F_B)h = (19.759 - 12.096)(6) = 46.0 \text{ ft}\cdot\text{lb}$$

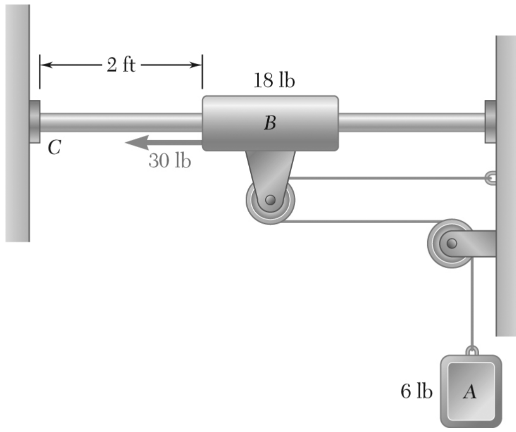
PROBLEM 13.19 (Continued)

Since the pulley is assumed to be massless, it cannot acquire kinetic energy; hence,

(*a*) Energy dissipated by the pulley: $E_p = 46.0 \text{ ft} \cdot \text{lb}$ ◀

(b) Tension in each portion of the cord: $A: 19.76 \text{ lb}$ ◀

$B: 12.10 \text{ lb}$ ◀



PROBLEM 13.20

The system shown is at rest when a constant 30 lb force is applied to collar *B*. (*a*) If the force acts through the entire motion, determine the speed of collar *B* as it strikes the support at *C*. (*b*) After what distance *d* should the 30 lb force be removed if the collar is to reach support *C* with zero velocity?

SOLUTION

Let *F* be the cable tension and v_B be the velocity of collar *B* when it strikes the support. Consider the collar *B*. Its movement is horizontal so only horizontal forces acting on *B* do work. Let *d* be the distance through which the 30 lb applied force moves.

$$(T_1)_B + (U_{1\to 2})_B = (T_2)_B$$

$$0 + 30d - (2F)(2) = \frac{1}{2}\frac{18}{32.2}v_B^2$$

$$30d - 4F = 0.27950 v_B^2 \qquad (1)$$

Now consider the weight *A*. When the collar moves 2 ft to the left, the weight moves 4 ft up, since the cable length is constant. Also, $v_A = 2v_B$.

$$(T_1)_A + (U_{1-2})_A = (T_2)_B$$

$$0 + (F - W_A)(4) = \frac{1}{2}\frac{W_A}{g}v_A^2$$

$$4F - (6)(4) = \frac{1}{2}\frac{6}{32.2}(2v_B)^2$$

$$4F - 24 = 0.37267\, v_B^2 \qquad (2)$$

Add Eqs. (1) and (2) to eliminate *F*.

$$30d - 24 = 0.65217 v_B^2 \qquad (3)$$

(*a*) Case *a*: $d = 2$ ft, $v_B = ?$

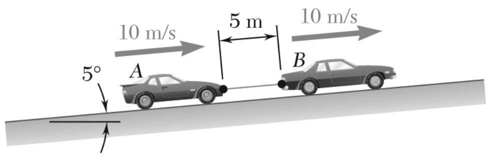
$$(30)(2) - (24) = 0.65217 v_B^2$$

$$v_B^2 = 55.2 \text{ ft}^2/\text{s}^2 \qquad v_B = 7.43 \text{ ft/s} \blacktriangleleft$$

(*b*) Case *b*: $d = ?$, $v_B = 0$.

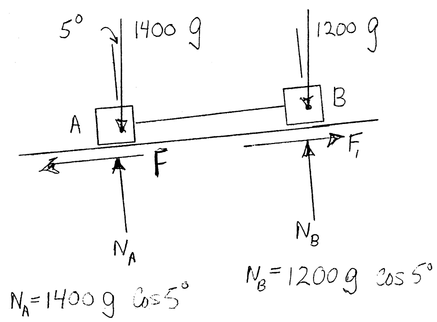
$$30d - 24 = 0 \qquad d = 0.800 \text{ ft} \blacktriangleleft$$

PROBLEM 13.21



Car B is towing car A at a constant speed of 10 m/s on an uphill grade when the brakes of car A are fully applied causing all four wheels to skid. The driver of car B does not change the throttle setting or change gears. The masses of the cars A and B are 1400 kg and 1200 kg, respectively, and the coefficient of kinetic friction is 0.8. Neglecting air resistance and rolling resistance, determine (a) the distance traveled by the cars before they come to a stop, (b) the tension in the cable.

SOLUTION



$F = 0.8N_A$

Given: Car B tows car A at 10 m/s uphill.

Car A brakes so 4 wheels skid. $\mu_k = 0.8$

Car B continues in same gear and throttle setting.

Find: (a) Distance d, traveled to stop

(b) Tension in cable

(a) F_1 = traction force (from equilibrium)

$$F_1 = (1400g)\sin 5° + (1200g)\sin 5°$$

$$= 2600(9.81)\sin 5°$$

For system: $A + B$

$$U_{1-2} = [(F_1 - 1400g\sin 5° - 1200g\sin 5°) - F]d$$

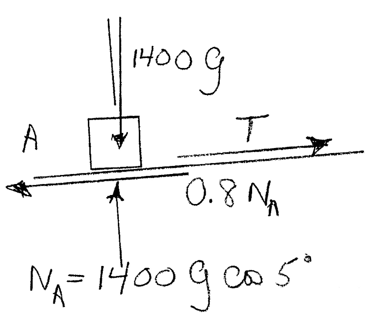
$$= T_2 - T_1 = 0 - \frac{1}{2}m_{A+B}v^2 = -\frac{1}{2}(2600)(10)^2$$

Since $(F_1 - 1400g\sin 5° - 1200g\sin 5°) = 0$

$$-Fd = -0.8[1400(9.81)\cos 5°]d = -130{,}000 \text{ N}\cdot\text{m}$$

$$d = 11.88 \text{ m} \blacktriangleleft$$

(b) Cable tension, T



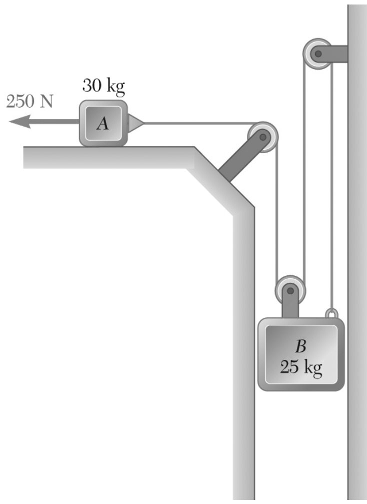
$$U_{1-2} = [T - 0.8N_A](11.88) = T_2 - T_1$$

$$(T - 0.8(1400)(9.81)\cos 5°)11.88 = -\frac{1400}{2}(10)^2$$

$$(T - 10945) = -5892$$

$$= 5.053 \text{ kN}$$

$$T = 5.05 \text{ kN} \blacktriangleleft$$

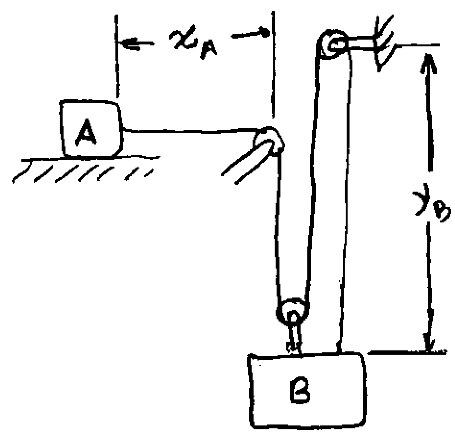


PROBLEM 13.22

The system shown is at rest when a constant 250-N force is applied to block A. Neglecting the masses of the pulleys and the effect of friction in the pulleys and between block A and the horizontal surface, determine (a) the velocity of block B after block A has moved 2 m, (b) the tension in the cable.

SOLUTION

Constraint of cable:



$$x_A + 3y_B = \text{constant}$$

$$\Delta x_A + 3\Delta y_B = 0$$

$$v_A + 3v_B = 0$$

Let F be the tension in the cable.

Block A: $\quad m_A = 30 \text{ kg}, \quad P = 250 \text{ N}, \quad (T_1)_A = 0$

$$(T_1)_A + (U_{1\to 2})_A = (T_2)_A$$

$$0 + (P - F)(\Delta x_A) = \frac{1}{2} m_A v_A^2$$

$$0 + (250 - F)(2) = \frac{1}{2}(30)(3v_B)^2$$

$$500 - 2F = 135 v_B^2 \tag{1}$$

Block B: $\quad m_B = 25 \text{ kg}, \quad W_B = m_B g = 245.25 \text{ N}$

$$(T_1)_B + (U_{1\to 2})_B = (T_2)_B$$

$$0 + (3F - W_B)(-\Delta y_B) = \frac{1}{2} m_B v_B^2$$

$$(3F) - 245.25)\left(\frac{2}{3}\right) = \frac{1}{2}(25)\, v_B^2$$

$$2F - 163.5 = 12.5\, v_B^2 \tag{2}$$

PROBLEM 13.22 (Continued)

Add Eqs. (1) and (2) to eliminate F.

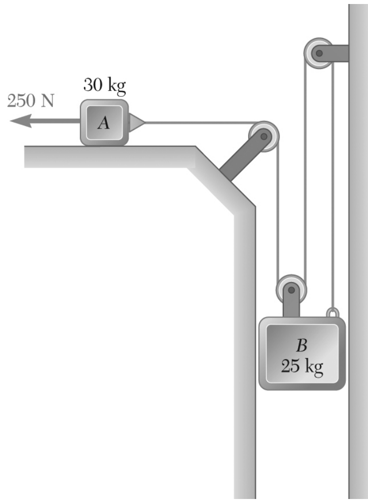
$$500 - 163.5 = 147.5 v_B^2$$

$$v_B^2 = 2.2814 \text{ m}^2/\text{s}^2$$

(a) Velocity of B. $\mathbf{v}_B = 1.510 \text{ m/s} \longleftarrow$ ◀

(b) Tension in the cable.

From Eq. (2), $2F - 163.5 = (12.5)(2.2814)$ $F = 96.0 \text{ N}$ ◀



PROBLEM 13.23

The system shown is at rest when a constant 250-N force is applied to block A. Neglecting the masses of the pulleys and the effect of friction in the pulleys and assuming that the coefficients of friction between block A and the horizontal surface are $\mu_s = 0.25$ and $\mu_k = 0.20$, determine (*a*) the velocity of block B after block A has moved 2 m, (*b*) the tension in the cable.

SOLUTION

Check the equilibrium position to see if the blocks move. Let F be the tension in the cable.

Block B: $3F - m_B g = 0$

$$F = \frac{m_B g}{3} = \frac{(25)(9.81)}{3} = 81.75 \text{ N}$$

Block A: $+\uparrow \Sigma F_y = 0$: $\quad N_A - m_A g = 0$

$$N_A = m_A g = (30)(9.81) = 294.3 \text{ N}$$

$\overset{+}{\leftarrow} \Sigma F_x = 0$: $\quad 250 - F_A - F = 0$

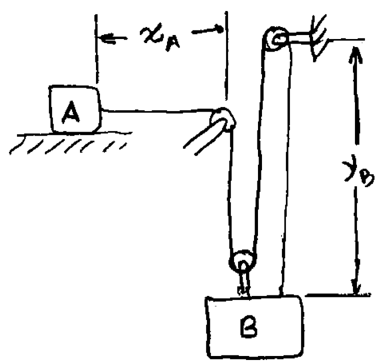
$$F_A = 250 - 81.75 = 168.25 \text{ N}$$

Available static friction force: $\quad \mu_s N_A = (0.25)(294.3) = 73.57 \text{ N}$

Since $F_A > \mu_s N_A$, the blocks move.

The friction force, F_A, during sliding is

$$F_A = \mu_k N_A = (0.20)(294.3) = 58.86 \text{ N}$$



Constraint of cable:

$$x_A + 3y_B = \text{constant}$$
$$\Delta x_A + 3\Delta y_B = 0$$
$$v_A + 3v_B = 0$$

Block A: $m_A = 30$ kg, $P = 250$ N, $(T_1)_A = 0.$

$$(T_1)_A + (U_{1\to 2})_A = (T_2)_A$$

$$0 + (P - F_A - F)(\Delta x_A) = \frac{1}{2} m_A v_A^2$$

$$0 + (250 - 58.86 - F)(2) = \frac{1}{2}(30)(3v_B)^2$$

$$382.28 - 2F = 135 v_B^2 \quad (1)$$

Block B: $M_B = 25$ kg, $W_B = m_B g = 245.25$ N

$$(T_1)_B + (U_{1\to 2})_B = (T_2)_B$$

$$0 + (3F - W_B)(-\Delta y_B) = \frac{1}{2} m_B v_B^2$$

$$(3F - 245.25)\left(\frac{2}{3}\right) = \frac{1}{2}(25) v_B^2$$

$$2F - 163.5 = 12.5 v_B^2 \quad (2)$$

Add Eqs. (1) and (2) to eliminate F.

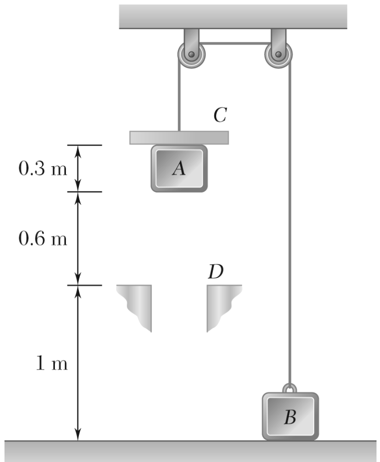
$$382.28 - 163.5 = 147.5 v_B^2$$

$$v_B^2 = 1.48325 \text{ m}^2/\text{s}^2$$

(*a*) Velocity of B: $\mathbf{v}_B = 1.218$ m/s $\longleftarrow$ ◀

(*b*) Tension in the cable:

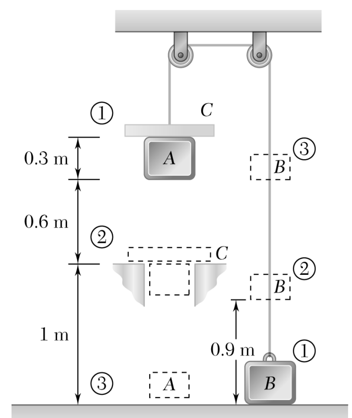
From Eq. (2), $2F - 163.5 = (12.5)(1.48325)$ $F = 91.0$ N ◀



PROBLEM 13.24

Two blocks A and B, of mass 4 kg and 5 kg, respectively, are connected by a cord which passes over pulleys as shown. A 3 kg collar C is placed on block A and the system is released from rest. After the blocks have moved 0.9 m, collar C is removed and blocks A and B continue to move. Determine the speed of block A just before it strikes the ground.

SOLUTION



Position ① to Position ②. $v_1 = 0 \quad T_1 = 0$

At ② before C is removed from the system

$$T_2 = \frac{1}{2}(m_A + m_B + m_C)v_2^2 = \frac{1}{2}(12\text{ kg})v_2^2 = 6v_2^2$$

$$U_{1-2} = (m_A + m_C - m_B)g(0.9\text{ m})$$

$$U_{1-2} = (4+3-5)(g)(0.9\text{ m}) = (2\text{ kg})(9.81\text{ m/s}^2)(0.9\text{ m})$$

$$U_{1-2} = 17.658\text{ J}$$

$$T_1 + U_{1-2} = T_2:$$

$$0 + 17.658 = 6v_2^2 \qquad v_2^2 = 2.943$$

At Position ②, collar C is removed from the system.

Position ② to Position ③. $T_2' = \frac{1}{2}(m_A + m_B)v_2^2 = \left(\frac{9}{2}\text{ kg}\right)(2.943) = 13.244\text{ J}$

$$T_3 = \frac{1}{2}(m_A + m_B)(v_3)^2 = \frac{9}{2}v_3^2$$

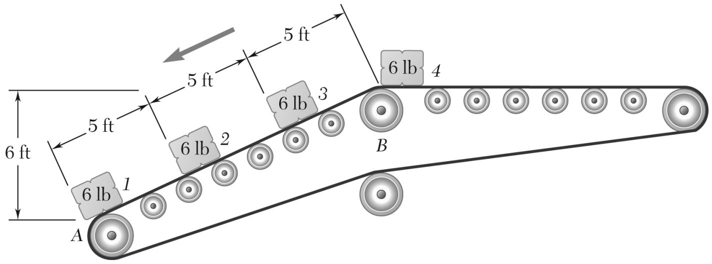
$$U_{2'-3} = (m_A - m_B)(g)(0.7\text{ m}) = (-1\text{ kg})(9.81\text{ m/s}^2)(0.7\text{ m}) = -6.867\text{ J}$$

$$T_2' + U_{2-3} = T_3$$

$$13.244 - 6.867 = 4.5v_3^2 \qquad v_3^2 = 1.417$$

$$v_A = v_3 = 1.190\text{ m/s}$$

$v_A = 1.190\text{ m/s}$ ◀

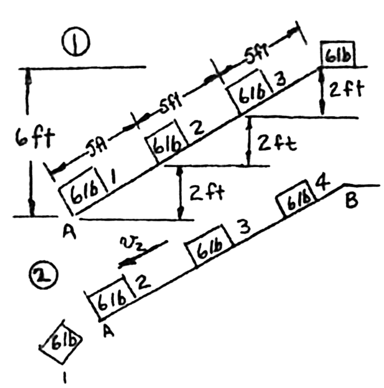


PROBLEM 13.25

Four packages, each weighing 6 lb, are held in place by friction on a conveyor which is disengaged from its drive motor. When the system is released from rest, package 1 leaves the belt at A just as package 4 comes onto the inclined portion of the belt at B. Determine (a) the speed of package 2 as it leaves the belt at A, (b) the speed of package 3 as it leaves the belt at A. Neglect the mass of the belt and rollers.

SOLUTION

Slope angle: $\sin\ \beta = \dfrac{6\text{ ft}}{15\text{ ft}}$ $\quad \beta = 23.6°$



(a) Package falls off the belt and 2, 3, 4 move down

$$\frac{6}{3} = 2\text{ ft.}$$

$$T_2 = 3\left[\frac{1}{2}mv_2^2\right] = \frac{3}{2}\left(\frac{6\text{ lb}}{32.2\text{ ft/s}^2}\right)v_2^2 = 0.2795v_2^2$$

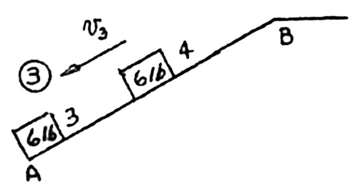
$$U_{1-2} = (3)(W)(R) = (3)(6\text{ lb})(2\text{ ft}) = 36\text{ lb}\cdot\text{ft}$$

$$T_1 + U_{1-2} = T_2$$

$$0 + 36 = 0.2795v_2^2 \qquad v_2^2 = 128.8$$

$\mathbf{v}_2 = 11.35\text{ ft/s} \measuredangle\ 23.6°$ ◀

(b) Package 2 falls off the belt and its energy is lost to the system and 3 and 4 move down 2 ft.



$$T_2' = (2)\left[\frac{1}{2}mv_2^2\right] = \left(\frac{6\text{ lb}}{32\text{ ft/s}^2}\right)(128.8)$$

$$T_2' = 24\text{ lb}\cdot\text{ft}$$

$$T_3 = (2)\left[\frac{1}{2}mv_3^2\right] = \left(\frac{6\text{ lb}}{32.2\text{ ft/s}^2}\right)(v_3^2) = 0.18634v_3^2$$

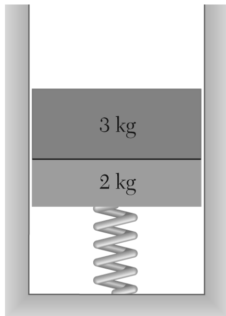
$$U_{2-3} = (2)(W)(2) = (2)(6\text{ lb})(2\text{ ft}) = 24\text{ lb}\cdot\text{ft}$$

$$T_2 + U_{2-3} = T_3$$

$$24 + 24 = 0.18634v_3^2 \qquad v_3^2 = 257.6$$

$\mathbf{v}_3 = 16.05\text{ ft/s} \measuredangle\ 23.6°$ ◀

PROBLEM 13.26



A 3-kg block rests on top of a 2-kg block supported by, but not attached to, a spring of constant 40 N/m. The upper block is suddenly removed. Determine (*a*) the maximum speed reached by the 2-kg block, (*b*) the maximum height reached by the 2-kg block.

SOLUTION

Call blocks *A* and *B*. $m_A = 2 \text{ kg}, \quad m_B = 3 \text{ kg}$

(*a*) Position 1: Block *B* has just been removed.

Spring force: $F_S = -(m_A + m_B)g = -kx \uparrow$

Spring stretch: $x_1 = -\dfrac{(m_A + m_B)g}{k} = -\dfrac{(5 \text{ kg})(9.81 \text{ m/s}^2)}{40 \text{ N/m}} = -1.22625 \text{ m}$

Let position 2 be a later position while the spring still contacts block *A*.

Work of the force exerted by the spring: $(U_{1\to 2})_e = -\int_{x_1}^{x_2} kx\,dx$

$$= -\frac{1}{2}kx^2\bigg|_{x_1}^{x_2} = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2$$

$$= \frac{1}{2}(40)(-1.22625)^2 - \frac{1}{2}(40)x_2^2 = 30.074 - 20x_2^2$$

Work of the gravitational force: $(U_{1\to 2})_g = -m_A g(x_2 - x_1)$

$$= -(2)(9.81)(x_2 + 1.22625) = -19.62x_2 - 24.059$$

Total work: $U_{1\to 2} = -20x_2^2 + 19.62x_2 + 6.015$

Kinetic energies: $T_1 = 0$

$$T_2 = \frac{1}{2}m_A v_2^2 = \frac{1}{2}(2)v_2^2 = v_2^2$$

Principle of work and energy: $T_1 + U_{1\to 2} = T_2$

$$0 + 20x_2^2 - 19.62x_2 + 6.015 = v_2^2$$

Speed squared: $v_2^2 = -20x_2^2 - 19.62x_2 + 6.015$ (1)

At maximum speed, $\dfrac{dv_2}{dx_2} = 0$

Differentiating Eq. (1), and setting equal to zero,

$$2v_2 \frac{dv_2}{dx} = -40x_2 = -19.62 = 0$$

$$x_2 = -\frac{19.62}{40} = -0.4905 \text{ m}$$

Substituting into Eq. (1), $\quad v_2^2 = -(20)(-0.4905)^2 - (19.62)(-0.4905) + 6.015 = 10.827 \text{ m}^2/\text{s}^2$

Maximum speed: $\quad v^2 = 3.29 \text{ m/s}$ ◀

(*b*) Position 3: Block *A* reaches maximum height. Assume that the block has separated from the spring. Spring force is zero at separation.

Work of the force exerted by the spring:

$$(U_{1\to 3})_e = -\int_{x_1}^{0} kx dx = \frac{1}{2}kx_1^2 = \frac{1}{2}(40)(1.22625)^2 = 30.074 \text{ J}$$

Work of the gravitational force:

$$(U_{1\to 3})_g = -m_A gh = -(2)(9.81)h = -19.62\ h$$

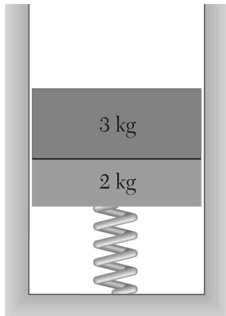
Total work: $\quad U_{1\to 3} = 30.074 - 19.62\ h$

At maximum height, $\quad v_3 = 0, \quad T_3 = 0$

Principle of work and energy: $\quad T_1 + U_{1\to 3} = T_3$

$$0 + 30.074 - 19.62\ h = 0$$

Maximum height: $\quad h = 1.533 \text{ m}$ ◀



PROBLEM 13.27

Solve Problem 13.26, assuming that the 2-kg block is attached to the spring.

PROBLEM 13.26 A 3-kg block rests on top of a 2-kg block supported by, but not attached to, a spring of constant 40 N/m. The upper block is suddenly removed. Determine (*a*) the maximum speed reached by the 2-kg block, (*b*) the maximum height reached by the 2-kg block.

SOLUTION

Call blocks *A* and *B*. $m_A = 2\text{ kg}, \quad m_B = 3\text{ kg}$

(*a*) Position 1: Block *B* has just been removed.

Spring force: $F_S = -(m_A + m_B)g = -kx_1$

Spring stretch: $$x_1 = -\frac{(m_A + m_B)g}{k} = -\frac{(5\text{ kg})(9.81\text{ m/s}^2)}{40\text{ N/m}} = -1.22625\text{ m}$$

Let position 2 be a later position. Note that the spring remains attached to block *A*.

Work of the force exerted by the spring: $$(U_{1\to 2})_e = -\int_{x_1}^{x_2} kx\,dx$$

$$= -\frac{1}{2}kx^2\Big|_{x_1}^{x_2} = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2$$

$$= \frac{1}{2}(40)(-1.22625)^2 - \frac{1}{2}(40)x_2^2 = 30.074 - 20x_2^2$$

Work of the gravitational force: $$(U_{1\to 2})_g = -m_A g(x_2 - x_1)$$

$$= -(2)(9.81)(x_2 + 1.22625) = -19.62x_2 - 24.059$$

Total work: $U_{1\to 2} = -20x_2^2 - 19.62x_2 + 6.015$

Kinetic energies: $T_1 = 0$

$$T_2 = \frac{1}{2}m_A v_2^2 = \frac{1}{2}(2)v_2^2 = v_2^2$$

Principle of work and energy: $T_1 + U_{1\to 2} = T_2$

$$0 + 20x_2^2 - 19.62x_2 + 6.015 = v_2^2$$

Speed squared: $v_2^2 = -20x_2^2 - 19.62x_2 + 6.015$ (1)

At maximum speed, $\dfrac{dv_2}{dx_2} = 0$

PROBLEM 13.27 (Continued)

Differentiating Eq. (1) and setting equal to zero,

$$2v_2 \frac{dv_2}{dx_2} = -40x_2 = -19.62 = 0$$

$$x_2 = -\frac{19.62}{40} = -0.4905 \text{ m}$$

Substituting into Eq. (1), $\quad v_2^2 = -(20)(-0.4905)^2 - (19.62)(-0.4905) + 6.015 = 10.827 \text{ m}^1/\text{s}^2$

Maximum speed: $\quad v_2 = 3.29 \text{ m/s}$ ◀

(*b*) Maximum height occurs when $v_2 = 0$.

Substituting into Eq. (1), $\quad 0 = -20x_2^2 - 19.62x_2 + 6.015$

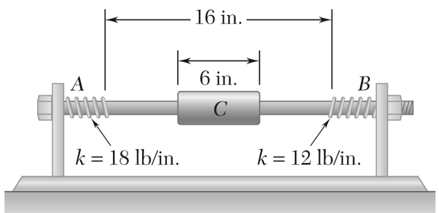
Solving the quadratic equation

$$x_2 = -1.22625 \text{ m} \quad \text{and} \quad 0.24525 \text{ m}$$

Using the larger value, $\quad x_2 = 0.24525 \text{ m}$

Maximum height: $\quad h = x_2 - x_1 = 0.24525 + 1.22625 \qquad h = 1.472 \text{ m}$ ◀

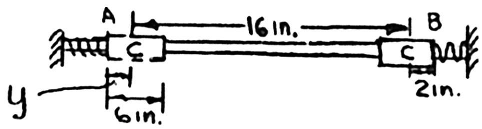
PROBLEM 13.28



An 8-lb collar C slides on a horizontal rod between springs A and B. If the collar is pushed to the right until spring B is compressed 2 in. and released, determine the distance through which the collar will travel assuming (a) no friction between the collar and the rod, (b) a coefficient of friction $\mu_k = 0.35$.

SOLUTION

(a)



$$k_B = 144 \text{ lb/ft}$$
$$k_A = 216 \text{ lb/ft}$$

Since the collar C leaves the spring at B and there is no friction, it must engage the spring at A.

$$T_A = 0 \qquad T_B = 0$$

$$U_{A-B} = \int_0^{2/12} k_B x dx - \int_0^{y} k_A x dx$$

$$U_{A-B} = \left(\frac{144 \text{ lb/ft}}{2}\right)\left(\frac{2}{12}\text{ ft}\right)^2 - \left(\frac{216 \text{ lb/ft}}{2}\right)(y)^2$$

$$T_A + U_{A-B} = T_B: \qquad 0 + 2 - 108y^2 = 0$$

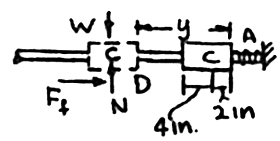
$$y = 0.1361 \text{ ft} = 1.633 \text{ in.}$$

Total distance $\qquad d = 2 + 16 - (6 - 1.633)$

$$d = 13.63 \text{ in.} \blacktriangleleft$$

(b) Assume that C does not reach the spring at B because of friction.

$$N = W = 6 \text{ lb}$$
$$F_f = (0.35)(8 \text{ lb}) = 2.80 \text{ lb}$$
$$T_A = T_D = 0$$



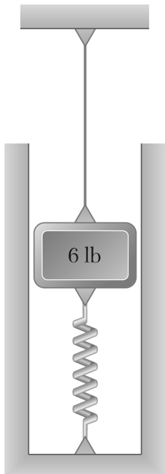
$$U_{A-D} = \int_0^{2/12} 144 \times dx - F_f(y) = 2 - 2.80y$$

$$T_A + U_{A-D} = T_D \quad 0 + 2 - 2.80y = 0$$

$$y = 0.714 \text{ ft} = 8.57 \text{ in.}$$

The collar must travel $16 - 6 + 2 = 12$ in. before it engages the spring at B. Since $y = 8.57$ in., it stops before engaging the spring at B.

Total distance $\qquad d = 8.57 \text{ in.} \blacktriangleleft$

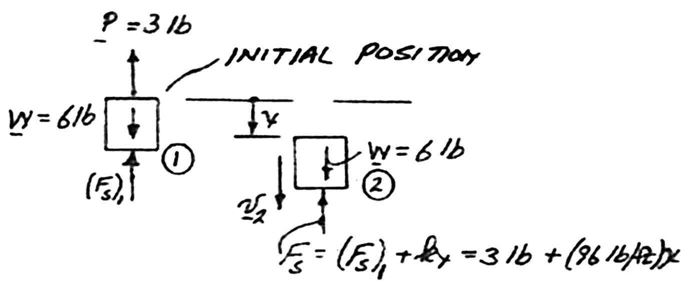


PROBLEM 13.29

A 6-lb block is attached to a cable and to a spring as shown. The constant of the spring is $k = 8$ lb/in. and the tension in the cable is 3 lb. If the cable is cut, determine (*a*) the maximum displacement of the block, (*b*) the maximum speed of the block.

SOLUTION

$$k = 8 \text{ lb/in.} = 96 \text{ lb/ft}$$



$$\Sigma F_y = 0: \quad (F_s)_1 = 6 - 3 = 3 \text{ lb } C$$

$$v_1 = 0 \quad T_1 = 0: \quad T_2 = \frac{1}{2}\left(\frac{6 \text{ lb}}{32.2}\right)v_2^2 = 0.09317 v_2^2$$

For weight: $$U_{1-2} = (6 \text{ lb})x = 6x$$

For spring: $$U_{1-2} = -\int_0^x (3 + 96x)dx = -3x - 48x^2$$

$$T_1 + U_{1-2} = T_2: \quad 0 + 6x - 3x - 48x^2 = 0.09317 v_2^2$$

$$3x - 48x^2 = 0.09317 v_2^2 \qquad (1)$$

(*a*) For x_m, $v_2 = 0$: $$3x - 48x^2 = 0$$

$$x = 0, \quad x_m = \frac{3}{48} = \frac{1}{16} \text{ ft}$$

$\mathbf{x}_m = 0.75$ in.↓ ◀

PROBLEM 13.29 (Continued)

(*b*) For v_m we see maximum of $U_{1-2} = 3x - 48x^2$

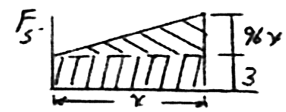
$$\frac{dU_{1-2}}{dx} = 3 - 96x = 0 \qquad x = \frac{3}{96}\text{ ft} = \frac{1}{32}\text{ ft}$$

Eq. (1): $$3\left(\frac{1}{32}\text{ ft}\right) - 48\left(\frac{1}{32}\text{ ft}\right)^2 = 0.09317v_m^2$$

$$v_m^2 = 0.5031 \qquad v_m = 0.7093\text{ ft/s}$$

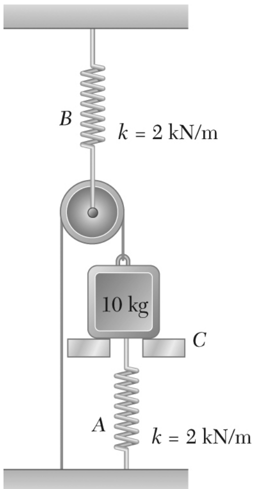
$\mathbf{v}_m = 8.51\text{ in./s} \updownarrow$ ◀

Note: U_{1-2} for the spring may be computed using $F_6 - x$ curve



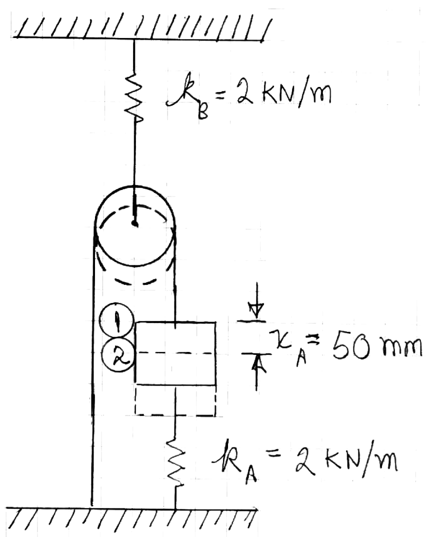
$$U_{1-2} = \text{area}$$
$$= 3x + \frac{1}{2}96x^2$$

PROBLEM 13.30



A 10-kg block is attached to spring A and connected to spring B by a cord and pulley. The block is held in the position shown with both springs unstretched when the support is removed and the block is released with no initial velocity. Knowing that the constant of each spring is 2 kN/m, determine (a) the velocity of the block after it has moved down 50 mm, (b) the maximum velocity achieved by the block.

SOLUTION



(a) W = weight of the block $= 10\,(9.81) = 98.1$ N

$$x_B = \frac{1}{2}x_A$$

$$U_{1-2} = W(x_A) - \frac{1}{2}k_A(x_A)^2 - \frac{1}{2}k_B(x_B)^2$$

(Gravity) (Spring A) (Spring B)

$$U_{1-2} = (98.1\text{ N})(0.05\text{ m}) - \frac{1}{2}(2000\text{ N/m})(0.05\text{ m})^2$$

$$-\frac{1}{2}(2000\text{ N/m})(0.025\text{ m})^2$$

$$U_{1-2} = \frac{1}{2}(m)v^2 = \frac{1}{2}(10\text{ kg})v^2$$

$$4.905 - 2.5 - 0.625 = \frac{1}{2}(10)v^2$$

$v = 0.597$ m/s ◀

(b) Let x = distance moved down by the 10 kg block

$$U_{1-2} = W(x) - \frac{1}{2}k_A(x)^2 - \frac{1}{2}k_B\left(\frac{x}{2}\right)^2 = \frac{1}{2}(m)v^2$$

$$\frac{d}{dx}\left[\frac{1}{2}(m)v^2\right] = 0 = W - k_A(x) - \frac{k_B}{8}(2x)$$

PROBLEM 13.30 (Continued)

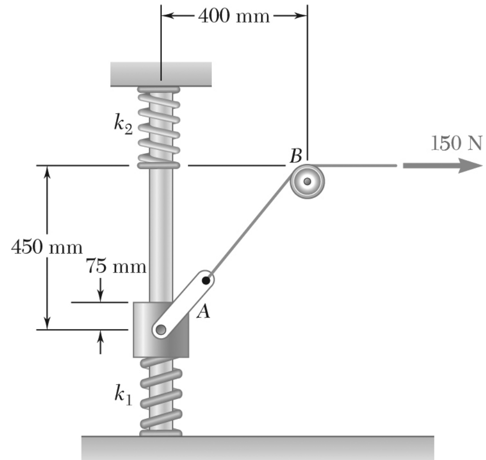
$$0 = 98.1 - 2000\,(x) - \frac{2000}{8}\,(2x) = 98.1 - (2000 + 250)x$$

$$x = 0.0436 \text{ m } (43.6 \text{ mm})$$

For $\quad x = 0.0436, \; U = 4.2772 - 1.9010 - 0.4752 = \frac{1}{2}(10)v^2$

$$v_{max} = 0.6166 \text{ m/s}$$

$v_{max} = 0.617 \text{ m/s}$ ◀



PROBLEM 13.31

A 5-kg collar A is at rest on top of, but not attached to, a spring with stiffness k_1 = 400 N/m; when a constant 150-N force is applied to the cable. Knowing A has a speed of 1 m/s when the upper spring is compressed 75 mm, determine the spring stiffness k_2. Ignore friction and the mass of the pulley.

SOLUTION

Use the method of work and energy applied to the collar A.

$$T_1 + U_{1\to2} = T_2$$

Since collar is initially at rest, $T_1 = 0.$

In position 2, where the upper spring is compressed 75 mm and $v_2 = 1.00$ m/s, the kinetic energy is

$$T_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}(5\text{ kg})(1.00\text{ m/s})^2 = 2.5\text{ J}$$

As the collar is raised from level A to level B, the work of the weight force is

$$(U_{1\to2})_g = -mgh$$

where $m = 5$ kg, $g = 9.81$ m/s^2 and $h = 450$ mm $= 0.450$ m

Thus, $(U_{1\to2})_g = -(5)(9.81)(0.450) = -22.0725$ J

In position 1, the force exerted by the lower spring is equal to the weight of collar A.

$$F_1 = mg = -(5\text{ kg})(9.81\text{ m/s}) = -49.05\text{ N}$$

As the collar moves up a distance x_1, the spring force is

$$F = F_1 - k_1 x_2$$

until the collar separates from the spring at

$$x_f = \frac{F_1}{k_1} = \frac{49.05\text{ N}}{400\text{ N/m}} = 0.122625\text{ m} = 122.625\text{ mm}$$

PROBLEM 13.31 (Continued)

Work of the force exerted by the lower spring:

$$(U_{1\to 2})_1 = \int_0^{x_f} (F_1 - k_1 x)dx$$
$$= F_1 x_f - \frac{1}{2}kx_f^2 = k_1 x_f^2 - \frac{1}{2}k_1 x_f^2 = \frac{1}{2}k_1 x_f^2$$
$$= \frac{1}{2}(400 \text{ N/m})(0.122625)^2 = 3.0074 \text{ J}$$

In position 2, the upper spring is compressed by $y = 75 \text{ mm} = 0.075 \text{ m}$. The work of the force exerted by this spring is

$$(U_{1\to 2})_2 = -\frac{1}{2}k_2 y^2 = -\frac{1}{2}k_2 (0.075)^2 = -0.0028125 \text{ k}_2$$

Finally, we must calculate the work of the 150 N force applied to the cable. In position 1, the length AB is

$$(l_{AB})_1 = \sqrt{(450)^2 + (400)^2} = 602.08 \text{ mm}$$

In position 2, the length AB is $(l_{AB})_2 = 400$ mm.

The displacement d of the 150 N force is

$$d = (l_{AB})_1 - (l_{AB})_2 = 202.08 \text{ mm} = 0.20208 \text{ m}$$

The work of the 150 N force P is

$$(U_{1\to 2})_P = Pd = (150 \text{ N})(0.20208 \text{ m}) = 30.312 \text{ J}$$

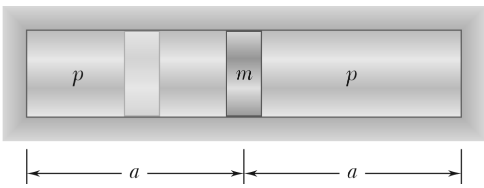
Total work:

$$U_{1\to 2} = -22.0725 + 3.0074 - 0.0028125k_2 + 30.312$$
$$= 11.247 - 0.0028125k_2$$

Principle of work and energy:

$$T_1 + U_{1\to 2} = T_2$$
$$0 + 11.247 - 0.0028125k_2 = 2.5$$
$$k_2 = 3110 \text{ N/m}$$

$k_2 = 3110$ N/m ◀

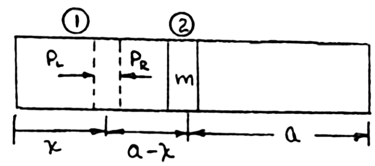


PROBLEM 13.32

A piston of mass m and cross-sectional area A is equilibrium under the pressure p at the center of a cylinder closed at both ends. Assuming that the piston is moved to the left a distance $a/2$ and released, and knowing that the pressure on each side of the piston varies inversely with the volume, determine the velocity of the piston as it again reaches the center of the cylinder. Neglect friction between the piston and the cylinder and express your answer in terms of m, a, p, and A.

SOLUTION

Pressures vary inversely as the volume



$$\frac{p_L}{P} = \frac{Aa}{Ax} \qquad p_L = \frac{pa}{x}$$

$$\frac{p_R}{P} = \frac{Aa}{A(2a-x)} \qquad p_R = \frac{pa}{(2a-x)}$$

Initially at ①, $\quad v = 0 \quad x = \dfrac{a}{2}$

$$T_1 = 0$$

At ②, $\quad x = a, \quad T_2 = \dfrac{1}{2}mv^2$

$$U_{1-2} = \int_{a/2}^{a} (p_L - p_R)A\,dx = \int_{a/2}^{a} paA\left[\frac{1}{x} - \frac{1}{2a-x}\right]dx$$

$$U_{1-2} = paA[\ln x + \ln(2a-x)]_{a/2}^{a}$$

$$U_{1-2} = paA\left[\ln a + \ln a - \ln\left(\frac{a}{2}\right) - \ln\left(\frac{3a}{2}\right)\right]$$

$$U_{1-2} = paA\left[\ln a^2 - \ln\frac{3a^2}{4}\right] = paA\ \ln\left(\frac{4}{3}\right)$$

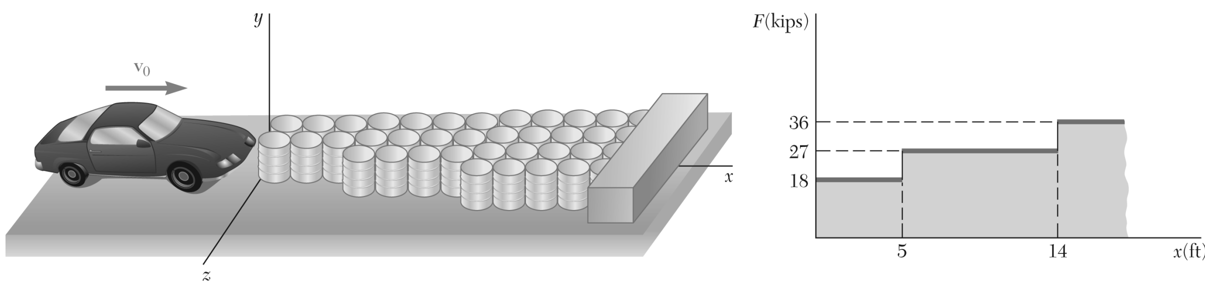
$$T_1 + U_{1-2} = T_2 \qquad 0 + paA\ \ln\left(\frac{4}{3}\right) = \frac{1}{2}mv^2$$

$$v^2 = \frac{2paA\ \ln\left(\frac{4}{3}\right)}{m} = 0.5754\frac{paA}{m}$$

$$v = 0.759\sqrt{\frac{paA}{m}} \blacktriangleleft$$

PROBLEM 13.33

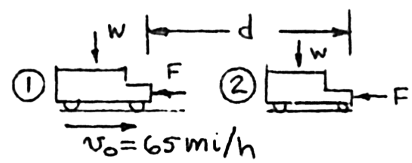
An uncontrolled automobile traveling at 65 mph strikes squarely a highway crash cushion of the type shown in which the automobile is brought to rest by successively crushing steel barrels. The magnitude F of the force required to crush the barrels is shown as a function of the distance x the automobile has moved into the cushion. Knowing that the weight of the automobile is 2250 lb and neglecting the effect of friction, determine (a) the distance the automobile will move into the cushion before it comes to rest, (b) the maximum deceleration of the automobile.

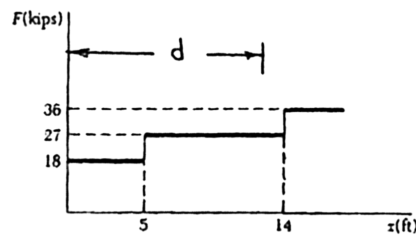


SOLUTION

(a) 65 mi/h = 95.3 ft/s

Assume auto stops in $5 \le d \le 14$ ft.





$$v_1 = 95.33 \text{ ft/s}$$

$$T_1 = \frac{1}{2}mv_1^2 \frac{1}{2}\left(\frac{2250 \text{ lb}}{32.2 \text{ ft/s}^2}\right)(95.3 \text{ ft/s})^2$$

$$T_1 = 317{,}530 \text{ lb} \cdot \text{ft}$$

$$= 317.63 \text{ k} \cdot \text{ft}$$

$$v_2 = 0$$

$$T_2 = 0$$

$$U_{1-2} = (18 \text{ k})(5 \text{ ft}) + (27 \text{ k})(d - 5)$$

$$= 90 + 27d - 135$$

$$= 27d - 45 \text{ k} \cdot \text{ft}$$

$$T_1 + U_{1-2} = T_2$$

$$317.53 = 27d - 45$$

$d = 13.43$ ft ◀

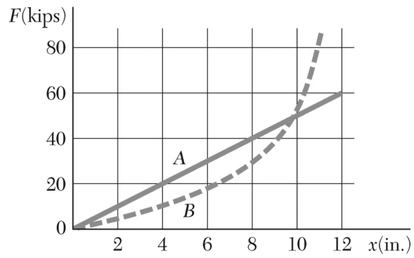
Assumption that $d \le 14$ ft is ok.

(b) Maximum deceleration occurs when F is largest. For $d = 13.43$ ft, $F = 27$ k. Thus, $F = ma_D$

$$(27{,}000 \text{ lb}) = \left(\frac{2250 \text{ lb}}{32.2 \text{ ft/s}^2}\right)(a_D)$$

$a_D = 386 \text{ ft/s}^2$ ◀

PROBLEM 13.34



Two types of energy-absorbing fenders designed to be used on a pier are statically loaded. The force-deflection curve for each type of fender is given in the graph. Determine the maximum deflection of each fender when a 90-ton ship moving at 1 mi/h strikes the fender and is brought to rest.

SOLUTION

Weight: $W_1 = (90 \text{ ton})(2000 \text{ lb/ton}) = 180\times 10^3 \text{ lb}$

Mass: $m = \dfrac{W}{g} = \dfrac{180\times 10^3}{32.2} = 5590 \text{ lb}\cdot\text{s}^2/\text{ft}$

Speed: $v_1 = 1 \text{ mi/h} = \dfrac{5280 \text{ ft}}{3600 \text{ s}} = 1.4667 \text{ ft/s}$

Kinetic energy: $T_1 = \dfrac{1}{2}mv_1^2 = \dfrac{1}{2}(5590)(1.4667)^2$

$$= 6012 \text{ ft}\cdot\text{lb}$$

$$T_2 = 0 \qquad \text{(rest)}$$

Principle of work and energy: $T_1 + U_{1\to 2} = T_2$

$$6012 + U_{1\to 2} = 0$$

$$U_{1\to 2} = -6012 \text{ ft}\cdot\text{lb} = -72.15 \text{ kip}\cdot\text{in.}$$

The area under the force-deflection curve up to the maximum deflection is equal to $72.15 \text{ kip}\cdot\text{in.}$

Fender A: From the force-deflection curve $F = kx$ $\quad k = \dfrac{F_{max}}{x_{max}} = \dfrac{60}{12} = 5 \text{ kip/in.}$

$$\text{Area} = \int_0^x f dx = \int_0^x kx\, dx = \frac{1}{2}kx^2$$

$$\frac{1}{2}(5)x^2 = 72.51$$

$$x^2 = 28.86 \text{ in.}^2 \qquad\qquad x = 5.37 \text{ in.} \blacktriangleleft$$

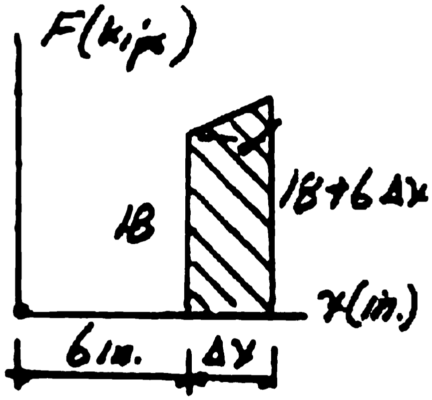
Fender B: We divide area under curve B into trapezoids

		Partial area	Total Area
From $x = 0$	to $x = 2$ in.:	$\frac{1}{2}(2 \text{ in.})(4 \text{ kips}) = 4 \text{ kip}\cdot\text{in.}$	$4 \text{ kip}\cdot\text{in.}$
From $x = 2$ in.	to $x = 4$ in.:	$\frac{1}{2}(2 \text{ in.})(4+10) = 14 \text{ kip}\cdot\text{in.}$	$18 \text{ kip}\cdot\text{in.}$
From $x = 4$ in.	to $x = 6$ in.:	$\frac{1}{2}(2 \text{ in.})(10+18) = 28 \text{ kip}\cdot\text{in.}$	$46 \text{ kip}\cdot\text{in.}$

PROBLEM 13.34 (Continued)

We still need $\Delta U = 72.15 - 46 = 26.15 \text{ kip} \cdot \text{in.}$

Equation of straight line approximating curve B from $x = 6$ in. to $x = 8$ in. is



$$\frac{\Delta x}{2} = \frac{F - 18}{30 - 18} \quad F = 18 + 6\Delta x$$

$$\Delta U = 18\Delta x + \frac{1}{2}(6\Delta x)\Delta x = 26.15 \text{ kip} \cdot \text{in.}$$

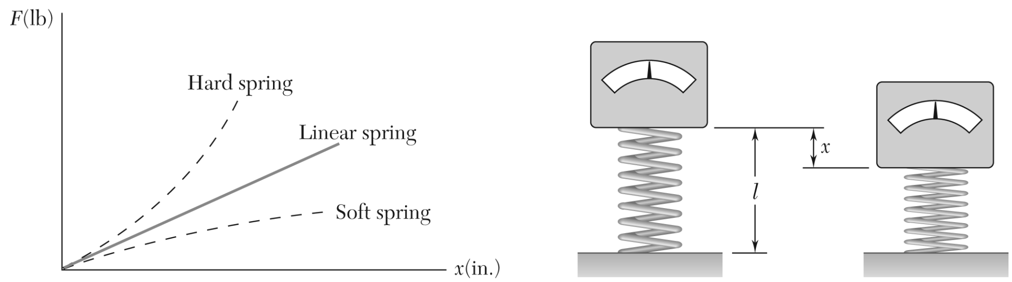
$$(\Delta x)^2 + 6\Delta x - 8.716 = 0$$

$$\Delta x = 1.209 \text{ in.}$$

Thus: $x = 6 \text{ in.} + 1.209 \text{ in.} = 7.209 \text{ in.}$ $\quad x = 7.21 \text{ in.}$ ◀

PROBLEM 13.35

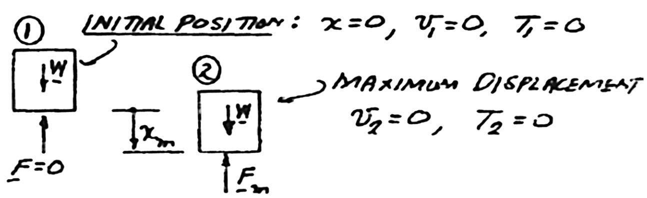
Nonlinear springs are classified as hard or soft, depending upon the curvature of their force-deflection curve (see figure). If a delicate instrument having a mass of 5 kg is placed on a spring of length l so that its base is just touching the undeformed spring and then inadvertently released from that position, determine the maximum deflection x_m of the spring and the maximum force F_m exerted by the spring, assuming (*a*) a linear spring of constant $k = 3$ kN/m, (*b*) a hard, nonlinear spring, for which $F = (3 \text{ kN/m})(x + 160x^2)$.



SOLUTION

$W = mg = (5 \text{ kg})g$

$W = 49.05 \text{ N}$



Since $\quad T_1 = T_2 = 0, \quad T_1 + U_{1-2} = T_2$ yields $U_{1-2} = 0$

$$U_{1-2} = Wx_m - \int_0^{x_m} F dx = 49.05x_m - \int_0^{x_m} F dx = 0 \qquad (1)$$

(*a*) For $F = kx = (300 \text{ N/m})x$

Eq. (1): $\quad 49.05x_m - \int_0^{x_m} 3000x\,dx = 0$

$$49.05x_m - 1500x_m^2 = 0 \qquad x_m = 32.7 \times 10^{-3} \text{ m} = 32.7 \text{ mm} \blacktriangleleft$$

$$F_m = 3000x_m = 3000(32.7 \times 10^{-3}) \qquad \mathbf{F}_m = 98.1 \text{ N} \uparrow \blacktriangleleft$$

(*b*) For $\quad F = (3000 \text{ N/m})x(1 + 160x^2)$

Eq. (1) $\quad 49.05x_m - \int_0^{x_m} 3000(x + 160x^3)dx = 0$

$$49.05x_m - 3000\left(\frac{1}{2}x_m^2 + 40x_m^4\right) = 0 \qquad (2)$$

Solve by trial: $\quad x_m = 30.44 \times 10^{-3}$ m $\qquad x_m = 30.4 \text{ mm} \blacktriangleleft$

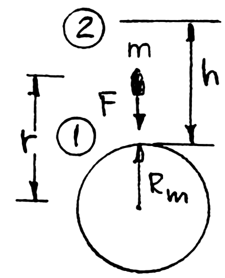
$$F_m = (3000)(30.44 \times 10^{-3})[1 + 160(30.44 \times 10^{-3})^2] \qquad \mathbf{F}_m = 104.9 \text{ N} \uparrow \blacktriangleleft$$

PROBLEM 13.36

A rocket is fired vertically from the surface of the moon with a speed v_0. Derive a formula for the ratio h_n/h_u of heights reached with a speed v, if Newton's law of gravitation is used to calculate h_n and a uniform gravitational field is used to calculate h_u. Express your answer in terms of the acceleration of gravity g_m on the surface of the moon, the radius R_m of the moon, and the speeds v and v_0.

SOLUTION

Newton's law of gravitation



$$T_1 = \frac{1}{2}mv_0^2 \qquad T_2 = \frac{1}{2}mv^2$$

$$U_{1-2} = \int_{R_m}^{R_m+h_n}(-F_n)dr \qquad F_n = \frac{mg_m R_m^2}{r^2}$$

$$U_{1-2} = -mg_m R_m^2 \int_{R_m}^{R_m+h_n}\frac{dr}{r^2}$$

$$U_{1-2} = mg_m R_m^2\left(\frac{1}{R_m} - \frac{1}{R_m+h_n}\right)$$

$$T_1 + U_{1-2} = T_2$$

$$\frac{1}{2}mv_0^2 + mg_m\left(R_m - \frac{R_m}{R_m+h_n}\right) = \frac{1}{2}mv^2$$

$$h_n = \frac{\left(v_0^2 - v^2\right)}{2g_m}\left[\frac{R_m}{R_m - \frac{\left(v_0^2 - v^2\right)}{2_{gm}}}\right] \qquad (1)$$

Uniform gravitational field

$$T_1 = \frac{1}{2}mv_0^2 \quad T_2 = mv^2$$

$$U_{1-2} = \int_{R_m}^{R_m+h_n}(-F_u)dr = -mg_m(R_m + h_u - R_m) = -mgh_u$$

$$T_1 + U_{1-2} = T_2 \quad \frac{1}{2}mv_0^2 - mg_m h_u = \frac{1}{2}mv^2$$

$$h_u = \frac{\left(v_0^2 - v^2\right)}{2g_m} \qquad (2)$$

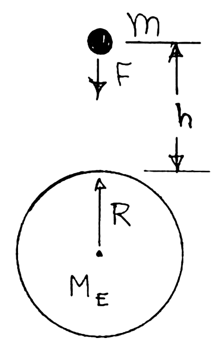
Dividing (1) *by* (2)

$$\frac{h_n}{h_u} = \frac{1}{1 - \frac{\left(v_0^2 - v^2\right)}{(2g_m R_m)}} \blacktriangleleft$$

PROBLEM 13.37

Express the acceleration of gravity g_h at an altitude h above the surface of the earth in terms of the acceleration of gravity g_0 at the surface of the earth, the altitude h and the radius R of the earth. Determine the percent error if the weight that an object has on the surface of earth is used as its weight at an altitude of (*a*) 1 km, (*b*) 1000 km.

SOLUTION



$$F = \frac{GM_E m}{(h+R)^2} = \frac{GM_E m/R^2}{\left(\frac{h}{R}+1\right)^2} mg_h$$

At earth's surface, ($h = 0$) $\qquad \dfrac{GM_E m}{R^2} = mg_0$

$$\frac{GM_E}{R^2} = g_0 \qquad g_h = \frac{\frac{GM_E}{R^2}}{\left(\frac{h}{R}+1\right)^2}$$

Thus, $\qquad g_h = \dfrac{g_0}{\left(\frac{h}{R}+1\right)^2}$

$$R = 6370 \text{ km}$$

At altitude h, "true" weight $\qquad F = mg_h = W_T$

Assumed weight $\qquad W_0 = mg_0$

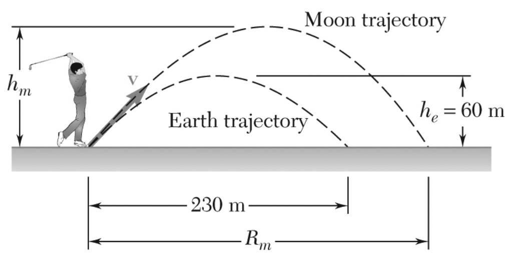
$$\text{Error} = E = \frac{W_0 - W_T}{W_0} = \frac{mg_0 - mg_h}{mg_0} = \frac{g_0 - g_h}{g_0}$$

$$g_h = \frac{g_0}{\left(\frac{h}{R}+1\right)^2} \qquad E = \frac{g_0 - \frac{g_0}{\left(1+\frac{h}{R}\right)^2}}{g_0} = \left[1 - \frac{1}{\left(1+\frac{h}{R}\right)^2}\right]$$

(*a*) $h = 1$ km: $\qquad P = 100E = 100\left[1 - \dfrac{1}{\left(1+\frac{1}{6370}\right)^2}\right]$ $\qquad P = 0.0314\%$ ◀

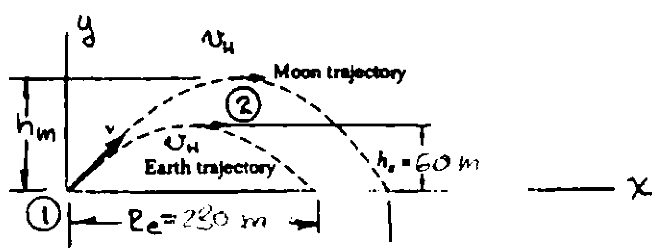
(*b*) $h = 1000$ km: $\qquad P = 100E = 100\left[1 - \dfrac{1}{\left(1+\frac{1000}{6370}\right)^2}\right]$ $\qquad P = 25.3\%$ ◀

PROBLEM 13.38



A golf ball struck on earth rises to a maximum height of 60 m and hits the ground 230 m away. How high will the same golf ball travel on the moon if the magnitude and direction of its velocity are the same as they were on earth immediately after the ball was hit? Assume that the ball is hit and lands at the same elevation in both cases and that the effect of the atmosphere on the earth is neglected, so that the trajectory in both cases is a parabola. The acceleration of gravity on the moon is 0.165 times that on earth.

SOLUTION



Solve for h_m.

At maximum height, the total velocity is the horizontal component of the velocity, which is constant and the same in both cases.

$$T_1 = \frac{1}{2}mv^2 \qquad T_2 = \frac{1}{2}mv_H^2$$

$$U_{1-2} = -mg_e h_e \quad \text{Earth}$$

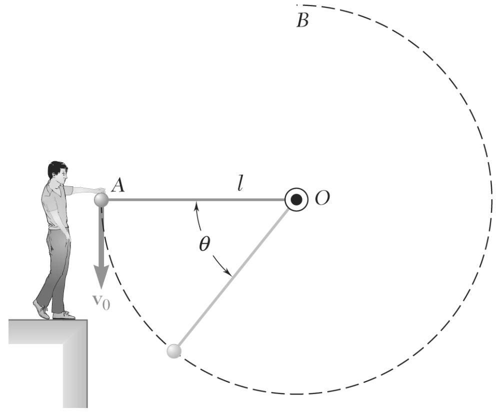
$$U_{1-2} = -mg_m h_m \quad \text{Moon}$$

Earth $$\frac{1}{2}mv^2 - mg_e h_e = \frac{1}{2}mv_H^2$$

Moon $$\frac{1}{2}mv^2 - mg_m h_m = \frac{1}{2}mv_H^2$$

$$-g_e h_e + g_m h_m = 0 \qquad \frac{h_m}{h_e} = \frac{g_e}{g_m}$$

Subtracting $$h_m = (60 \text{ m})\left(\frac{g_e}{0.165 g_e}\right)$$ $h_m = 364$ m ◀



PROBLEM 13.39

The sphere at A is given a downward velocity $\mathbf{v}_0$ of magnitude 5 m/s and swings in a vertical plane at the end of a rope of length $l = 2$ m attached to a support at O. Determine the angle θ at which the rope will break, knowing that it can withstand a maximum tension equal to twice the weight of the sphere.

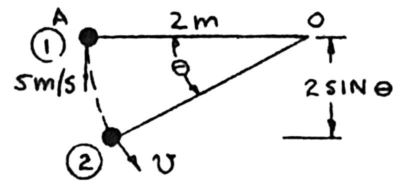
SOLUTION

$$T_1 = \frac{1}{2}mv_0^2 = \frac{1}{2}m\,(5)^2$$

$$T_1 = 12.5 \text{ m}$$

$$T_2 = \frac{1}{2}mv^2$$

$$U_{1-2} = mg(l)\sin\theta$$



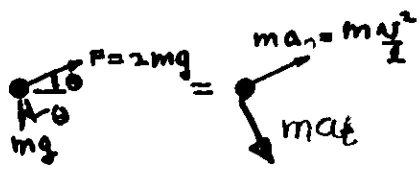
$$T_1 + U_{1-2} = T_2 \qquad 12.5m + 2mg\sin\theta = \frac{1}{2}mv^2$$

$$25 + 4g\sin\theta = v^2 \tag{1}$$

Newton's law at ②.

$$+\!\nearrow \; 2mg - mg\sin\theta = m\frac{v^2}{\ell} = m\frac{v^2}{2} \tag{2}$$

$$v^2 = 4g - 2g\sin\theta$$



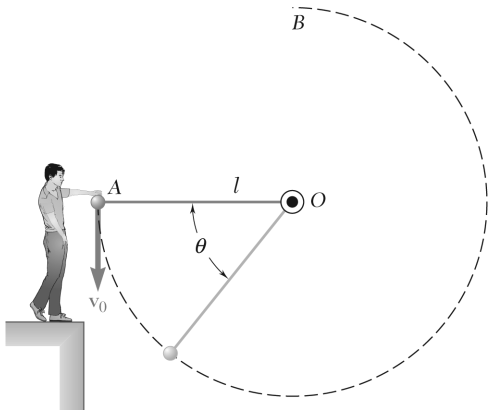
Substitute for v^2 from Eq. (2) into Eq. (1)

$$25 + 4g\sin\theta = 4g - 2g\sin\theta$$

$$\sin\theta = \frac{(4)(9.81) - 25}{(6)(9.81)} = 0.2419$$

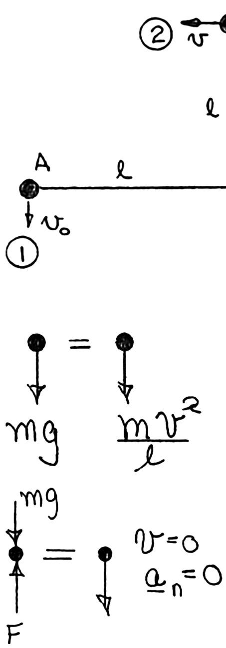
$\theta = 14.00°$ ◀

PROBLEM 13.40



The sphere at A is given a downward velocity $\mathbf{v}_0$ and swings in a vertical circle of radius l and center O. Determine the smallest velocity $\mathbf{v}_0$ for which the sphere will reach Point B as it swings about Point O (*a*) if AO is a rope, (*b*) if AO is a slender rod of negligible mass.

SOLUTION



$$T_1 = \frac{1}{2}mv_0^2$$

$$T_2 = \frac{1}{2}mv^2$$

$$U_{1-2} = -mgl$$

$$T_1 + U_{1-2} = T_2 \qquad \frac{1}{2}mv_0^2 - mgl = \frac{1}{2}mv^2$$

$$v_0^2 = v^2 + 2gl$$

Newton's law at ②

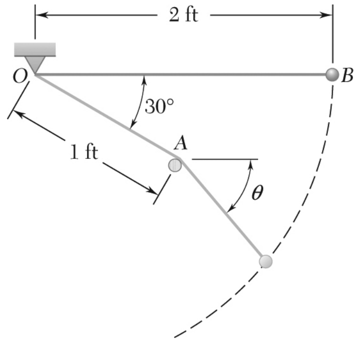
(*a*) For minimum v, tension in the cord must be zero.

Thus, $v^2 = gl$

$$v_0^2 = v^2 + 2gl = 3gl \qquad v_0 = \sqrt{3gl} \blacktriangleleft$$

(*b*) Force in the rod can support the weight so that v can be zero.

Thus, $$v_0^2 = 0 + 2gl \qquad v_0 = \sqrt{2gl} \blacktriangleleft$$

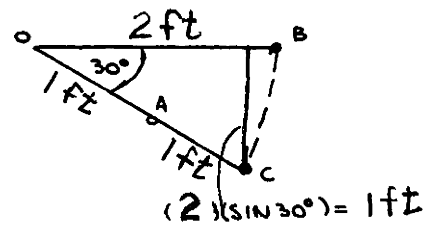


PROBLEM 13.41

A small sphere B of weight W is released from rest in the position shown and swings freely in a vertical plane, first about O and then about the peg A after the cord comes in contact with the peg. Determine the tension in the cord (a) just before the sphere comes in contact with the peg, (b) just after it comes in contact with the peg.

SOLUTION

Velocity of the sphere as the cord contacts A



$$v_B = 0 \qquad T_B = 0$$

$$T_C = \frac{1}{2} m v_C^2$$

$$U_{B-C} = (mg)(1)$$

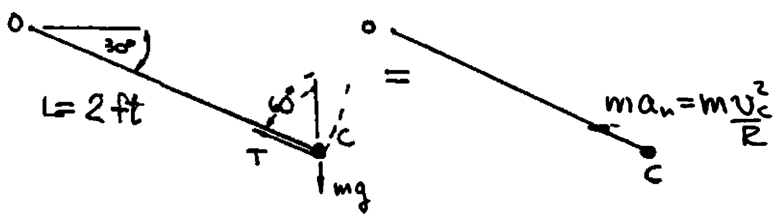
$$T_B + U_{B-C} = T_C$$

$$0 + 1mg = \frac{1}{2} m v_C^2$$

$$v_C^2 = (2)(g)$$

Newton's law

(a) Cord rotates about Point O ($R = L$)



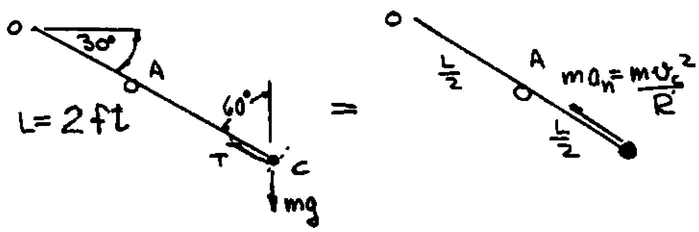
$$+\nwarrow T - mg(\cos 60°) = m\frac{v_C^2}{L}$$

$$T = mg(\cos 60°) + \frac{m(2)g}{2}$$

$$T = \frac{3}{2} mg \qquad\qquad T = 1.5\ W \blacktriangleleft$$

PROBLEM 13.41 (Continued)

(*b*) Cord rotates about $A\left(R = \frac{L}{2}\right)$

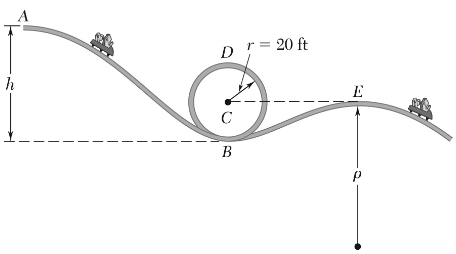


$$T - mg(\cos 60°) = \frac{mv_C^2}{\frac{L}{2}}$$

$$T = \frac{mg}{2} + \frac{m(2)(g)}{1}$$

$$T = \left(\frac{1}{2} + 2\right)mg = \frac{5}{2}mg$$

$T = 2.5W$ ◀



PROBLEM 13.42

A roller coaster starts from rest at A, rolls down the track to B, describes a circular loop of 40-ft diameter, and moves up and down past Point E. Knowing that h = 60 ft and assuming no energy loss due to friction, determine (a) the force exerted by his seat on a 160-lb rider at B and D, (b) the minimum value of the radius of curvature at E if the roller coaster is not to leave the track at that point.

SOLUTION

Let y_p be the vertical distance from Point A to any Point P on the track. Let position 1 be at A and position 2 be at P. Apply the principle of work and energy.

$$T_1 = 0 \qquad T_2 = \frac{1}{2}mv_P^2$$

$$U_{1\to 2} = mgy_P$$

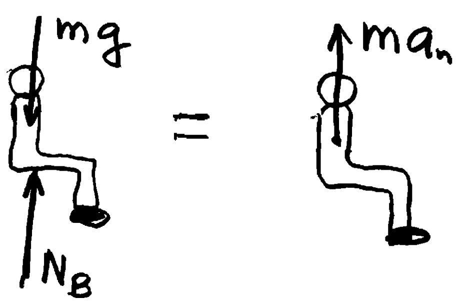
$$T_1 + U_{1\to 2} = T_2: \quad 0 + mgy_P = \frac{1}{2}mv_P^2$$

$$v_P^2 = 2gy_P$$

Magnitude of normal acceleration at P:

$$(a_P)_n = \frac{v_P^2}{\rho_P} = \frac{2gy_P}{\rho_P}$$

(a) Rider at Point B.



$$y_B = h = 60 \text{ ft}$$

$$\rho_B = r = 20 \text{ ft}$$

$$a_n = \frac{(2g)(60)}{20} = 6g$$

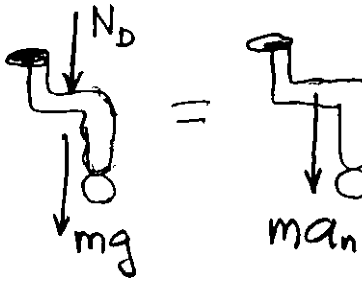
$$+\uparrow \Sigma F = ma:$$

$$N_B - mg = m(6g)$$

$$N_B = 7mg = 7W = (7)(160 \text{ lb})$$

$$\mathbf{N}_B = 1120 \text{ lb} \uparrow \blacktriangleleft$$

Rider at Point D.



$$y_D = h - 2r = 20 \text{ ft}$$
$$\rho_D = 20 \text{ ft}$$
$$a_n = \frac{(2g)(20)}{20} = 2g$$

$+\downarrow \Sigma F = ma$:

$$N_D + mg = m(2g)$$
$$N_D = mg = W = 160 \text{ lb}$$

$\mathbf{N}_D = 160 \text{ lb} \downarrow$ ◀

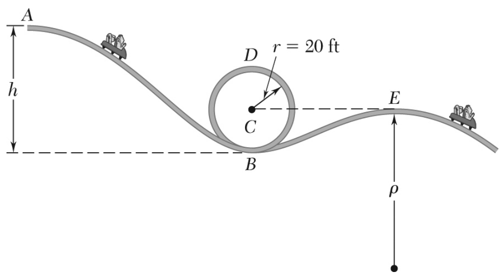
(*b*) Car at Point E.

$$y_E = h - r = 40 \text{ ft}$$
$$N_E = 0$$

$+\uparrow \Sigma F = ma_n$:

$$mg = m \cdot \frac{2gy_E}{\rho_E}$$
$$\rho_E = 2y_E$$

$\rho = 80.0 \text{ ft}$ ◀



PROBLEM 13.43

In Problem 13.42, determine the range of values of h for which the roller coaster will not leave the track at D or E, knowing that the radius of curvature at E is $\rho = 75$ ft. Assume no energy loss due to friction.

PROBLEM 13.42 A roller coaster starts from rest at A, rolls down the track to B, describes a circular loop of 40-ft diameter, and moves up and down past Point E. Knowing that $h = 60$ ft and assuming no energy loss due to friction, determine (a) the force exerted by his seat on a 160-lb rider at B and D, (b) the minimum value of the radius of curvature at E if the roller coaster is not to leave the track at that point.

SOLUTION

Let y_p be the vertical distance from Point A to any Point P on the track. Let position 1 be at A and position 2 be at P. Apply the principle of work and energy.

$$T_1 = 0 \qquad T_2 = \frac{1}{2}mv_P^2$$

$$U_{1\to 2} = mg\,y_P$$

$$T_1 + U_{1\to 2} = T_2: \quad 0 + mgy_p = \frac{1}{2}mv_P^2$$

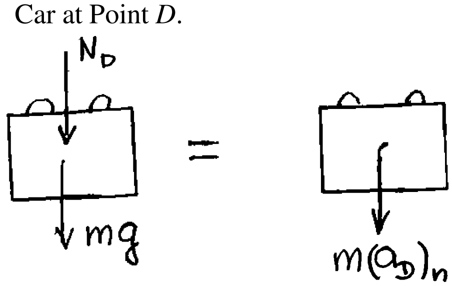
$$v_P^2 = 2g\,y_P$$

Magnitude of normal acceleration of P:

$$(a_P)_n = \frac{v_P^2}{\rho_P} = \frac{2g\,y_P}{\rho_P}$$

The condition of loss of contact with the track at P is that the curvature of the path is equal to ρ_p and the normal contact force $N_P = 0$.

Car at Point D.



$$\rho_D = r = 20 \text{ ft}$$

$$y_D = h - 2r$$

$$(a_D)_n = \frac{2\,g\,(h-2r)}{r}$$

$$+\!\downarrow \Sigma F = ma$$

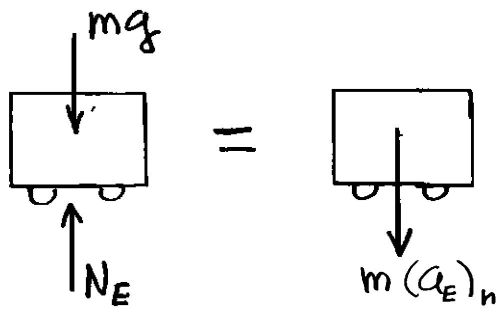
$$N_D + mg = m\frac{2g(h-2r)}{r}$$

$$N_D = mg\frac{2h-5r}{r}$$

For $N_D > 0$

$$2h - 5r > 0$$
$$h > \frac{5}{2}r = 50 \text{ ft}$$

Car at Point E.

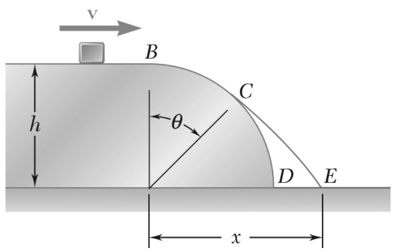


$$\rho_E = \rho = 75 \text{ ft}$$
$$y_E = h - r = h - 20 \text{ ft}$$
$$(a_E)_n = \frac{2g(h-20)}{75}$$
$$+\uparrow \Sigma F = ma$$
$$N_E - mg = -\frac{2mg(h-20)}{75}$$
$$N_E = mg\frac{115-2h}{75}$$

For $\qquad N_E > 0, \qquad 115 - 2h > 0$

$$h < 57.5 \text{ ft}$$

Range of values for h: $\qquad 50.0 \text{ ft} \le h \le 57.5 \text{ ft}$ ◀

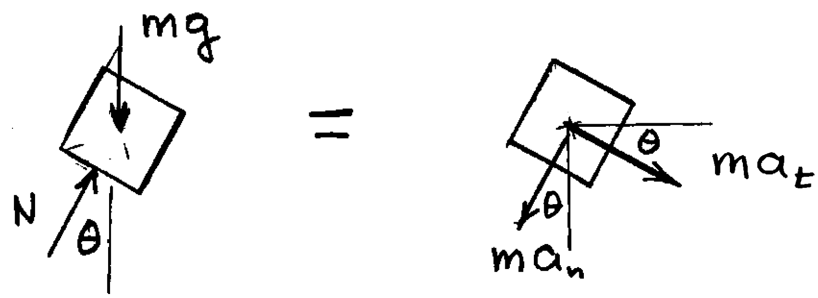


PROBLEM 13.44

A small block slides at a speed v on a horizontal surface. Knowing that $h = 0.9$ m, determine the required speed of the block if it is to leave the cylindrical surface BCD when $\theta = 30°$.

SOLUTION

At Point C where the block leaves the surface BCD the contact force is reduced to zero. Apply Newton's second law at Point C.



n-direction: $$N - mg\cos\theta = -ma_n = -\frac{mv_C^2}{h}$$

With $N = 0$, we get $$v_c^2 = gh\cos\theta$$

Apply the work-energy principle to the block sliding over the path BC. Let position 1 correspond to Point B and position 2 to C.

$$T_1 = \frac{1}{2}mv_B^2 \qquad T_2 = \frac{1}{2}\ mv_C^2 = \frac{1}{2}mgh\cos\theta$$

$$\begin{aligned} U_{1\to 2} &= \text{ weight} \times \text{change in vertical distance} \\ &= mgh(1-\cos\theta) \end{aligned}$$

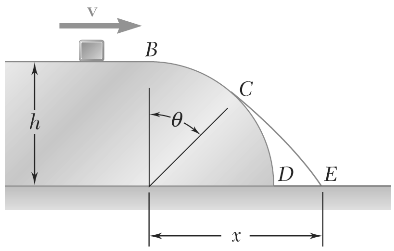
$$T_1 + U_{1\to 2} = T_2 : \frac{1}{2}mv_B^2 + mg(1-\cos\theta) = \frac{1}{2}mgh\cos\theta$$

$$v_B^2 = gh\cos\theta - 2gh(1-\cos\theta) = gh(3\cos\theta - 2)$$

Data: $$g = 9.81 \text{ m/s}^2,\ h = 0.9 \text{ m},\ \theta = 30°.$$

$$v_B^2 = (9.81)(0.9)(3\cos 30° - 2) = 5.2804 \text{ m}^2/\text{s}^2$$

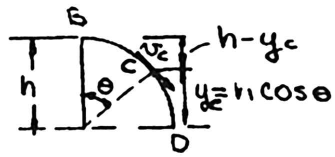
$$v_B = 2.30 \text{ m/s} \blacktriangleleft$$

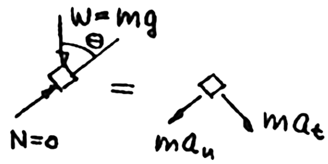


PROBLEM 13.45

A small block slides at a speed $v = 8$ ft/s on a horizontal surface at a height $h = 3$ ft above the ground. Determine (*a*) the angle θ at which it will leave the cylindrical surface *BCD*, (*b*) the distance *x* at which it will hit the ground. Neglect friction and air resistance.

SOLUTION





Block leaves surface at *C* when the normal force $N = 0$.

$$+\swarrow\ mg\cos\theta = ma_n$$

$$g\cos\theta = \frac{v_C^2}{h}$$

$$v_C^2 = gh\cos\theta = gy \qquad (1)$$

Work-energy principle.

(*a*)

$$T_B = \frac{1}{2}mv^2 = \frac{1}{2}m(8)^2 = 32m$$

$$T_C = \frac{1}{2}mv_C^2 \qquad U_{B-C} = W(h-g) = mg(h-y_C)$$

$$T_B + U_{B-C} = T_C$$

$$32m + mg(h-y) = \frac{1}{2}mv_C^2$$

Use Eq. (1)

$$32 + g(h-y_C) = \frac{1}{2}gy_C \qquad (2)$$

$$32 + gh = \frac{3}{2}gy_C$$

$$y_C = \frac{(32+gh)}{\left(\frac{3}{2}g\right)}$$

$$y_C = \frac{(32+(32.2)(3))}{\frac{3}{2}(32.2)}$$

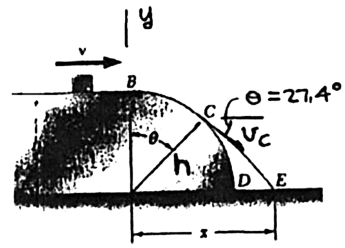
$$y_C = 2.6625\text{ ft} \qquad (3)$$

$$y_C = h\cos\theta \qquad \cos\theta = \frac{y_C}{h} = \frac{2.6625}{3} = 0.8875$$

$\theta = 27.4°$ ◀

PROBLEM 13.45 (Continued)

(*b*) From (1) and (3)



$$v_C = \sqrt{gy}$$
$$v_C = \sqrt{(32.2)(2.6625)}$$
$$v_C = 9.259 \text{ ft/s}$$

At *C*: $(v_C)_x = v_C \cos\theta = (9.259)(\cos 27.4°) = 8.220 \text{ ft/s}$

$(v_C)_y = -v_C \sin\theta = -(9.259)(\sin 27.4°) = 4.261 \text{ ft/s}$

$$y = y_C + (v_C)_y t - \frac{1}{2}gt^2 = 2.6625 - 4.261t - 16.1t^2$$

At *E*: $y_E = 0$: $t^2 + 0.2647t - 0.1654 = 0$

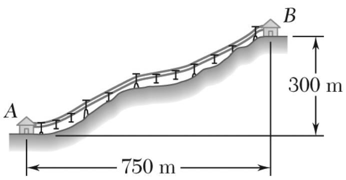
$t = 0.2953 \text{ s}$

At *E*: $x = h(\sin\theta) + (v_C)_x t = (3)(\sin 27.4°) + (8.220)(0.2953)$

$x = 1.381 + 2.427 = 3.808 \text{ ft}$

$x = 3.81 \text{ ft}$ ◀

PROBLEM 13.46



A chair-lift is designed to transport 1000 skiers per hour from the base A to the summit B. The average mass of a skier is 70 kg and the average speed of the lift is 75 m/min. Determine (a) the average power required, (b) the required capacity of the motor if the mechanical efficiency is 85 percent and if a 300 percent overload is to be allowed.

SOLUTION

Note: Solution is independent of speed.

(a)

$$\text{Average power } = \frac{\Delta U}{\Delta t} = \frac{(1000)(70 \text{ kg})(9.81 \text{ m/s}^2)(300 \text{ m})}{3600 \text{ s}} = 57,225 \frac{\text{N} \cdot \text{m}}{\text{s}}$$

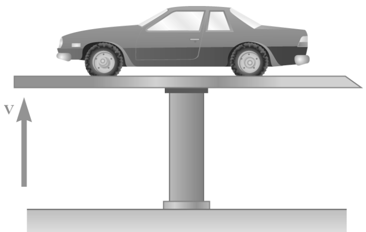
$$\text{Average power } = 57.2 \text{ kW} \blacktriangleleft$$

(b) Maximum power required with 300% over load

$$= \frac{100 + 300}{100}(57.225 \text{ kW}) = 229 \text{ kW}$$

Required motor capacity (85% efficient)

$$\text{Motor capacity } = \frac{229 \text{ kW}}{0.85} = 269 \text{ kW} \blacktriangleleft$$



PROBLEM 13.47

It takes 15 s to raise a 1200-kg car and the supporting 300-kg hydraulic car-lift platform to a height of 2.8 m. Determine (*a*) the average output power delivered by the hydraulic pump to lift the system, (*b*) the average power electric required, knowing that the overall conversion efficiency from electric to mechanical power for the system is 82 percent.

SOLUTION

(*a*)

$$(P_P)_A = (F)(v_A) = (m_C + m_L)(g)(v_A)$$

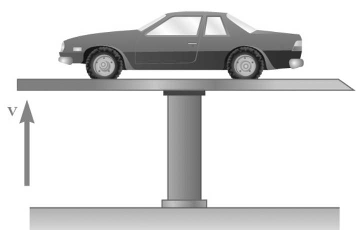
$$v_A = \text{s/t} = (2.8 \text{ m})/(15 \text{ s}) = 0.18667 \text{ m/s}$$

$$(P_P)_A = [(1200 \text{ kg}) + (300 \text{ kg})](9.81 \text{ m/s}^2)(0.18667 \text{ m/s})^3$$

$$(P_P)_A = 2.747 \text{ kJ/s} \qquad (P_P)_A = 2.75 \text{ kW} \blacktriangleleft$$

(*b*)

$$(P_E)_A = (P_P)/\eta = (2.75 \text{ kW})/(0.82) \qquad (P_E)_A = 3.35 \text{ kW} \blacktriangleleft$$



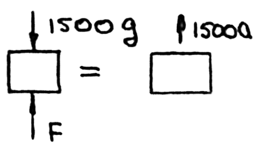
PROBLEM 13.48

The velocity of the lift of Problem 13.47 increases uniformly from zero to its maximum value at mid-height 7.5 s and then decreases uniformly to zero in 7.5 s. Knowing that the peak power output of the hydraulic pump is 6 kW when the velocity is maximum, determine the maximum life force provided by the pump.

PROBLEM 13.47 It takes 15 s to raise a 1200-kg car and the supporting 300-kg hydraulic car-lift platform to a height of 2.8 m. Determine (*a*) the average output power delivered by the hydraulic pump to lift the system, (*b*) the average power electric required, knowing that the overall conversion efficiency from electric to mechanical power for the system is 82 percent.

SOLUTION

Newton's law



$$Mg = (M_C + M_L)g = (1200 + 300)g$$

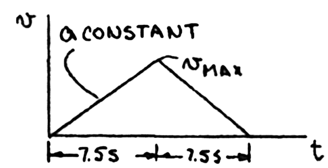
$$Mg = 1500g$$

$$+\!\uparrow \Sigma F = F - 1500g = 1500a \qquad (1)$$

Since motion is uniformly accelerated, a = constant

Thus, from (1), F is constant and peak power occurs when the velocity is a maximum at 7.5 s.

$$a = \frac{v_{\max}}{7.5 \text{ s}}$$



$$P = (6000 \text{ W}) = (F)(v_{\max})$$

$$v_{\max} = (6000)/F$$

Thus, $$a = (6000)/(7.5)(F) \qquad (2)$$

Substitute (2) into (1)

$$F - 1500g = (1500)(6000)/(7.5)(F)$$

$$F^2 - (1500 \text{ kg})(9.81 \text{ m/s}^2)F - \frac{(1500 \text{ kg})(6000 \text{ N}\cdot\text{m/s})}{(7.5 \text{ s})} = 0$$

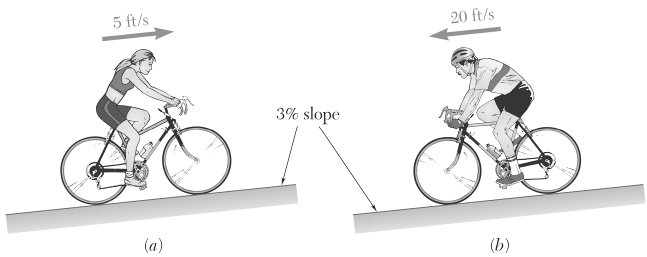
$$F^2 - 14{,}715F - 1.2\times 10^6 = 0$$

$$F = 14{,}800 \text{ N}$$

$F = 14.8$ kN ◀

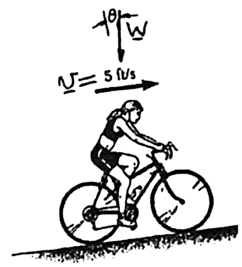
PROBLEM 13.49

(a) A 120-lb woman rides a 15-lb bicycle up a 3-percent slope at a constant speed of 5 ft/s. How much power must be developed by the woman? (*b*) A 180-lb man on an 18-lb bicycle starts down the same slope and maintains a constant speed of 20 ft/s by braking. How much power is dissipated by the brakes? Ignore air resistance and rolling resistance.



SOLUTION

$$\tan\theta = \frac{3}{100} \qquad \theta = 1.718°$$



(*a*)

$$W = W_B + W_W = 15 + 120$$

$$W = 135 \text{ lb}$$

$$P_W = \mathbf{W}\cdot\mathbf{v} = (W\sin\theta)(v)$$

$$P_W = (135)(\sin 1.718°)(5)$$

$$P_W = 20.24 \text{ ft}\cdot\text{lb/s}$$

(*a*)

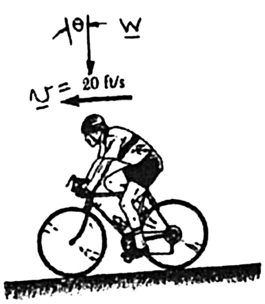
$$P_W = 20.2 \text{ ft}\cdot\text{lb/s} \blacktriangleleft$$

(*b*)

$$W = W_B + W_m = 18 + 180$$

$$W = 198 \text{ lb}$$

Brakes must dissipate the power generated by the bike and the man going down the slope at 20 ft/s.

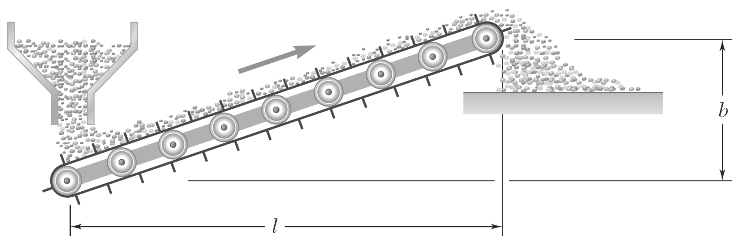


(*b*)

$$P_B = \mathbf{W}\cdot\mathbf{v} = (W\sin\theta)(v)$$

$$P_B = (198)(\sin 1.718)(20)$$

$$P_B = 118.7 \text{ ft}\cdot\text{lb/s} \blacktriangleleft$$



PROBLEM 13.50

A power specification formula is to be derived for electric motors which drive conveyor belts moving solid material at different rates to different heights and distances. Denoting the efficiency of the motors by η and neglecting the power needed to drive the belt itself, derive a formula (*a*) in the SI system of units for the power *P* in kW, in terms of the mass flow rate *m* in kg/h, the height *b* and horizontal distance *l* in meters, and (*b*) in U.S. customary units, for the power in hp, in terms of the material flow rate *w* in tons/h, and the height *b* and horizontal distance *l* in feet.

SOLUTION

(*a*) Material is lifted to a height *b* at a rate, $(m\ \text{kg/h})(g\ \text{m/s}^2) = [mg(\text{N/h})]$

Thus,

$$\frac{\Delta U}{\Delta t} = \frac{[mg(\text{N/h})][b(m)]}{(3600\ \text{s/h})} = \left(\frac{mgb}{3600}\right) \text{N}\cdot\text{m/s}$$

$$1000\ \text{N}\cdot\text{m/s} = 1\ \text{kW}$$

Thus, including motor efficiency, η

$$P(\text{kw}) = \frac{mgb(\text{N}\cdot\text{m/s})}{(3600)\left(\dfrac{1000\ \text{N}\cdot\text{m/s}}{\text{kW}}\right)(\eta)}$$

$$P(\text{kW}) = 0.278\times10^{-6}\,\frac{mgb}{\eta} \blacktriangleleft$$

(*b*)

$$\frac{\Delta U}{\Delta t} = \frac{[W(\text{tons/h})(2000\ \text{lb/ton})][b(\text{ft})]}{3600\ \text{s/h}}$$

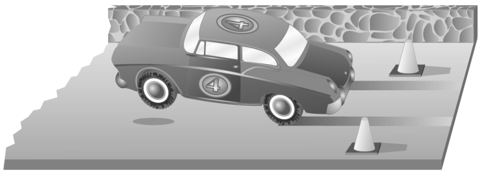
$$= \frac{Wb}{1.8}\ \text{ft}\cdot\text{lb/s}; \quad 1\,\text{hp} = 550\ \text{ft}\cdot\text{lb/s}$$

With η,

$$hp = \left[\frac{Wb}{1.8}(\text{ft}\cdot\text{lb/s})\right]\left[\frac{1\ \text{hp}}{550\ \text{ft}\cdot\text{lb/s}}\right]\left[\frac{1}{\eta}\right]$$

$$hp = \frac{1.010\times10^{-3}Wb}{\eta} \blacktriangleleft$$

PROBLEM 13.51



In an automobile drag race, the rear (drive) wheels of a 1000 kg car skid for the first 20 m and roll with sliding impending during the remaining 380 m. The front wheels of the car are just off the ground for the first 20 m, and for the remainder of the race 80 percent of the weight is on the rear wheels. Knowing that the coefficients of friction are $\mu_s = 0.90$ and $\mu_k = 0.68$, determine the power developed by the car at the drive wheels (*a*) at the end of the 20-m portion of the race, (*b*) at the end of the race: Give your answer in kW and in hp. Ignore the effect of air resistance and rolling friction.

SOLUTION

(*a*) First 20 m. (Calculate velocity at 20 m.) Force generated by rear wheels $= \mu_k W$, since car skids.

Thus,
$$F_s = (0.68)(1000)(g)$$
$$F_s = (0.68)(1000 \text{ kg})(9.81 \text{ m/s}^2) = 6670.8 \text{ N}$$

Work and energy.
$$T_1 = 0, \quad T_2 = \frac{1}{2}mv_{20}^2 = 500v_{20}^2$$
$$T_1 + U_{1-2} = T_2$$
$$U_{1-2} = (20 \text{ m})(F_s) = (20 \text{ m})(6670.8 \text{ N})$$
$$U_{1-2} = 133,420 \text{ J}$$
$$0 + 133,420 = 500v_{20}^2$$
$$v_{20}^2 = \frac{133,420}{500} = 266.83$$
$$v_{20} = 16.335 \text{ m/s}$$
$$\text{Power} = (F_s)(v_{20}) = (6670.8 \text{ N})(16.335 \text{ m/s})$$
$$\text{Power} = 108,970 \text{ J/s} = 108.97 \text{ kJ/s}$$
$$1 \text{ kJ/s} = 1 \text{ kW}$$
$$1 \text{ hp} = 0.7457 \text{ kW} \qquad \text{Power} = 109.0 \text{ kJ/s} = 109.0 \text{ kW} \blacktriangleleft$$
$$\text{Power} = \frac{(109.0 \text{ kW})}{(0.7457 \text{ kW/hp})} = 146.2 \text{ hp} \blacktriangleleft$$

(*b*) End of race. (Calculate velocity at 400 m.) For remaining 380 m, with 80% of weight on rear wheels, the force generated at impending sliding is $(\mu_s)(0.80)(mg)$

$$F_I = (0.90)(0.80)(1000 \text{ kg})(9.81 \text{ m/s}^2)$$

$$F_I = 7063.2 \text{ N}$$

Work and energy, from 20 m ② to 28 m ③.

$$v_2 = 16.335 \text{ m/s} \text{ [from part } (a)]$$

$$T_2 = \frac{1}{2}(1000 \text{ kg})(16.335 \text{ m/s})^2$$

$$T_2 = 133,420 \text{ J}$$

$$T_3 = \frac{1}{2}mv_{380}^2 = 500v_{380}^2$$

$$U_{2-3} = (F_I)(380 \text{ m}) = (7063.2 \text{ N})(380 \text{ m})$$

$$U_{2-3} = 2,684,000 \text{ J}$$

$$T_2 + U_{2-3} = T_3$$

$$(133,420 \text{ J}) + (2,684,000 \text{ J}) = 500\,v_{30}^2$$

$$v_{30} = 75.066 \text{ m/s}$$

$$\text{Power } = (F_I)(v_{30}) = (7063.2 \text{ N})(75.066 \text{ m/s})$$

$$= 530,200 \text{ J/s}$$

$$\text{kW Power } = 530,200 \text{ J} = 530 \text{ kW} \quad ◀$$

$$\text{hp Power } = \frac{530 \text{ kW}}{(0.7457 \text{ kW/hp})} = 711 \text{ hp} \quad ◀$$

PROBLEM 13.52

The frictional resistance of a ship is known to vary directly as the 1.75 power of the speed v of the ship. A single tugboat at full power can tow the ship at a constant speed of 4.5 km/h by exerting a constant force of 300 kN. Determine (*a*) the power developed by the tugboat, (*b*) the maximum speed at which two tugboats, capable of delivering the same power, can tow the ship.

SOLUTION

(*a*) Power developed by tugboat at 4.5 km/h.

$$v_0 = 4.5 \text{ km/h} = 1.25 \text{ m/s}$$
$$F_0 = 300 \text{ kN}$$

$$P_0 = F_0 v_0 = (300 \text{ kN})(1.25 \text{ m/s}) \qquad P_0 = 375 \text{ kW} \blacktriangleleft$$

(*b*) Maximum speed.

Power required to tow ship at speed v:

$$F = F_0 \left(\frac{v}{v_0}\right)^{1.75} \qquad P = Fv = F_0 v \left(\frac{v}{v_0}\right)^{1.75} = F_0 v_0 \left(\frac{v}{v_0}\right)^{2.75} \qquad (1)$$

Since we have two tugboats, the available power is twice maximum power $F_0 v_0$ developed by one tugboat.

$$2F_0 v_0 = F_0 v_0 \left(\frac{v}{v_0}\right)^{2.75}$$

$$\left(\frac{v}{v_0}\right)^{2.75} = 2 \qquad v = v_0 (2)^{1/2.75} = v_0 (1.2867)$$

Recalling that $\quad v_0 = 4.5 \text{ km/h}$

$$v = (4.5 \text{ km/h})(1.2867) = 5.7902 \text{ km/h} \qquad v = 5.79 \text{ km/h} \blacktriangleleft$$

PROBLEM 13.53

A train of total mass equal to 500 Mg starts from rest and accelerates uniformly to a speed of 90 km/h in 50 s. After reaching this speed, the train travels with a constant velocity. The track is horizontal and axle friction and rolling resistance result in a total force of 15 kN in a direction opposite to the direction of motion. Determine the power required as a function of time.

SOLUTION

Let F_P be the driving force and F_R be the resisting force due to axle friction and rolling resistance.

Uniformly accelerated motion. $(t < 50 \text{ s})$:

$$v = v_0 + at \qquad v_0 = 0$$

At $\quad t = 50s, \qquad v = 90 \text{ km/h} = 25 \text{ m/s}$

$$25 \text{ m/s} = 0 + a(50)$$
$$a = 0.5 \text{ m/s}^2$$
$$v = (0.5 \text{ m/s}^2)t$$

Newton's second law: $\quad F_P - F_R = ma$

where $\quad F_R = 15 \text{ kN} = 15\times10^3 \text{ N}$

$$m = 500 \text{ Mg} = 500\times10^3 \text{ kg}$$
$$a = 0.5 \text{ m/s}^2$$

$$F_P = F_R + ma = 15\times10^3 + (500\times10^3)(0.5)$$
$$= 265\times10^3 \text{ N} = 265 \text{ kN}$$

Power: $\quad F_P v = (265\times10^3)(0.5t)$

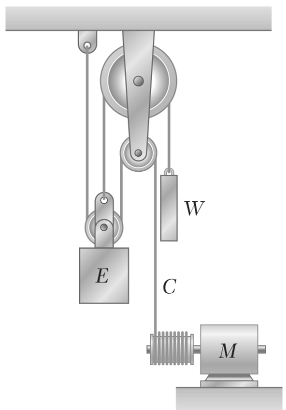
$(0 < 50\text{s}) \qquad$ Power = (132.5 kW/s)t ◀

Uniform motion. $\quad (t > 50 \text{ s})$: $\quad a = 0$

$$F_P = F_R = 15\times10^3 \text{ N}; \; v = 25 \text{ m/s}$$

Power: $\quad F_P v = (15\times10^3)(25 \text{ m/s}) = 375\times10^3 W$

$(t > 50 \text{ s}) \qquad$ Power = 375 kW ◀



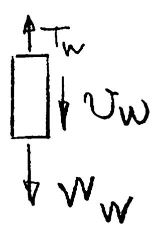
PROBLEM 13.54

The elevator E has a weight of 6600 lbs when fully loaded and is connected as shown to a counterweight W of weight of 2200 lb. Determine the power in hp delivered by the motor (a) when the elevator is moving down at a constant speed of 1 ft/s, (b) when it has an upward velocity of 1 ft/s and a deceleration of $0.18\ \text{ft/s}^2$.

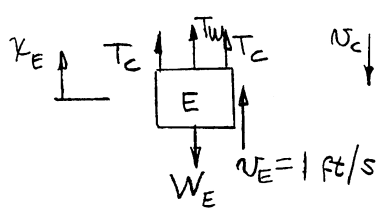
SOLUTION

(a) Acceleration = 0

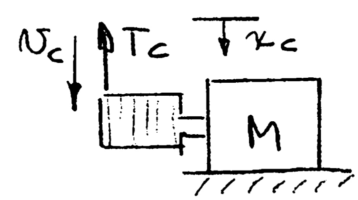
Counterweight



Elevator



Motor



$$+\uparrow \Sigma F_y = 0: \quad T_W - W_W = 0$$

$$T_W = 2200 \text{ lb}$$

$$+\uparrow \Sigma F = 0: \quad 2T_C + T_W - 6600 = 0$$

$$T_C = 2200 \text{ lb}$$

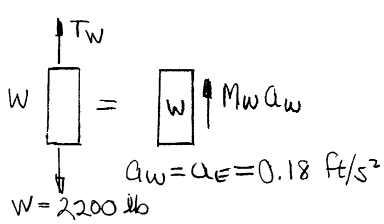
Kinematics: $\quad 2x_E = x_C,\ 2\dot{x}_E = \dot{x}_C,\ v_C = 2v_E = 2 \text{ ft/s}$

$$P = T_C \cdot v_C = (2200 \text{ lb})(2 \text{ ft/s}) = 4400 \text{ lb}\cdot\text{ft/s} = 8.00 \text{ hp}$$

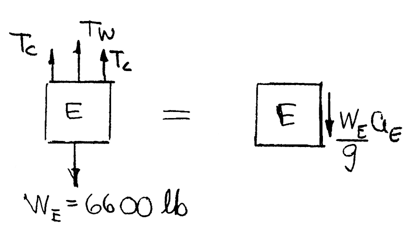
$P = 8.00$ hp ◀

(b) $\quad a_E = 0.18\ \text{ft/s}^2 \uparrow,\ v_E = 1 \text{ ft/s} \downarrow$

Counterweight



Elevator



Counterweight: $\quad +\uparrow \Sigma F = Ma: \quad T_W - W = \dfrac{W}{g}(a_W)$

PROBLEM 13.54 (Continued)

$$T_W = (2200 \text{ lb}) + \frac{(2200 \text{ lb})(0.18 \text{ ft/s}^2)}{(32.2 \text{ ft/s}^2)}$$

$$T_W = 2212 \text{ lb}$$

Elevator

$$+\uparrow \Sigma F = ma \qquad 2T_C + T_W - W_E = \frac{-W_E}{g}(a_E)$$

$$2T_C = (-2212 \text{ lb}) + (6600 \text{ lb}) - \frac{(6600 \text{ lb})(0.18 \text{ ft/s}^2)}{(32.2 \text{ ft/s}^2)}$$

$$2T_C = 4351 \text{ lb}$$

$$T_C = 2175.6 \text{ lb}$$

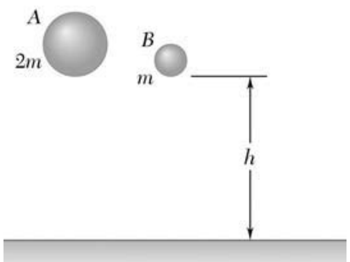
$$v_C = 2 \text{ ft/s (see part}(a))$$

$$P = T_C \cdot v_C = (2175.6 \text{ lb})(2 \text{ ft/s}) = 4351.2 \text{ lb}\cdot\text{ft/s}$$

$$= 7.911 \text{ hp}$$

$P = 7.91$ hp ◀

PROBLEM 13.CQ2



Two small balls A and B with masses $2m$ and m respectively are released from rest at a height h above the ground. Neglecting air resistance, which of the following statements are true when the two balls hit the ground?

(a) The kinetic energy of A is the same as the kinetic energy of B.

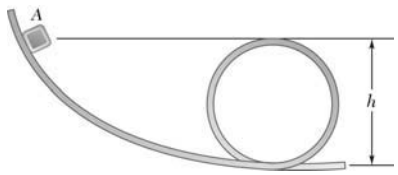
(b) The kinetic energy of A is half the kinetic energy of B.

(c) The kinetic energy of A is twice the kinetic energy of B.

(d) The kinetic energy of A is four times the kinetic energy of B.

SOLUTION

Answer: (c)



PROBLEM 13.CQ3

Block A is released from rest and slides down the frictionless ramp to the loop. The maximum height h of the loop is the same as the initial height of the block. Will A make it completely around the loop without losing contact with the track?

(a) Yes

(b) No

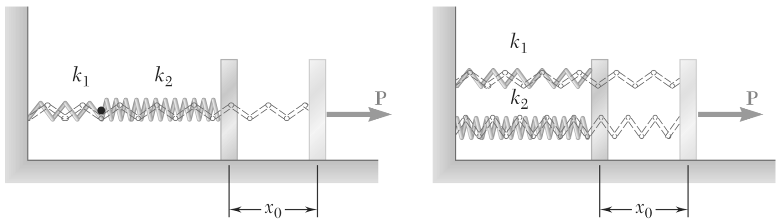
(c) need more information

SOLUTION

Answer: (b) In order for A to not maintain contact with the track, the normal force must remain greater than zero, which requires a non-zero speed at the top of the loop.

PROBLEM 13.55

A force **P** is slowly applied to a plate that is attached to two springs and causes a deflection x_0. In each of the two cases shown, derive an expression for the constant k_e, in terms of k_1 and k_2, of the single spring equivalent to the given system, that is, of the single spring which will undergo the same deflection x_0 when subjected to the same force **P**.

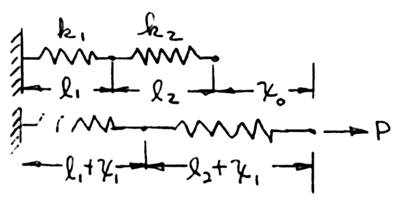


(*a*) (*b*)

SOLUTION

System is in equilibrium in deflected x_0 position.

Case (*a*) Force in both springs is the same = P



$$x_0 = x_1 + x_2$$

$$x_0 = \frac{P}{k_e}$$

$$x_1 = \frac{P}{k_1} \qquad x_2 = \frac{P}{k_2}$$

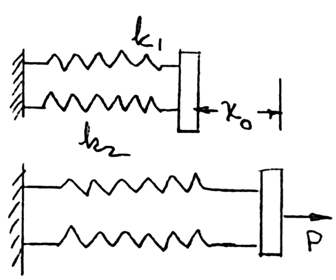
Thus,

$$\frac{P}{k_e} = \frac{P}{k_1} + \frac{P}{k_2}$$

$$\frac{1}{k_e} = \frac{1}{k_1} + \frac{1}{k_2}$$

$$k_e = \frac{k_1 k_2}{k_1 + k_2} \blacktriangleleft$$

Case (*b*) Deflection in both springs is the same = x_0



$$P = k_1 x_0 + k_2 x_0$$

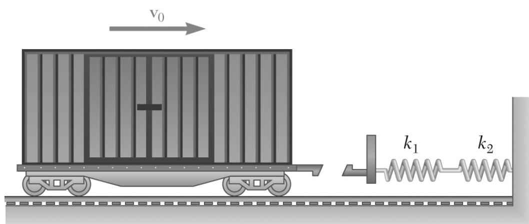
$$P = (k_1 + k_2) x_0$$

$$P = k_e x_0$$

Equating the two expressions for

$$P = (k_1 + k_2) x_0 = k_e x_0$$

$$k_e = k_1 + k_2 \blacktriangleleft$$



PROBLEM 13.56

A loaded railroad car of mass m is rolling at a constant velocity $\mathbf{v_0}$ when it couples with a massless bumper system. Determine the maximum deflection of the bumper assuming the two springs are (*a*) in series (as shown), (*b*) in parallel.

SOLUTION

Let position A be at the beginning of contact and position B be at maximum deflection.

$$T_A = \frac{1}{2}mv_0^2$$

$$V_A = 0 \qquad \text{(zero force in springs)}$$

$$T_B = 0 \qquad (v = 0 \text{ at maximum deflection})$$

$$V_B = \frac{1}{2}k_1x_1^2 + \frac{1}{2}k_2x_2^2$$

where x_1 is deflection of spring k_1 and x_2 is that of spring k_2.

Conservation of energy: $T_A + V_A = T_B + V_B$

$$\frac{1}{2}mv_0^2 + 0 = 0 + \frac{1}{2}k_1x_1^2 + \frac{1}{2}k_2x_2^2$$

$$k_1x_1^2 + k_2x_2^2 = mv_0^2 \qquad (1)$$

(*a*) *Springs are in series.*

Let F be the force carried by the two springs.

Then, $x_1 = \dfrac{F}{k_1}$ and $x_2 = \dfrac{F}{k_2}$

Eq. (1) becomes $F^2\left(\dfrac{1}{k_1} + \dfrac{1}{k_2}\right) = mv_0^2$

so that $F = v_0\sqrt{m/\left(\dfrac{1}{k_1} + \dfrac{1}{k_2}\right)}$

The maximum deflection is $\delta = x_1 + x_2 = \left(\dfrac{1}{k_1} + \dfrac{1}{k_2}\right)F$

$$= \left(\frac{1}{k_1} + \frac{1}{k_2}\right)v_0\sqrt{m/\left(\frac{1}{k_1} + \frac{1}{k_2}\right)}$$

$$= v_0\sqrt{m\left(\frac{1}{k_1} + \frac{1}{k_2}\right)}$$

$$\delta = v_0\sqrt{m(k_1 + k_2)/k_1k_2} \blacktriangleleft$$

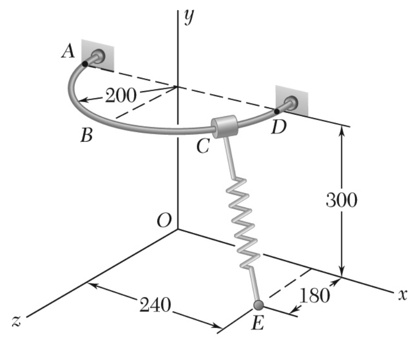
PROBLEM 13.56 (Continued)

(*b*) *Springs are in parallel.*

$$x_1 = x_2 = \delta$$

Eq. (1) becomes $(k_1 + k_2)\delta^2 = mv_0^2$ $\delta = v_0\sqrt{\dfrac{m}{k_1 + k_2}}$ ◀

PROBLEM 13.57



Dimensions in mm

A 600-g collar C may slide along a horizontal, semicircular rod ABD. The spring CE has an undeformed length of 250 mm and a spring constant of 135 N/m. Knowing that the collar is released from rest at A and neglecting friction, determine the speed of the collar (*a*) at B, (*b*) at D.

SOLUTION

First calculate the lengths of the spring when the collar is at positions A, B, and D.

$$l_A = \sqrt{440^2 + 300^2 + 180^2} = 562.14 \text{ mm}$$
$$l_B = \sqrt{240^2 + 300^2 + 20^2} = 384.71 \text{ mm}$$
$$l_D = \sqrt{40^2 + 300^2 + 180^2} = 352.14 \text{ mm}$$

The elongations of springs are given by $e = l - l_0$.

$$e_A = 562.14 - 250 = 312.14 \text{ mm} = 0.31214 \text{ m}$$
$$e_B = 384.71 - 250 = 134.71 \text{ mm} = 0.13471 \text{ m}$$
$$e_D = 352.14 - 250 = 102.14 \text{ mm} = 0.10214 \text{ m}$$

Potential energies: $$V = \frac{1}{2}ke^2$$

$$V_A = \frac{1}{2}(135 \text{ N/m})(0.31214 \text{ m})^2 = 6.5767 \text{ J}$$
$$V_B = \frac{1}{2}(135 \text{ N/m})(0.13471 \text{ m})^2 = 1.2249 \text{ J}$$
$$V_D = \frac{1}{2}(135 \text{ N/m})(0.10214 \text{ m})^2 = 0.7042 \text{ J}$$

Since the semicircular rod ABC is horizontal, there is no change in gravitational potential energy.

Mass of collar: $$m = 600 \text{ g} = 0.600 \text{ kg}$$

Kinetic energies: $$T_A = \frac{1}{2}mv_A^2 = 0.300\,v_A^2 = 0$$
$$T_B = \frac{1}{2}mv_B^2 = 0.300\,v_B^2$$
$$T_D = \frac{1}{2}mv_D^2 = 0.300\,v_D^2$$

PROBLEM 13.57 (Continued)

(*a*) *Speed of collar at B.*

Conservation of energy: $T_A + V_A = T_B + V_B$

$$0 + 6.5767 = 0.300 v_B^2 + 1.2249$$

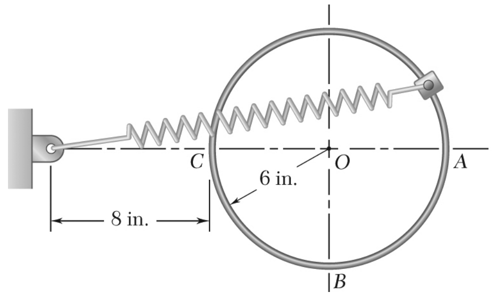
$$v_B^2 = 17.839 \text{ m}^2/\text{s}^2 \qquad v_B = 4.22 \text{ m/s} \blacktriangleleft$$

(*b*) *Speed of collar at D.*

Conservation of energy: $T_A + V_A = T_D + V_D$

$$0 + 6.5767 = 0.300 v_0^2 + 0.7042$$

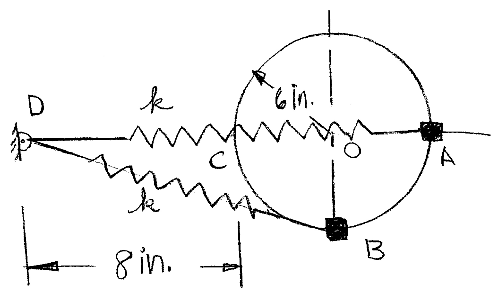
$$v_D^2 = 19.575 \text{ m}^2/\text{s}^2 \qquad v_D = 4.42 \text{ m/s} \blacktriangleleft$$



PROBLEM 13.58

A 3-lb collar is attached to a spring and slides without friction along a circular rod in a *horizontal* plane. The spring has an undeformed length of 7 in. and a constant $k = 1.5$ lb/in. Knowing that the collar is in equilibrium at A and is given a slight push to get it moving, determine the velocity of the collar (a) as it passes through B, (b) as it passes through C.

SOLUTION



$$L_0 = 7 \text{ in.}, L_{DA} = 20 \text{ in.}$$

$$L_{DB} = \sqrt{(8+6)^2 + 6^2} = 15.23 \text{ in.}$$

$$L_{DC} = 8 \text{ in.}$$

$$\Delta L_{DA} = 20 - 7 = 13 \text{ in.}$$

$$\Delta L_{DB} = 15.23 - 7 = 8.23 \text{ in.}$$

$$\Delta L_{DC} = 8 - 7 = 1 \text{ in.}$$

(a)

$$T_A = 0,\ V_A = \frac{1}{2}k(\Delta L_{DA})^2 = \frac{1}{2}(1.5)(13)^2 = 126.75 \text{ lb}\cdot\text{in.}$$

$$= 10.5625 \text{ lb}\cdot\text{ft}$$

$$T_B = \frac{1}{2}mv_B^2 = \frac{1.5}{g}\ v_B^2$$

$$V_B = \frac{1}{2}(1.5)(8.23)^2 = 50.8 \text{ lb}\cdot\text{in.} = 4.233 \text{ lb}\cdot\text{ft}$$

$$T_A + V_A = T_B + V_B: \quad 0 + 10.5625 = \frac{1.5v_B^2}{32.2} + 4.233$$

$v_B = 11.66$ ft/s ◀

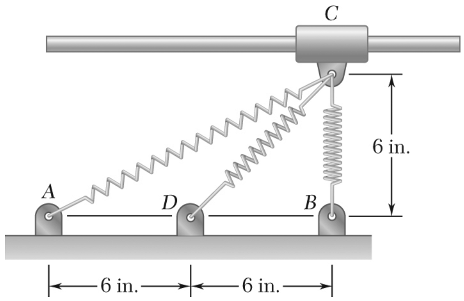
PROBLEM 13.58 (Continued)

(*b*)
$$T_A = 0,\ V_A = 10.5625 \text{ lb}\cdot\text{ft},\ T_C = \frac{1.5}{32.2}\ v_C^2$$

$$V_C = \frac{1}{2}(1.5)(1)^2 = 0.75 \text{ lb}\cdot\text{in.} = 0.0625 \text{ lb}\cdot\text{ft}$$

$$T_A + V_A = T_C + V_C:\quad 0 + 10.5625 = \frac{1.5}{32.2}v_c^2 + 0.0625$$

$v_C = 15.01$ ft/s ◀



PROBLEM 13.59

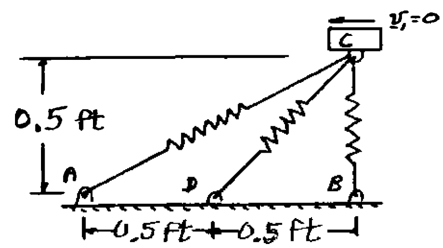
A 3-lb collar C may slide without friction along a horizontal rod. It is attached to three springs, each of constant $k = 2$ lb/in. and 6 in. undeformed length. Knowing that the collar is released from rest in the position shown, determine the maximum speed it will reach in the ensuing motion.

SOLUTION

Maximum velocity occurs at E where collar is passing through position of equilibrium.

Position ①

$$T_1 = 0$$



Note: Undeformed length of springs is 6 in. = 0.5 ft.

Spring AC: $L = \sqrt{(1\text{ ft})^2 + (0.5\text{ ft})^2} = 1.1180$ ft

$\Delta = 1.1180 - 0.50 = 0.6180$ ft

Spring CD: $L = \sqrt{(0.5\text{ ft})^2 + (0.5\text{ ft})^2} = 0.70711$ ft

$\Delta = 0.70711 - 0.50 = 0.20711$ ft

Spring BD: $L = 0.50$ ft, $\Delta = 0$

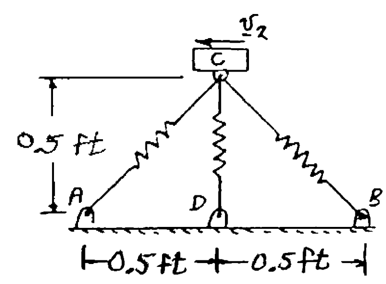
Potential energy. ($k = 2$ lb/in. $= 24$ lb/ft for each spring)

$$V_1 = \Sigma\frac{1}{2}k\Delta^2 = \frac{1}{2}k\Sigma\Delta^2 = \frac{1}{2}(24\text{ lb/ft})[(0.6180\text{ ft})^2 + (0.20711\text{ ft})^2 + 0]$$

$$V_1 = 5.0983\text{ lb}\cdot\text{ft}$$

Position ②

$$m = \frac{3.0\text{ lb}}{32.2\text{ ft/s}^2} = 0.093168\text{ slug};\quad T_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}(0.093168\text{ slug})\,v_2^2$$



Spring AC: $L = \sqrt{(0.5\text{ ft})^2 + (0.5\text{ ft})^2} = 0.7071067$ ft

$\Delta = 0.70711 - 0.50 = 0.20711$ ft

Spring CD: $L = 0.50$ ft

$\Delta = 0$

Spring BC: $L = \sqrt{(0.5\text{ ft})^2 + (0.5\text{ ft})^2} = 0.7071067$ ft

$\Delta = 0.70711 - 0.50 = 0.20711$ ft

PROBLEM 13.59 (Continued)

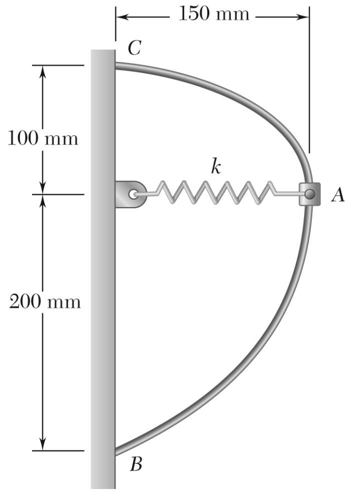
Potential energy. $V_2 = \Sigma \frac{1}{2} k \Delta^2 = \frac{1}{2} k \Sigma \Delta^2$

$$V_2 = \frac{1}{2}(24 \text{ lb/ft})[(0.20711 \text{ ft})^2 + 0 + (0.20711 \text{ ft})^2] = 1.0294 \text{ lb} \cdot \text{ft}$$

Conservation of energy. $T_1 + V_1 = T_2 + V_2$: $\quad 0 + 5.0983 \text{ lb} \cdot \text{ft} = \frac{1}{2}(0.093168 \text{ slug}) v_2^2 + 1.0294 \text{ lb} \cdot \text{ft}$

$$v_2^2 = 87.345$$

$\mathbf{v}_2 = 9.35 \text{ ft/s} \leftrightarrow$ ◀

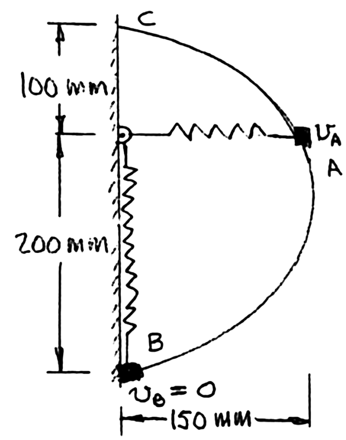


PROBLEM 13.60

A 500-g collar can slide without friction on the curved rod BC in a *horizontal* plane. Knowing that the undeformed length of the spring is 80 mm and that $k = 400\,\text{kN/m}$, determine (a) the velocity that the collar should be given at A to reach B with zero velocity, (b) the velocity of the collar when it eventually reaches C.

SOLUTION

(a) Velocity at A:



$$T_A = \frac{1}{2}mv_A^2 = \left(\frac{0.5}{2}\text{ kg}\right)v_A^2$$

$$T_A = (0.25)v_A^2$$

$$\Delta L_A = 0.150\text{ m} - 0.080\text{ m}$$

$$\Delta L_A = 0.070\text{ m}$$

$$V_A = \frac{1}{2}k(\Delta L_A)^2$$

$$V_A = \frac{1}{2}(400\times 10^3\text{ N/m})(0.070\text{ m})^2$$

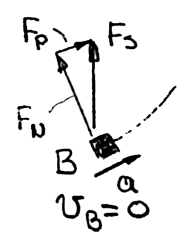
$$V_A = 980\text{ J}$$

$$v_B = 0 \qquad T_B = 0$$

$$\Delta L_B = 0.200\text{ m} - 0.080\text{ m} = 0.120\text{ m}$$

$$V_B = \frac{1}{2}k(\Delta L_B)^2 = \frac{1}{2}(400\times 10^3\text{ N/m})(0.120\text{ m})^2$$

$$V_B = 2880\text{ J}$$



Substitute into conservation of energy.

$$T_A + V_A = T_B + V_B \qquad 0.25v_A^2 + 980 = 0 + 2880$$

$$v_A^2 = \frac{(2880 - 980)}{(0.25)}$$

$$v_A^2 = 7600\text{ m}^2/\text{s}^2$$

$$v_A = 87.2\text{ m/s} \blacktriangleleft$$

PROBLEM 13.60 (Continued)

(*b*) Velocity at *C*:

Since slope at *B* is positive, the component of the spring force F_P, parallel to the rod, causes the block to move back toward *A*.

$$T_B = 0, \quad V_B = 2880 \text{ J} \quad \text{[from part } (a)\text{]}$$

$$T_C = \frac{1}{2}mv_C^2 = \frac{(0.5 \text{ kg})}{2}v_C^2 = 0.25v_C^2$$

$$\Delta L_C = 0.100 \text{ m} - 0.080 \text{ m} = 0.020 \text{ m}$$

$$V_C = \frac{1}{2}k(\Delta L_C)^2 = \frac{1}{2}(400\times 10^3 \text{ N/m})(0.020 \text{ m})^2 = 80.0 \text{ J}$$

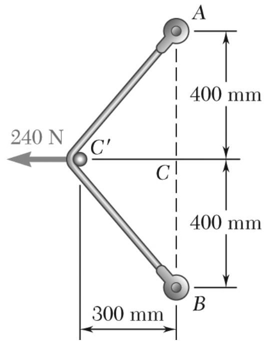
Substitute into conservation of energy.

$$T_B + V_B = T_C + V_C \quad 0 + 2880 = 0.25v_C^2 + 80.0$$

$$v_C^2 = 11,200 \text{ m}^2/\text{s}^2$$

$$v_C = 105.8 \text{ m/s} \blacktriangleleft$$

PROBLEM 13.61



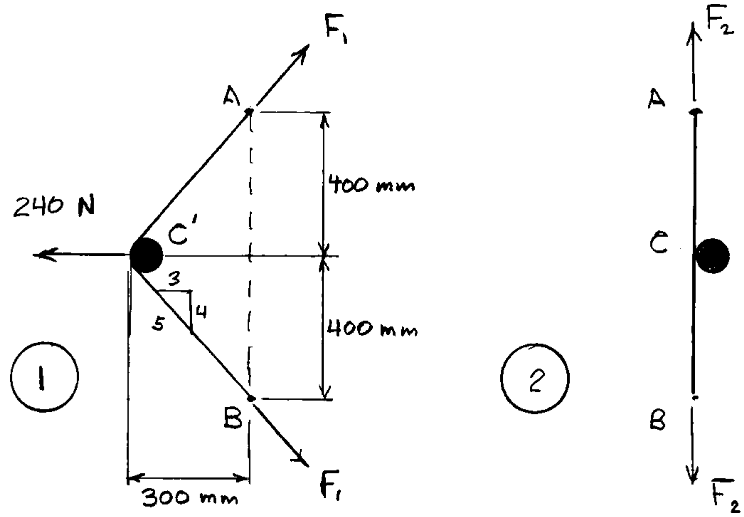
An elastic cord is stretched between two Points A and B, located 800 mm apart in the same horizontal plane. When stretched directly between A and B, the tension is 40 N. The cord is then stretched as shown until its midpoint C has moved through 300 mm to C'; a force of 240 N is required to hold the cord at C'. A 0.1 kg pellet is placed at C', and the cord is released. Determine the speed of the pellet as it passes through C.

SOLUTION

Let ℓ = undeformed length of cord.

Position 1. Length $AC'B = 1.0$ m; Elongation $= x_1 = 1.0 - \ell$

$$\Sigma F_x = 0: \quad 2\left(\frac{3}{5}F_1\right) - 240 \text{ N} = 0 \quad F_1 = 200 \text{ N}$$



Position 2. Length $ACB = 0.8$ m; Elongation $= x_2 = 0.8 - \ell$

Given $F_2 = 40$ N

$$F_1 = kx, \quad F_2 = kx_2$$

$$F_1 - F_2 = k(x_1 - x_2)$$

$$200 - 40 = k[(1.0 - \ell) - (0.8 - \ell)] = 0.2k$$

$$k = \frac{160}{0.2} = 800 \text{ N/m}$$

$$x_1 = \frac{F_1}{k} = \frac{200\text{ N}}{800\text{ N/m}} = 0.25\text{ m}$$

$$x_2 = \frac{F_2}{k} = \frac{40\text{ N}}{800\text{ N/m}} = 0.05\text{ m}$$

Position ①: $T_1 = 0 \qquad V_1 = \frac{1}{2}kx_1^2 = \frac{1}{2}(800\text{ N/m})(0.25\text{ m})^2 = 25.0\text{ N}\cdot\text{m}$

Position②: $m = 0.10\text{ kg}$

$$T_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}(0.1\text{ kg})v_2^2 = 0.05v_2^2$$

$$V_2 = \frac{1}{2}kx_2^2 = \frac{1}{2}(800\text{ N/m})(0.05\text{ m})^2 = 1\text{ N}\cdot\text{m}$$

Conservation of energy:

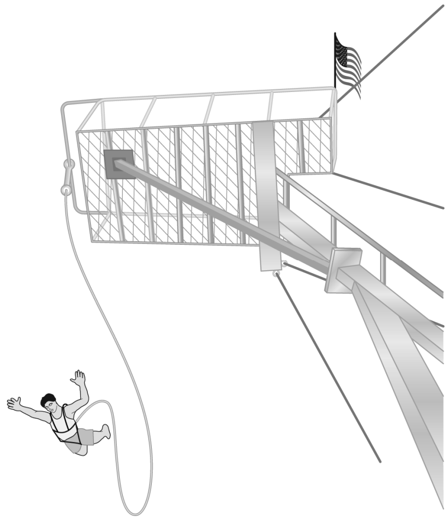
$$T_1 + V_1 = T_2 + V_2$$

$$0 + 25.0\text{ N}\cdot\text{m} = 0.05v_2^2 + 1.0\text{ N}\cdot\text{m}$$

$$24.0 = 0.05v_2^2 \qquad v_2 = 21.909\text{ m/s}$$

$v_2 = 21.9\text{ m/s}$ ◀

Note: The horizontal force applied at the midpoint of the cord is not proportional to the horizontal distance $C'C$. A solution based on the work of the horizontal force would be rather involved.



PROBLEM 13.62

An elastic cable is to be designed for bungee jumping from a tower 130 ft high. The specifications call for the cable to be 85 ft long when unstretched, and to stretch to a total length of 100 ft when a 600-lb weight is attached to it and dropped from the tower. Determine (*a*) the required spring constant k of the cable, (*b*) how close to the ground a 186-lb man will come if he uses this cable to jump from the tower.

SOLUTION

(*a*) Conservation of energy:

$$V_1 = 0 \quad T_1 = 0 \quad V_1 = 100\,W$$

Datum at ②:

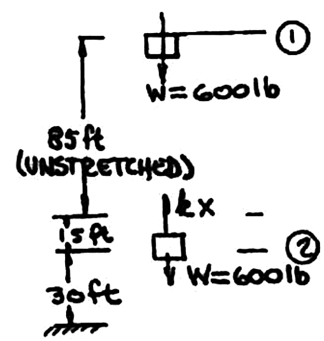
$$V_1 = (100\text{ ft})(600\text{ lb})$$
$$= 6\times 10^4\text{ ft}\cdot\text{lb}$$

$$V_2 = 0 \quad T_2 = 0$$

$$V_2 = V_g + V_e = 0 + \frac{1}{2}k\,(15\text{ ft})^2$$

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 6\times 10^4 = 0 + (112.5)k$$



$k = 533$ lb/ft ◀

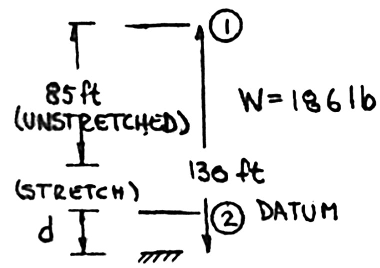
(*b*) From (*a*), $k = 533$ lb/ft

$$T_1 = 0$$
$$W = 186\text{ lb}$$
$$V_1 = (186)(130 - d)$$
$$T_2 = 0$$

Datum:

$$V_2 = V_g + V_e = 0 + \frac{1}{2}(533)(130 - 85 - d)^2$$

$$V_2 = (266.67)(45 - d)^2$$



PROBLEM 13.62 (Continued)

$d =$ distance from the ground $\qquad T_1 + V_1 = T_2 + V_2$

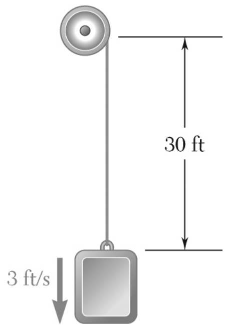
$$0 + (186)(130 - d) = 0 + (266.67)(45 - d)^2$$

$$266.7d^2 - 23815d + 515827 = 0$$

$$d = \frac{23815 \mp \sqrt{(23815)^2 - 4(266.7)(515827)}}{(2)(266.7)} = \begin{matrix} 36.99 \text{ ft} \\ 52.3 \text{ ft} \end{matrix}$$

Discard 52.3 ft (since the cord acts in compression when rebound occurs).

$d = 37.0$ ft ◀



PROBLEM 13.63

It is shown in mechanics of materials that the stiffness of an elastic cable is $k = AE/L$ where A is the cross sectional area of the cable, E is the modulus of elasticity and L is the length of the cable. A winch is lowering a 4000-lb piece of machinery using at a constant speed of 3ft/s when the winch suddenly stops. Knowing that the steel cable has a diameter of 0.4 in., $E = 29 \times 10^6$ lb/in^2, and when the winch stops $L = 30$ ft, determine the maximum downward displacement of the piece of machinery from the point it was when the winch stopped.

SOLUTION

Mass of machinery:
$$m = \frac{W}{g} = \frac{4000}{32.2} = 124.22 \text{ lb}\cdot\text{s}^2/\text{ft}$$

Let position 1 be the state just before the winch stops and the gravitational potential V_g be equal to zero at this state.

For the cable,
$$A = \frac{\pi}{4}(\text{diameter})^2 = \frac{\pi}{4}(0.4 \text{ in.})^2 = 0.12566 \text{ in}^2$$

$$AE = (0.12556 \text{ in.}^2)(29\times10^6 \text{ lb/in}^2) = 3.6442\times10^6 \text{ lb}$$

For
$$L = 30 \text{ ft}, \quad k = \frac{AE}{L} = \frac{3.6442\times10^6 \text{ lb}}{30 \text{ ft}} = 121.47\times10^3 \text{ lb/ft}$$

Initial force in cable (equilibrium):
$$F_1 = W = 4000 \text{ lb.}$$

Elongation in position 1:
$$x_1 = \frac{F_1}{k} = \frac{4000}{121.47\times10^3} = 0.03293 \text{ ft}$$

Potential energy:
$$V_1 = \frac{1}{2}kx_1^2 = \frac{F_1^2}{2k}$$

$$V_1 = \frac{(4000 \text{ lb})^2}{(2)(121.47\times10^3 \text{ lb/ft})} = 65.860 \text{ ft}\cdot\text{lb}$$

Kinetic energy:
$$T_1 = \frac{1}{2}mv_1^2$$

$$T_1 = \frac{1}{2}(124.22 \text{ lb}\cdot\text{s}^2/\text{ft})(3 \text{ ft/s}^2) = 558.99 \text{ ft}\cdot\text{lb}$$

PROBLEM 13.63 (Continued)

Let position 2 be the position of maximum downward displacement. Let x_2 be the elongation in this position.

Potential energy: $$V_2 = \frac{1}{2}kx_2^2 - W(x_2 - x_1)$$

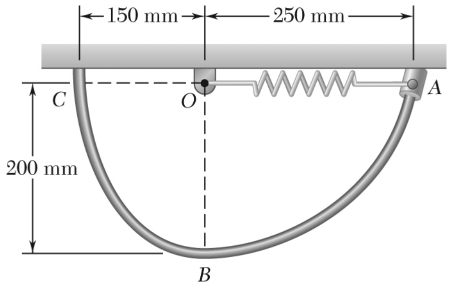
$$V_2 = \frac{1}{2}(121.47\times10^3)x_2^2 - (4000)(x_2 - 0.03293)$$
$$= 60.735\times10^3 x^2 - 4000x_2 + 131.72$$

Kinetic energy: $T_2 = 0 \qquad (\text{since } v_2 = 0)$

Principle of work and energy: $T_1 + V_1 = T_2 + V_2$

$$558.99 + 65.860 = 60.735\times10^3 x_2^2 - 4000x_2 + 131.72$$
$$60.735\times10^3 x_2^2 - 4000x_2 - 493.13 = 0$$
$$x_2 = 0.12887 \text{ ft}$$

Maximum displacement: $\delta = x_2 - x_1 = 0.09594$ ft $\qquad \delta = 1.151$ in.



PROBLEM 13.64

A 2-kg collar is attached to a spring and slides without friction in a vertical plane along the curved rod ABC. The spring is undeformed when the collar is at C and its constant is 600 N/m. If the collar is released at A with no initial velocity, determine its velocity (a) as it passes through B, (b) as it reaches C.

SOLUTION

Spring elongations:

At A, $x_A = 250 \text{ mm} - 150 \text{ mm} = 100 \text{ mm} = 0.100 \text{ m}$

At B, $x_B = 200 \text{ mm} - 150 \text{ mm} = 50 \text{ mm} = 0.050 \text{ m}$

At C, $x_C = 0$

Potential energies for springs.

$$(V_A)_e = \frac{1}{2}kx_A^2 = \frac{1}{2}(600)(0.100)^2 = 3.00 \text{ J}$$

$$(V_B)_e = \frac{1}{2}kx_B^2 = \frac{1}{2}(600)(0.050)^2 = 0.75 \text{ J}$$

$$(V_C)_e = 0$$

Gravitational potential energies: Choose the datum at level AOC.

$$(V_A)_g = (V_C)_g = 0$$

$$(V_B)_g = -mg\,y = -(2)(9.81)(0.200) = -3.924 \text{ J}$$

Kinetic energies: $T_A = 0$

$$T_B = \frac{1}{2}mv_B^2 = 1.00\,v_B^2$$

$$T_C = \frac{1}{2}mv_C^2 = 1.00\,v_C^2$$

(a) *Velocity as the collar passes through B.*

Conservation of energy: $T_A + V_A = T_B + V_B$

$$0 + 3.00 + 0 = 1.00\,v_B^2 + 0.75 - 3.924$$

$$v_B^2 = 6.174 \text{ m}^2/\text{s}^2$$

$\mathbf{v}_B = 2.48 \text{ m/s} \longleftarrow$ ◀

PROBLEM 13.64 (Continued)

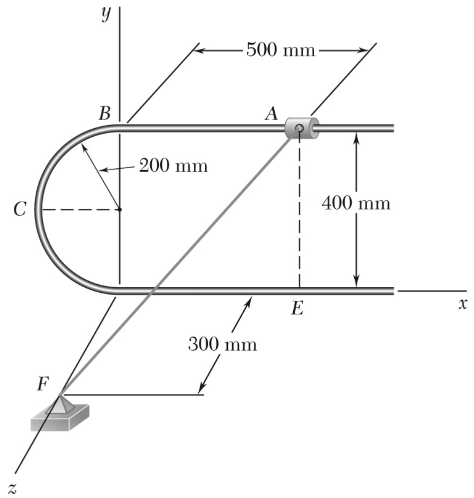
(*b*) *Velocity as the collar reaches C.*

Conservation of energy: $T_A + V_A = T_C + V_C$

$$0 + 3.00 + 0 = 1.00 v_C^2 + 0 + 0$$

$$v_C^2 = 3.00 \text{ m}^2/\text{s}^2$$

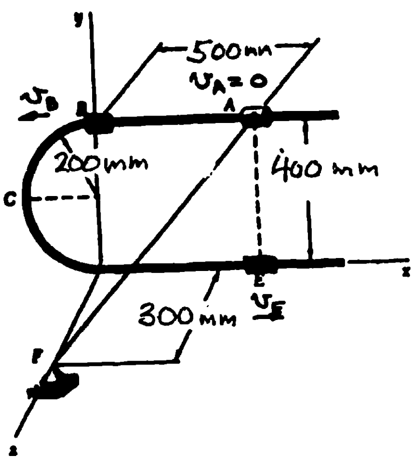
$\mathbf{v}_C = 1.732 \text{ m/s} \uparrow$ ◀



PROBLEM 13.65

A 1-kg collar can slide along the rod shown. It is attached to an elastic cord anchored at F, which has an undeformed length of 250 mm and a spring constant of 75 N/m. Knowing that the collar is released from rest at A and neglecting friction, determine the speed of the collar (a) at B, (b) at E.

SOLUTION



$$L_{AF} = \sqrt{(0.5)^2 + (0.4)^2 + (0.3)^2}$$
$$L_{AF} = 0.70711 \text{ m}$$
$$L_{BF} = \sqrt{(0.4)^2 + (0.3)^2}$$
$$L_{BF} = 0.5 \text{ m}$$
$$L_{FE} = \sqrt{(0.5)^2 + (0.3)^2}$$
$$L_{FE} = 0.58309 \text{ m}$$
$$V = V_e + V_g$$

(a) Speed at B: $\quad v_A = 0, \qquad T_A = 0$

Point A:

$$(V_A)_e = \frac{1}{2}k(\Delta L_{AF})^2 \qquad \Delta L_{AF} = L_{AF} - L_0 = 0.70711 - 0.25$$
$$\Delta L_{AF} = 0.45711 \text{ m}$$
$$(V_A)_e = \frac{1}{2}(75 \text{ N/m})(0.45711 \text{ m})^2$$
$$(V_A)_e = 7.8355 \text{ N} \cdot \text{m}$$
$$(V_A)_g = (mg)(0.4)$$
$$= (1.0 \text{ kg})(9.81 \text{ m/s}^2)(0.4 \text{ m})$$
$$= 3.9240 \text{ N} \cdot \text{m}$$
$$V_A = (V_A)_e + (V_A)_g$$
$$= 7.8355 + 3.9240$$
$$= 11.7595 \text{ N} \cdot \text{m}$$

PROBLEM 13.65 (Continued)

Point B: $T_B = \frac{1}{2}mv_B^2 = \frac{1}{2}(1.0 \text{ kg})v_B^2$

$$T_B = 0.5v_B^2$$

$$(V_B)_e = \frac{1}{2}k(\Delta L_{BF})^2 \qquad \Delta L_{BF} = L_{BF} - L_0 = 0.5 - 0.25$$

$$\Delta L_{BF} = 0.25 \text{ m}$$

$$(V_B)_e = \frac{1}{2}(75 \text{ N/m})(0.25 \text{ m})^2 = 2.3438 \text{ N}\cdot\text{m}$$

$$(V_B)_g = (mg)(0.4) = (1.0 \text{ kg})(9.81 \text{ m/s}^2)(0.4 \text{ m}) = 3.9240 \text{ N}\cdot\text{m}$$

$$V_B = (v_B)_e + (V_B)_g = 2.3438 + 3.9240 = 6.2678 \text{ N}\cdot\text{m}$$

$$T_A + V_A = T_B + V_B$$

$$0 + 11.7595 = 0.5v_B^2 + 6.2678$$

$$v_B^2 = (5.49169)/(0.5)$$

$$v_B^2 = 10.983 \text{ m}^2/\text{s}^2$$

$v_B = 3.31$ m/s ◀

(*b*) Speed at E:

Point A: $T_A = 0 \qquad V_A = 11.7595 \text{ N}\cdot\text{m}$ (from part (*a*))

Point E:

$$T_E = \frac{1}{2}mv_E^2 = \frac{1}{2}(1.0 \text{ kg})v_E^2 = 0.5v_E^2$$

$$(V_E)_e = \frac{1}{2}k(\Delta L_{FE})^2 \qquad \Delta L_{FE} = L_{FE} - L_0 = 0.5831 - 0.25$$

$$\Delta L_{FE} = 0.3331 \text{ m}$$

$$(V_E)_e = \frac{1}{2}(75 \text{ N/m})(0.3331 \text{ m})^2 = 4.1607 \text{ N}\cdot\text{m}$$

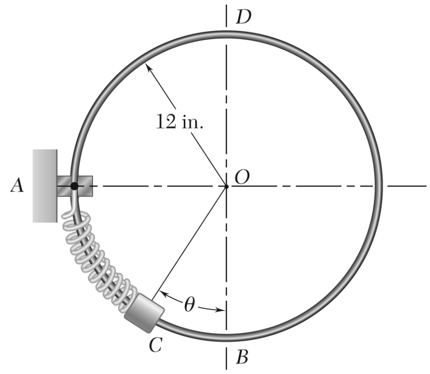
$$(V_E)_g = 0 \quad V_E = 4.1607 \text{ N}\cdot\text{m}$$

$$T_A + V_A = T_E + V_E \quad 0 + 11.7595 = 0.5v_E^2 + 4.1607$$

$$v_E^2 = 7.5988/0.5$$

$$v_E^2 = 15.1976 \text{ m}^2/\text{s}^2$$

$v_E = 3.90$ m/s ◀

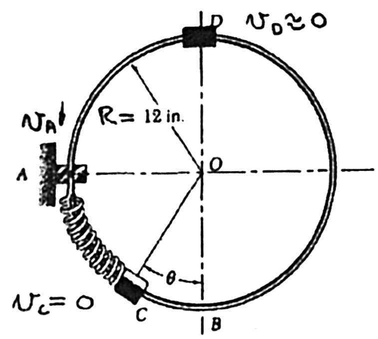


PROBLEM 13.66

A thin circular rod is supported in a *vertical plane* by a bracket at *A*. Attached to the bracket and loosely wound around the rod is a spring of constant $k = 3$ lb/ft and undeformed length equal to the arc of circle *AB*. An 8-oz collar *C*, not attached to the spring, can slide without friction along the rod. Knowing that the collar is released from rest at an angle θ with the vertical, determine (*a*) the smallest value of θ for which the collar will pass through *D* and reach Point *A*, (*b*) the velocity of the collar as it reaches Point *A*.

SOLUTION

(*a*) Smallest angle θ occurs when the velocity at *D* is close to zero.



$$v_C = 0 \qquad v_D = 0$$
$$T_C = 0 \qquad T_D = 0$$
$$V = V_e + V_g$$

$R = 12 \text{ in.} = 1 \text{ ft}$

Point *C*:

$$\Delta L_{BC} = (1 \text{ ft})(\theta) = \theta \text{ ft}$$
$$(V_C)_e = \frac{1}{2}k(\Delta L_{BC})^2$$
$$(V_C)_e = \frac{3}{2}\theta^2$$
$$(V_C)_g = WR(1-\cos\theta)$$
$$(V_C)_g = \left(\frac{8 \text{ oz}}{16 \text{ oz/lb}}\right)(1 \text{ ft})(1-\cos\theta)$$
$$(V_C)_g = \frac{1}{2}(1-\cos\theta)$$
$$V_C = (V_C)_e + (V_C)_g = \frac{3}{2}\theta^2 + \frac{1}{2}(1-\cos\theta)$$

Point *D*:

$$(V_D)_e = 0 \quad \text{(spring is unattached)}$$
$$(V_D)_g = W(2R) = (2)(0.5 \text{ lb})(1 \text{ ft}) = 1 \text{ lb}\cdot\text{ft}$$
$$T_C + V_C = T_D + V_D \qquad 0 + \frac{3}{2}\theta^2 + \frac{1}{2}(1-\cos\theta) = 1$$
$$(1.5)\theta^2 - (0.5)\cos\theta = 0.5$$

By trial, $\theta = 0.7592$ rad $\qquad \theta = 43.5° \blacktriangleleft$

PROBLEM 13.66 (Continued)

(*b*) Velocity at *A*:

Point *D*:

$$V_D = 0 \quad T_D = 0 \quad V_D = 1 \text{ lb} \cdot \text{ft} \,[\text{see Part } (a)]$$

Point *A*:

$$T_A = \frac{1}{2} m v_A^2 = \frac{1}{2} \frac{(0.5 \text{ lb})}{(32.2 \text{ ft/s}^2)} v_A^2$$

$$T_A = 0.0077640 v_A^2$$

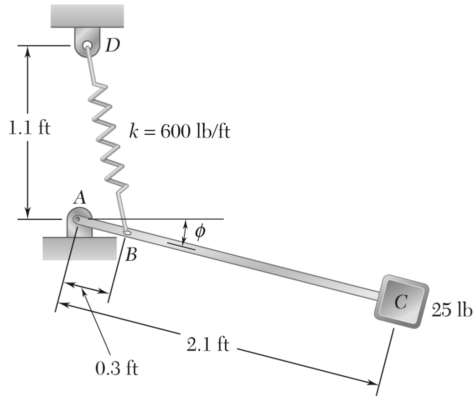
$$V_A = (V_A)_g = W(R) = (0.5 \text{ lb})(1 \text{ ft}) = 0.5 \text{ lb} \cdot \text{ft}$$

$$T_A + V_A = T_D + V_D$$

$$0.0077640 v_A^2 + 0.5 = 0 + 1$$

$$v_A^2 = 64.4 \text{ ft}^2/\text{s}^2$$

$\mathbf{v}_A = 8.02 \text{ ft/s} \downarrow$ ◀

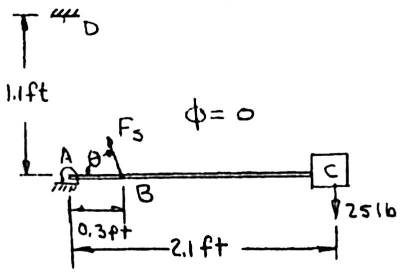


PROBLEM 13.67

The system shown is in equilibrium when $\phi = 0$. Knowing that initially $\phi = 90°$ and that block C is given a slight nudge when the system is in that position, determine the velocity of the block as it passes through the equilibrium position $\phi = 0$. Neglect the weight of the rod.

SOLUTION

Find the unstretched length of the spring.



$$\theta = \tan^{-1}\frac{1.1}{0.3}$$
$$= 1.3045 \text{ rad}$$
$$\theta = 74.745°$$
$$L_{BD} = \sqrt{(1.1)^2 + .3^2}$$
$$L_{BD} = 1.140 \text{ ft}$$

Equilibrium $\qquad \Sigma M_A = (0.3)(F_s \sin\theta) - (25)(2.1) = 0$

$$F_s = \frac{(25 \text{ lb})(2.1 \text{ ft})}{(0.3 \text{ ft})(\sin 74.745°)}$$
$$= 181.39 \text{ lb}$$
$$F_s = k\Delta L_{BD}$$

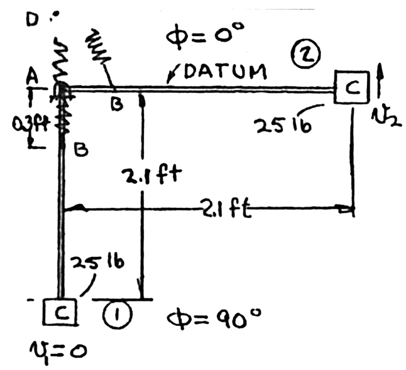
$$181.39 \text{ lb} = (600 \text{ lb/ft})(\Delta L_{BD})$$
$$\Delta L_{BD} = 0.30232 \text{ ft}$$

Unstretched length $\qquad L_0 = L_{BD} - \Delta L_{BD}$

$$L_0 = 1.140 - 0.3023$$
$$= 0.83768 \text{ ft}$$

Spring elongation, $\Delta L'_{BD}$, when $\phi = 90°$.

$$\Delta L'_{BD} = (1.1 \text{ ft} + 0.3 \text{ ft}) - L_0$$
$$\Delta L'_{BD} = 1.4 \text{ ft} - 0.8377 \text{ ft}$$
$$= 0.56232 \text{ ft}$$



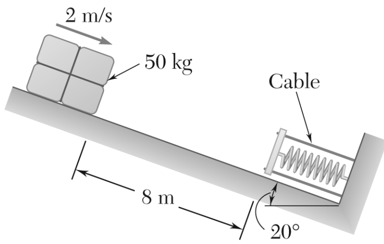
At ①, $(\phi = 90°)$

$$v_1 = 0, \quad T_1 = 0$$
$$V_1 = (V_1)_e + (V_1)_g$$
$$(V_1)_e = \frac{1}{2}k(\Delta L'_{BD})^2$$
$$(V_1)_e = \frac{1}{2}(600 \text{ lb/ft})(0.5623 \text{ ft})^2$$
$$(V_1)_e = 94.86 \text{ lb} \cdot \text{ft}$$
$$(V_1)_g = -(25 \text{ lb})(2.1 \text{ ft}) = -52.5 \text{ ft} \cdot \text{lb}$$
$$V_1 = 94.86 - 52.5 = 42.36 \text{ ft} \cdot \text{lb}$$

At ②, $(\phi = 0°)$

$$(V_2)_e = \frac{1}{2}k(\Delta L_{BD})^2 = \frac{1}{2}(600 \text{ lb/ft})(0.3023 \text{ ft})^2$$
$$(V_2)_e = 27.42 \text{ lb} \cdot \text{ft}$$
$$(V_2)_g = 0 \qquad V_2 = 27.42 \text{ ft} \cdot \text{lb}$$
$$T_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}\left(\frac{25 \text{ lb}}{32.2 \text{ ft/s}^2}\right)v_2^2 = 0.3882 v_2^2$$
$$T_1 + V_1 = T_2 + V_2$$
$$0 + 42.36 = 0.3882 v_2^2 + 27.42$$
$$v_2^2 = (14.941)/(0.3882)$$
$$v_2^2 = 38.48 \text{ ft}^2/\text{s}^2$$

$v_2 = 6.20 \text{ ft/s}$ ◀



PROBLEM 13.68

A spring is used to stop a 50-kg package which is moving down a 20º incline. The spring has a constant $k = 30$ kN/m and is held by cables so that it is initially compressed 50 mm. Knowing that the velocity of the package is 2 m/s when it is 8 m from the spring and neglecting friction, determine the maximum additional deformation of the spring in bringing the package to rest.

SOLUTION

Let position 1 be the starting position 8 m from the end of the spring when it is compressed 50 mm by the cable. Let position 2 be the position of maximum compression. Let x be the additional compression of the spring. Use the principle of conservation of energy. $T_1 + V_1 = T_2 + V_2$.

Position 1:
$$T_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(50)(2)^2 = 100 \text{ J}$$
$$V_{1g} = mgh_1 = (50)(9.81)(8\sin 20°) = 1342.09 \text{ J}$$
$$V_{1e} = \frac{1}{2}ke_1^2 = \frac{1}{2}(30\times 10^3)(0.050)^2 = 37.5 \text{ J}$$

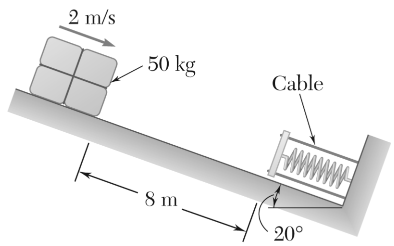
Position 2:
$$T_2 = \frac{1}{2}mv_2^2 = 0 \quad \text{since} \quad v_2 = 0.$$
$$V_{2g} = mgh_2 = (50)(9.81)(-x\sin 20°) = -167.76x$$
$$V_{2e} = \frac{1}{2}ke_2^2 = \frac{1}{2}(30\times 10^3)(0.05 + x)^2 = 37.5 + 1500x + 15000x^2$$

Principle of conservation of energy:
$$100 + 1342.09 + 37.5 = -167.61x + 37.5 + 1500x + 15000x^2$$
$$15,000x^2 + 1332.24x - 1442.09 = 0$$

Solving for x,
$$x = 0.26882 \quad \text{and} \quad -0.357\,64$$

$x = 0.269$ m ◀

PROBLEM 13.69



Solve Problem 13.68 assuming the kinetic coefficient of friction between the package and the incline is 0.2.

PROBLEM 13.68 A spring is used to stop a 50-kg package which is moving down a 20° incline. The spring has a constant $k = 30$ kN/m and is held by cables so that it is initially compressed 50 mm. Knowing that the velocity of the package is 2 m/s when it is 8 m from the spring and neglecting friction, determine the maximum additional deformation of the spring in bringing the package to rest.

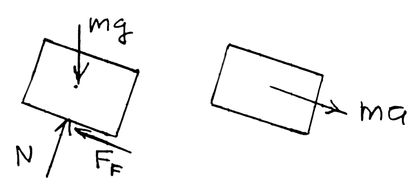
SOLUTION

Let position 1 be the starting position 8 m from the end of the spring when it is compressed 50 mm by the cable. Let position 2 be the position of maximum compression. Let x be the additional compression of the spring. Use the principle of work and energy. $T_1 + V_1 + U_{1\to 2} = T_2 + V_2$

Position 1.
$$T_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(50)(2)^2 = 100\ \text{J}$$
$$V_{1g} = mgh_1 = (50)(9.81)(8\sin 20°) = 1342.09\ \text{J}$$
$$V_{1e} = \frac{1}{2}ke_1^2 = \frac{1}{2}(30\times 10^3)(0.05)^2 = 37.5\ \text{J}$$

Position 2.
$$T_2 = \frac{1}{2}mv_2^2 = 0 \quad \text{since} \quad v_2 = 0.$$
$$V_{2g} = mgh_2 = (50)(9.81)(-x\sin 20°) = -167.76x$$
$$V_{2e} = \frac{1}{2}ke_2^2 = \frac{1}{2}(30\times 10^3)(0.05+x)^2 = 37.5 + 1500x + 15{,}000x^2$$

Work of the friction force.
$$+\!\nearrow\ \Sigma F_n = 0$$
$$N - mg\ \cos 20° = 0$$
$$\begin{aligned} N &= mg\cos 20° \\ &= (50)(9.81)\cos 20° \\ &= 460.92\ \text{N} \\ F_f &= \mu_k N \\ &= (0.2)(460.92) \\ &= 92.184 \\ U_{1\to 2} &= -F_f d \\ &= -92.184(8+x) \\ &= -737.47 - 92.184x \end{aligned}$$



PROBLEM 13.69 (Continued)

Principle of work and energy: $T_1 + V_1 + U_{1-2} = T_2 + V_2$

$$100 + 1342.09 + 37.5 - 737.47 - 92.184x$$

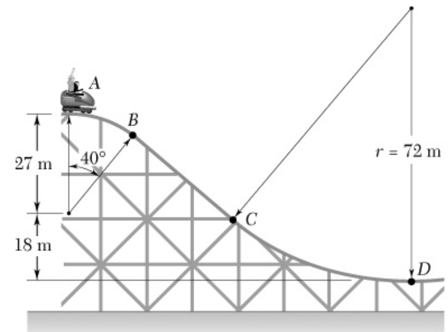
$$= -167.76x + 37.5 + 1500x + 15,000x^2$$

$$15,000\,x^2 + 1424.42x - 704.62 = 0$$

Solving for x,

$$x = 0.17440 \quad \text{and} \quad -0.26936$$

$x = 0.1744$ m ◀



PROBLEM 13.70

A section of track for a roller coaster consists of two circular arcs AB and CD joined by a straight portion BC. The radius of AB is 27 m and the radius of CD is 72 m. The car and its occupants, of total mass 250 kg, reach Point A with practically no velocity and then drop freely along the track. Determine the normal force exerted by the track on the car as the car reaches point B. Ignore air resistance and rolling resistance.

SOLUTION

Calculate the speed of the car as it reaches Point B using the principle of conservation of energy as the car travels from position A to position B.

Position A: $\quad v_A = 0, \qquad T_A = \dfrac{1}{2}mv_A^2 = 0, \quad V_A = 0 \text{ (datum)}$

Position B: $\quad V_B = -mgh$

where h is the decrease in elevation between A and B.

$$T_B = \frac{1}{2}mv_B^2$$

Conservation of energy: $\quad T_A + V_A = T_B + V_B$:

$$0 + 0 = \frac{1}{2}mv_B^2 - mgh$$

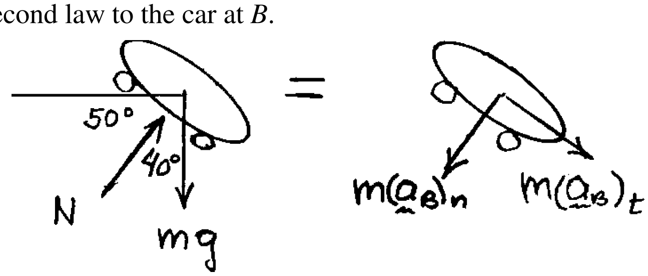
$$\begin{aligned} v_B^2 &= 2gh \\ &= (2)(9.81 \text{ m/s}^2)(27 \text{ m})(1 - \cos 40°) \\ &= 123.94 \text{ m}^2/\text{s}^2 \end{aligned}$$

Normal acceleration at B:

$$(a_B)_n = \frac{v_B^2}{\rho} = \frac{123.94 \text{ m}^2/\text{s}^2}{27 \text{ m}} = 4.59 \text{ m/s}^2$$

$$(\mathbf{a}_B)_n = 4.59 \text{ m/s}^2 \measuredangle 50°$$

Apply Newton's second law to the car at B.



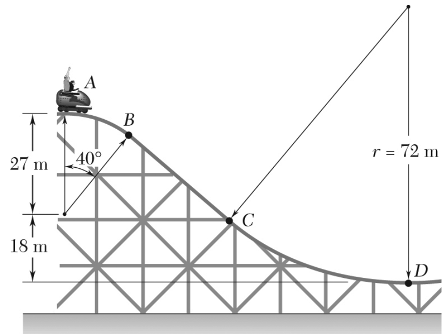
PROBLEM 13.70 (Continued)

$$\measuredangle 50° \Sigma F_n = ma_n: \quad N - mg\cos 40° = -ma_n$$

$$N = mg\cos 40° - ma_n = m(g\cos 40° - a_n)$$

$$= (250 \text{ kg})[(9.81 \text{ m/s}^2)\cos 40° - 4.59 \text{ m/s}^2]$$

$$= 1878.7 - 1147.5 \qquad N = 731 \text{ N} \blacktriangleleft$$



PROBLEM 13.71

A section of track for a roller coaster consists of two circular arcs AB and CD joined by a straight portion BC. The radius of AB is 27 m and the radius of CD is 72 m. The car and its occupants, of total mass 250 kg, reach Point A with practically no velocity and then drop freely along the track. Determine the maximum and minimum values of the normal force exerted by the track on the car as the car travels from A to D. Ignore air resistance and rolling resistance.

SOLUTION

Calculate the speed of the car as it reaches Point P, any point on the roller coaster track. Apply the principle of conservation of energy.

Position A: $\quad v_A = 0, \quad T_A = \frac{1}{2}mv_A^2 = 0, \quad V_A = 0$ (datum)

Position P: $\quad V_P = -mgh$

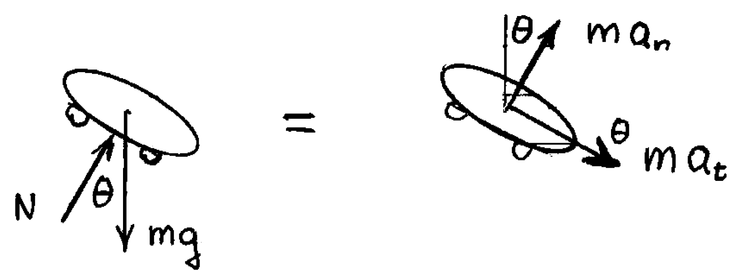
where h is the decrease in elevation along the track.

$$T_P = \frac{1}{2}\, mv^2$$

Conservation of energy: $\quad T_A + V_A = T_P + V_P$

$$0 + 0 = \frac{1}{2}mv^2 - mgh \qquad v^2 = 2gh \tag{1}$$

Calculate the normal force using Newton's second law. Let θ be the slope angle of the track.



$$\Sigma F_n = ma_n: \quad N - mg\cos\theta = ma_n$$

$$N = mg\cos\theta + ma_n \tag{2}$$

Over portion AB of the track, $\quad h = \rho(1-\cos\theta)$

and $\quad a_n = -\dfrac{mv^2}{\rho}$

where ρ is the radius of curvature. $(\rho = 27 \text{ m})$

$$N = mg\cos\theta - \frac{2mg\rho(1-\cos\theta)}{\rho} = mg(3\cos\theta - 2)$$

At Point A $(\theta = 0)$ $\quad N_A = mg = (250)(9.81) = 2452.5 \text{ N}$

At Point B $(\theta = 40°)$ $\quad N_B = (2452.5)(3\cos 40° - 2)$

$$N_B = 731 \text{ N}$$

Over portion BC, $\quad \theta = 40°, \quad a_n = 0 \quad$ (straight track)

$$N_{BC} = mg\cos 40° = 2452.5\cos 40°$$
$$N_{BC} = 1879 \text{ N}$$

Over portion CD, $\quad h = h_{\max} - r(1-\cos\theta)$

and $\quad a_n = \dfrac{mv^2}{r}$

where r is the radius of curvature. $(r = 72 \text{ m})$

$$\begin{aligned} N &= mg\cos\theta + \frac{2mgh}{r} \\ &= mg\cos\theta + 2mg\left(\frac{h_{\max}}{r} - 1 - \cos\theta\right) \\ &= mg\left(3\cos\theta - 2 + \frac{2h_{\max}}{r}\right) \end{aligned}$$

which is maximum at Point D, where

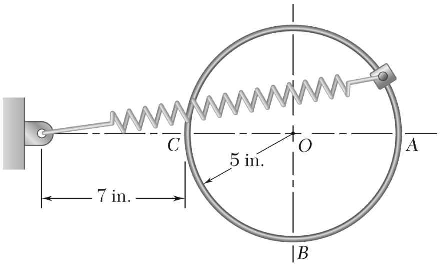
$$N_D = mg\left(1 + \frac{2h_{\max}}{r}\right)$$

Data: $\quad h_{\max} = 27 + 18 = 45 \text{ m}, \quad r = 72 \text{ m}$

$$N_D = (2452.5)\left[1 + \frac{(2)(45)}{72}\right] = 5520 \text{ N}$$

Summary: $\quad$ minimum (just above B): $\quad$ 731 N ◀

maximum (at D): $\quad$ 5520 N ◀



PROBLEM 13.72

A 1-lb collar is attached to a spring and slides without friction along a circular rod in a *vertical* plane. The spring has an undeformed length of 5 in. and a constant $k = 10$ lb/ft. Knowing that the collar is released from being held at A determine the speed of the collar and the normal force between the collar and the rod as the collar passes through B.

SOLUTION

For the collar, $$m = \frac{W}{g} = \frac{1}{32.2} = 0.031056 \text{ lb} \cdot \text{s}^2/\text{ft}$$

For the spring, $$k = 10 \text{ lb/ft} \quad l_0 = 5 \text{ in.}$$

At A: $$\ell_A = 7 + 5 + 5 = 17 \text{ in.}$$
$$\ell_\Delta - \ell_0 = 12 \text{ in.} = 1 \text{ ft}$$

At B: $$\ell_B = \sqrt{(7+5)^2 + 5^2} = 13 \text{ in.}$$
$$\ell_B - \ell_0 = 1.8 \text{ in.} = \frac{2}{3} \text{ ft}$$

Velocity of the collar at B.

Use the principle of conservation of energy.

$$T_A + V_A = T_B + V_B$$

Where $$T_A = \frac{1}{2} m v_A^2 = 0$$

$$V_A = \frac{1}{2} k(\ell_A - \ell_0)^2 + W(0)$$
$$= \frac{1}{2}(10)(1)^2 + 0 = 5 \text{ ft} \cdot \text{lb}$$
$$T_B = \frac{1}{2} m v_B^2 = \frac{1}{2}(0.031056) v_B^2 = 0.015528 v_B^2$$
$$V_B = \frac{1}{2} k(\ell_B - \ell_0)^2 + Wh$$
$$= \frac{1}{2}(10)\left(\frac{2}{3}\right)^2 + (1)\left(-\frac{5}{12}\right)$$
$$= 1.80556 \text{ ft} \cdot \text{lb}$$
$$0 + 5 = 0.015528 v_B^2 = 1.80556$$
$$v_B^2 = 205.72 \text{ ft}^2/\text{s}^2$$

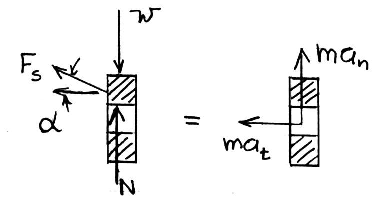
$v_B = 14.34$ ft/s ◀

Forces at B.

$$F_s = k(\ell_B - \ell_0) = (10)\left(\frac{2}{3}\right) = 6.6667 \text{ lb.}$$

$$\sin\alpha = \frac{5}{13}$$

$$\rho = 5 \text{ in.} = \frac{5}{12}\text{ft}$$



$$ma_n = \frac{mv_B^2}{\rho}$$

$$= \frac{(0.031056)(205.72)}{5/12}$$

$$= 15.3332 \text{ lb}$$

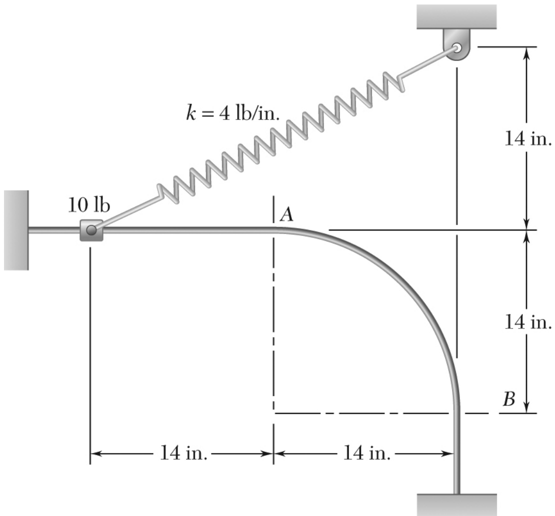
$$+\uparrow \Sigma F_y = ma_y: \quad F_s \sin\alpha - W + N = ma_n$$

$$N = ma_n + W - F_s \sin\alpha$$

$$= 15.3332 + 1 - (6.6667)\left(\frac{5}{13}\right)$$

$$N = 13.769 \text{ lb}$$

$\mathbf{N} = 13.77 \text{ lb} \uparrow$ ◀



PROBLEM 13.73

A 10-lb collar is attached to a spring and slides without friction along a fixed rod in a vertical plane. The spring has an undeformed length of 14 in. and a constant $k = 4$ lb/in. Knowing that the collar is released from rest in the position shown, determine the force exerted by the rod on the collar at (*a*) Point *A*, (*b*) Point *B*. Both these points are on the curved portion of the rod.

SOLUTION

Mass of collar: $$m = \frac{W}{g} = \frac{10}{32.2} = 0.31056 \text{ lb} \cdot \text{s}^2/\text{ft}$$

Let position 1 be the initial position shown, and calculate the potential energies of the spring for positions 1, *A*, and *B*. $l_0 = 14$ in.

$$l_1 = \sqrt{(14+14)^2 + (14)^2} = 31.305 \text{ in.}$$
$$x_1 = l_1 - l_0 = 31.305 - 14 = 17.305 \text{ in.}$$
$$(V_1)_e = \frac{1}{2}kx_1^2 = \frac{1}{2}(4)(17.305)^2 = 598.92 \text{ in} \cdot \text{lb} = 49.910 \text{ ft} \cdot \text{lb}$$

$$l_A = \sqrt{(14)^2 + (14)^2} = 19.799 \text{ in.}$$
$$x_A = l_A - l_0 = 19.799 - 14 = 5.799 \text{ in.}$$
$$(V_A)_e = \frac{1}{2}kx_A^2 = \frac{1}{2}(4)(5.799)^2 = 67.257 \text{ in} \cdot \text{lb} = 5.605 \text{ ft} \cdot \text{lb}$$

$$l_B = 14 + 14 = 28 \text{ in.}$$
$$x_B = l_B - l_0 = 28 - 14 = 14 \text{ in.}$$
$$(V_B)_e = \frac{1}{2}kx_B^2 = \frac{1}{2}(4)(14)^2 = 392 \text{ in} \cdot \text{lb} = 32.667 \text{ ft} \cdot \text{lb}$$

Gravitational potential energies: Datum at level *A*.

$$(V_1)_g = 0 \quad (V_A)_g = 0$$

$$(V_B)g = Wy = (10 \text{ lb})(-14 \text{ in.}) = -140 \text{ in} \cdot \text{lb} = -11.667 \text{ ft} \cdot \text{lb}$$

PROBLEM 13.73 (Continued)

Total potential energies: $V = V_e + V_g$

$V_1 = 49.910 \text{ ft} \cdot \text{lb}, \;\; V_A = 5.605 \text{ ft} \cdot \text{lb}, \;\; V_B = 21.0 \text{ ft} \cdot \text{lb}$

Kinetic energies: $T_1 = 0$

$$T_A = \frac{1}{2}mv_A^2 = \frac{1}{2}(0.31056)v_A^2 = 0.15528v_A^2$$

$$T_B = \frac{1}{2}mv_B^2 = \frac{1}{2}(0.31056)v_B^2 = 0.15528v_B^2$$

Conservation of energy: $T_1 + V_1 = T_A + V_A$:

$$0 + 49.910 = 0.15528v_A^2 + 5.605 \qquad v_A^2 = 285.32 \text{ ft}^2/\text{s}^2$$

Conservation of energy: $T_1 + V_1 = T_B + V_B$

$$0 + 49.910 = 0.15528v_B^2 + 21.0 \qquad v_B^2 = 186.18 \text{ ft}^2/\text{s}^2$$

Normal accelerations at A and B. $a_n = v^2/\rho$

$\rho = 14 \text{ in.} = 1.16667 \text{ ft}$

$$(a_A)_n = \frac{285.32 \text{ ft}^2/\text{s}^2}{1.16667 \text{ ft}} \qquad (\mathbf{a}_A)_n = 244.56 \text{ ft/s}^2 \downarrow$$

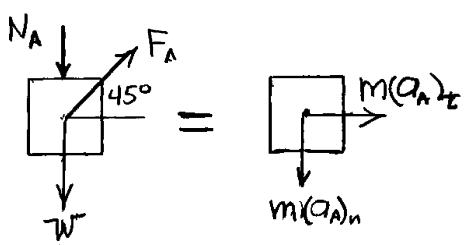
$$(a_B)_n = \frac{189.10 \text{ ft}^2/\text{s}^2}{1.16667 \text{ ft}} \qquad (\mathbf{a}_B)_n = 159.58 \text{ ft/s}^2 \longleftarrow$$

Spring forces at A and B: $F = kx$

$F_A = (4 \text{ lb/in.})(5.799 \text{ in.}) \qquad \mathbf{F}_A = 23.196 \text{ lb} \measuredangle 45°$

$F_B = (4 \text{ lb/in.})(14 \text{ in.}) \qquad \mathbf{F}_B = 56.0 \text{ lb} \uparrow$

To determine the forces ($\mathbf{N}_A$ and $\mathbf{N}_B$) exerted by the rod on the collar, apply Newton's second law.



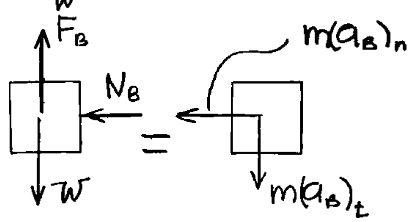
(*a*) At Point A:

$\downarrow \Sigma F = m(a_A)_n$:

$W + N_A - F_A \sin 45° = m(a_A)_n$

$10 + N_A - 23.196 \sin 45° = (0.31056)(244.56)$

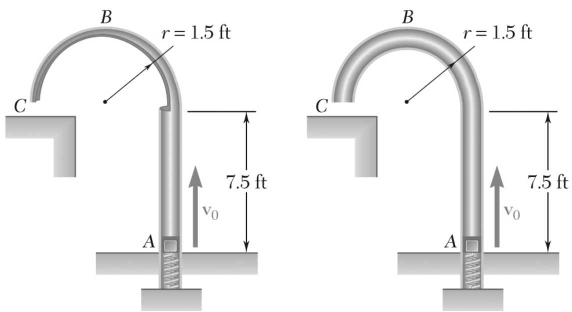
$\mathbf{N}_A = 82.4 \text{ lb} \downarrow$ ◀



(*b*) At Point B:

$\overset{+}{\longleftarrow} \Sigma F = m(a_B)_n$:

$N_B = (0.31056)(159.58) \qquad \mathbf{N}_B = 49.6 \text{ lb} \longleftarrow$ ◀

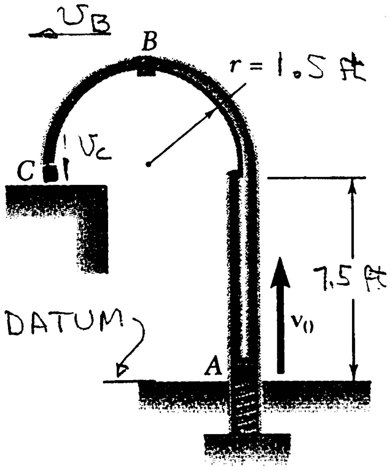


PROBLEM 13.74

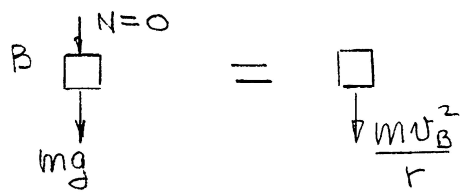
An 8-oz package is projected upward with a velocity $\mathbf{v}_0$ by a spring at A; it moves around a frictionless loop and is deposited at C. For each of the two loops shown, determine (*a*) the smallest velocity $\mathbf{v}_0$ for which the package will reach C, (*b*) the corresponding force exerted by the package on the loop just before the package leaves the loop at C.

SOLUTION

Loop 1



(*a*) The smallest velocity at B will occur when the force exerted by the tube on the package is zero.



$$+\downarrow \Sigma F = 0 + mg = \frac{mv_B^2}{r}$$

$$v_B^2 = rg = 1.5 \text{ ft}(32.2 \text{ ft/s}^2)$$

$$v_B^2 = 48.30$$

At A $\quad T_A = \frac{1}{2}mv_0^2$

$$V_A = 0 \quad \left(8 \text{ oz} = 0.5 \text{ lb} \Rightarrow = \frac{0.5}{32.2} = 0.01553\right)$$

At B $\quad T_B = \frac{1}{2}mv_B^2 = \frac{1}{2}m(48.30) = 24.15 \text{ m}$

$$V_B = mg(7.5 + 1.5) = 9mg = 9(0.5) = 4.5 \text{ lb}\cdot\text{ft}$$

$$T_A + V_A = T_B + V_B: \quad \frac{1}{2}(0.01553)v_0^2 = 24.15(0.01553) + 4.5$$

$$v_0^2 = 627.82 \qquad v_0 = 25.056 \qquad v_0 = 25.1 \text{ ft/s} \blacktriangleleft$$

At C

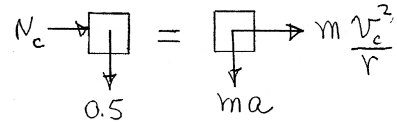
$$T_C = \frac{1}{2}mv_C^2 = 0.007765v_C^2 \qquad V_C = 7.5mg = 7.5(0.5) = 3.75$$

$$T_A + V_A = T_C + V_C: \quad 0.007765v_0^2 = 0.007765v_C^2 + 3.75$$

$$0.007765(25.056)^2 - 3.75 = 0.007765v_C^2$$

$$v_C^2 = 144.87$$

(*b*)

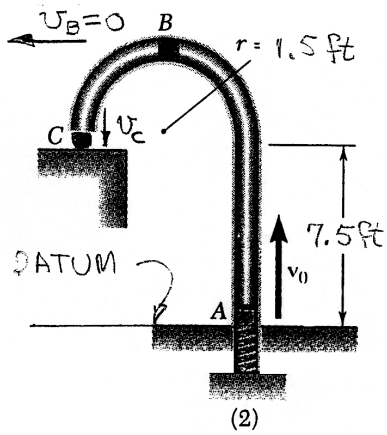


$$\xrightarrow{+} \Sigma F = ma_n: \quad N = 0.01553\frac{(144.87)}{1.5}$$

$$N = 1.49989$$

{Package in tube} $N_C = 1.500$ lb $\longleftarrow$ ◀

Loop 2



(*a*) At *B*, tube supports the package so,

$$v_B \approx 0$$

$$v_B = 0,\ T_B = 0 \qquad V_B = mg\,(7.5 + 1.5)$$

$$= 4.5 \text{ lb}\cdot\text{ft}$$

$$T_A + V_A = T_B + V_B$$

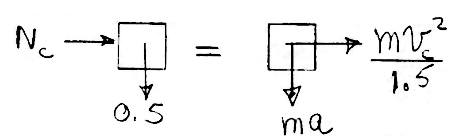
$$\frac{1}{2}(0.01553)v_A^2 = 4.5 \Rightarrow v_A = 24.073$$

$v_A = 24.1$ ft/s ◀

(*b*) At *C* $\quad T_C = 0.007765v_C^2,\ V_C = 7.5mg = 3.75$

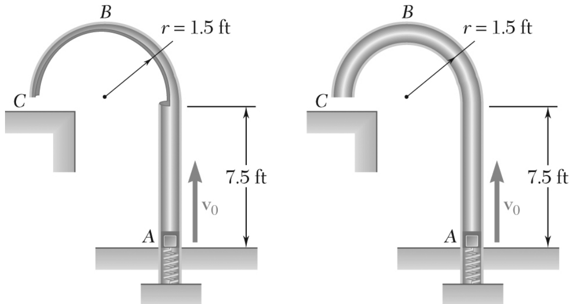
$$T_A + V_A = T_C + V_C: \quad 0.007765(24.073)^2 = 0.007765v_C^2 + 3.75$$

$$v_C^2 = 96.573$$



$$N_C = 0.01553\left(\frac{96.573}{1.5}\right) = 0.99985$$

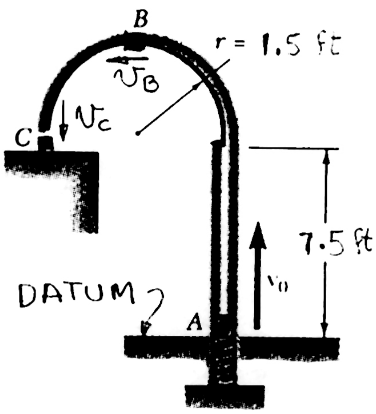
{Package on tube} $N_C = 1.000$ lb ◀



PROBLEM 13.75

If the package of Problem 13.74 is not to hit the horizontal surface at C with a speed greater than 10 ft/s, (a) show that this requirement can be satisfied only by the second loop, (b) determine the largest allowable initial velocity $\mathbf{v}_0$ when the second loop is used.

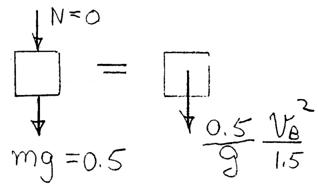
SOLUTION



(1)

(a) Loop 1

From Problem 13.74, at B



$$v_B^2 = gr = 48.3\ \text{ft}^2/\text{s}^2 \Rightarrow v_B = 6.9498\ \text{ft/s}$$

$$T_B = \frac{1}{2}mv_B^2 = \frac{1}{2}(0.01553)(48.3) = 0.37505$$

$$V_B = mg(7.5 + 1.5) = (0.5)(9) = 4.5\ \text{lb}\cdot\text{ft}$$

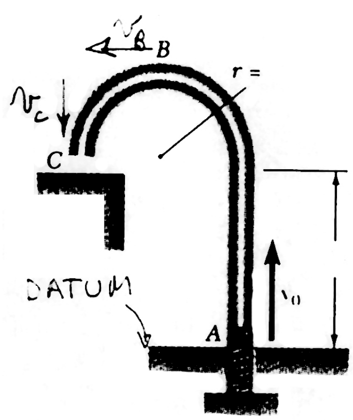
$$T_C = \frac{1}{2}mv_C^2 = \frac{1}{2}(0.01553)v_C^2 = 0.007765v_C^2$$

$$V_C = 7.5(0.5) = 3.75\ \text{lb}\cdot\text{ft}$$

$$T_B + V_B = T_C + V_C:\quad 0.37505 + 4.5 = 0.007765v_C^2 + 3.75$$

$$v_C^2 = 144.887 \Rightarrow v_C = 12.039\ \text{ft/s}$$

12.04 ft/s > 10 ft/s $\Rightarrow$ Loop (1) does not work ◀



(2)

(b) Loop 2 at A $\qquad T_A = \frac{1}{2}mv_0^2 = 0.007765v_0^2$

$$V_A = 0$$

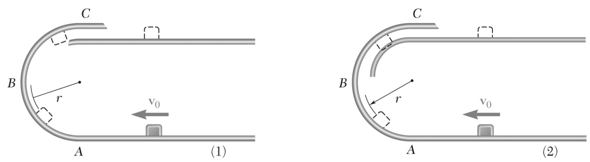
At C assume $\qquad v_C = 10$ ft/s

$$T_C = \frac{1}{2}mv_C^2 = 0.007765(10)^2 = 0.7765$$

$$v_C = 7.5(0.5) = 3.75$$

$$T_A + V_A = T_C + V_C:\quad 0.007765v_0^2 = 0.7765 + 3.75$$

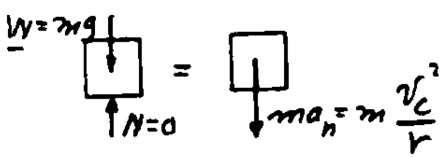
$$v_0 = 24.144 \qquad\qquad v_0 = 24.1\ \text{ft/s}$$ ◀



PROBLEM 13.76

A small package of weight W is projected into a vertical return loop at A with a velocity $\boldsymbol{v}_0$. The package travels without friction along a circle of radius r and is deposited on a horizontal surface at C. For each of the two loops shown, determine (*a*) the smallest velocity $\mathbf{v}_0$ for which the package will reach the horizontal surface at C, (*b*) the corresponding force exerted by the loop on the package as it passes Point B.

SOLUTION



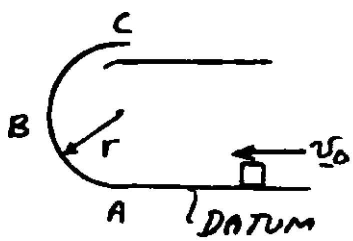
Loop 1:

(*a*) Newton's second law at position C:

$$+\downarrow \Sigma F = \text{ma}:$$

$$mg = m\frac{v_c^2}{r} \qquad v_c^2 = gr$$

Conservation of energy between position A and B.



$$T_A = \frac{1}{2}mv_0^2$$

$$V_A = 0$$

$$T_C = \frac{1}{2}mv_C^2 = \frac{1}{2}mgr$$

$$V_C = mg(2r) = 2mgr$$

$$T_A + V_A = T_C + V_C: \quad \frac{1}{2}mv_0^2 + 0 = \frac{1}{2}mgr + 2mgr$$

$$v_0^2 = 5gr$$

Smallest velocity $\mathbf{v}_0$: $\qquad v_0 = \sqrt{5gr} \longleftarrow$ ◀

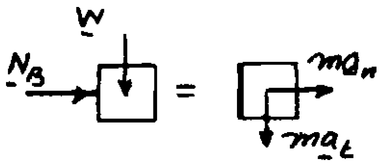
(*b*) Conservation of energy between positions A and B.

$$(b)\ T_B = \frac{1}{2}mv_B^2; \quad V_B = mg(r)$$

$$T_A + V_A = T_B + V_B: \quad \frac{1}{2}mv_A^2 + 0 = \frac{1}{2}mv_B^2 + mgr$$

$$\frac{1}{2}m(5\,gr) + 0 = \frac{1}{2}mv_B^2 + mgr \qquad v_B^2 = 3gr$$

PROBLEM 13.76 (Continued)



Newton's second law at position B.

$$ma_n = m\frac{v_B^2}{r} = m\frac{3gr}{r} = 3\ mg$$

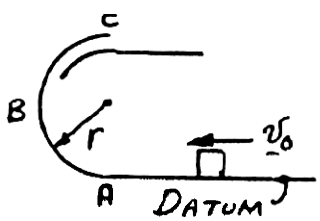
$\xrightarrow{+} \Sigma F = \Sigma F_{eff}$: $\quad N_B = 3\ mg$

Force exerted by the loop: $\quad N_B = 3W \longrightarrow$ ◀

Loop 2:

(a) At point C, $v_c = 0$

Conservation of energy between positions A and C.



$$T_C = \frac{1}{2}mv_C^2 = 0$$

$$V_C = mg(2r) = 2mgr$$

$$T_A + V_A = T_C + V_C:$$

$$\frac{1}{2}mv_0^2 + 0 = 0 + 2mgr$$

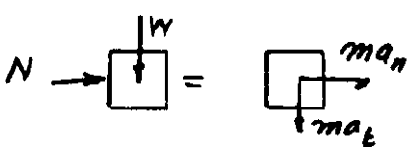
$$v_0^2 = 4gr$$

Smallest velocity $\mathbf{v}_0$: $\quad v_0 = \sqrt{4gr} \longleftarrow$ ◀

(b) Conservation of energy between positions A and B.

$$T_A + V_A - T_B + V_B:\ \frac{1}{2}mv_0^2 + 0 = \frac{1}{2}mv_B^2 + mgr$$

$$\frac{1}{2}m(4gr) = \frac{1}{2}mv_B^2 + mgr \qquad v_B^2 = 2gr$$

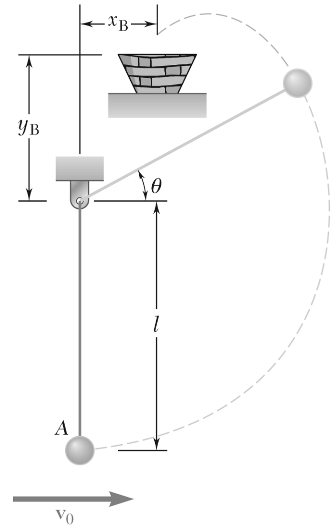


Newton's second law at position B.

$$ma_n = m\frac{v_B^2}{r} = m\frac{2gr}{r} = 2\ mg$$

$\xrightarrow{+} \Sigma F = \Sigma_{\text{eff}}$: $\ N = 2\ mg$

Force exerted by loop: $\quad \mathbf{N} = 2W \longrightarrow$ ◀



PROBLEM 13.77

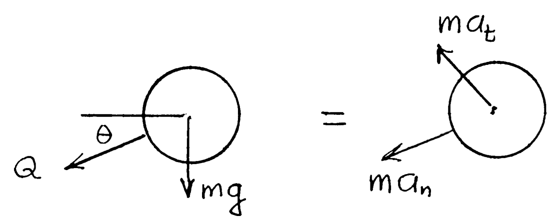
The 1 kg ball at A is suspended by an inextensible cord and given an initial horizontal velocity of 5 m/s. If $l = 0.6$ m and $x_B = 0,$ determine y_B so that the ball will enter the basket.

SOLUTION

Let position 1 be at A. $\qquad v_1 = v_0$

Let position 2 be the point described by the angle where the path of the ball changes from circular to parabolic. At position 2, the tension Q in the cord is zero.

Relationship between v_2 and θ based on $Q = 0.$ Draw the free body diagram.



$$+\swarrow \Sigma F = 0: \quad Q + mg\sin\theta = ma_n = \frac{mv_2^2}{l}$$

With $\qquad Q = 0, \quad v_2^2 = gl\sin\theta \quad$ or $\quad v_2 = \sqrt{gl\sin\theta} \qquad (1)$

Relationship among v_0, v_2 and θ based on conservation of energy.

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}mv_0^2 - mgl = \frac{1}{2}mv_2^2 + mgl\sin\theta$$

$$v_0^2 - v_2^2 = 2gl(1 + \sin\theta) \qquad (2)$$

Eliminating v_2 from Eqs. (1) and (2),

$$v_0^2 - gl\sin\theta = 2gl(1+\sin\theta)$$

$$\sin\theta = \frac{1}{3}\left[\frac{v_0^2}{gl} - 2\right] = \frac{1}{3}\left[\frac{(5)^2}{(9.81)(0.6)} - 2\right] = 0.74912$$

$$\theta = 48.514°$$

From Eq. (1), $$v_2^2 = (9.81)(0.6)\sin 48.514° = 4.4093 \text{ m}^2/\text{s}^2$$

$$v_2 = 2.0998 \text{ m/s}$$

x and y coordinates at position 2.

$$x_2 = l\cos\theta = 0.6\cos 48.514° = 0.39746 \text{ m}$$

$$y_2 = l\sin\theta = 0.6\sin 48.514° = 0.44947 \text{ m}$$

Let t_2 be the time when the ball is a position 2.

Motion on the parabolic path. The horizontal motion is

$$\dot{x} = -v_2\sin\theta = -2.0998 \sin 48.514°$$

$$= -1.5730 \text{ m/s}$$

$$x = x_2 - 1.5730(t - t_2)$$

At Point B, $$x_B = 0$$

$$0 = 0.39746 - 1.5730(t_B - t_2) \quad t_B - t_2 = 0.25267 \text{ s}$$

The vertical motion is

$$y = y_2 + v_2\cos\theta(t - t_2) - \frac{1}{2}g(t - t_2)^2$$

At Point B, $$y_B = y_2 + v_2\cos\theta(t_B - t_2) - \frac{1}{2}g(t_B - t_2)^2$$

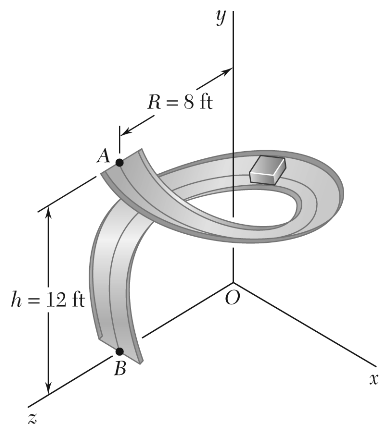
$$y_B = 0.44947 + (2.0998\cos 48.514°)(0.25267)$$

$$-\frac{1}{2}(9.81)(0.25267)^2$$

$$= 0.48779 \text{ m}$$

$y_B = 0.448$ m ◀

PROBLEM 13.78*



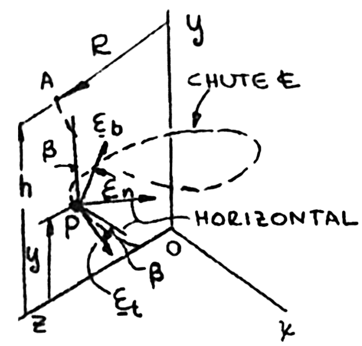
Packages are moved from Point A on the upper floor of a warehouse to Point B on the lower floor, 12 ft directly below A, by means of a chute, the centerline of which is in the shape of a helix of vertical axis y and radius $R = 8$ ft. The cross section of the chute is to be banked in such a way that each package, after being released at A with no velocity, will slide along the centerline of the chute without ever touching its edges. Neglecting friction, (*a*) express as a function of the elevation y of a given Point P of the centerline the angle ϕ formed by the normal to the surface of the chute at P and the principal normal of the centerline at that point, (*b*) determine the magnitude and direction of the force exerted by the chute on a 20-lb package as it reaches Point B. *Hint:* The principal normal to the helix at any Point P is horizontal and directed toward the y axis, and the radius of curvature of the helix is $\rho = R[1-(h/2\pi R)^2]$.

SOLUTION

(*a*) At Point A:

$$v_A = 0 \quad T_A = 0$$

$$V_A = mgh$$



At any Point P:

$$T_P = \frac{1}{2}mv^2$$

$$V_P = Wy = mgy$$

$$T_A + V_A = T_P + V_P$$

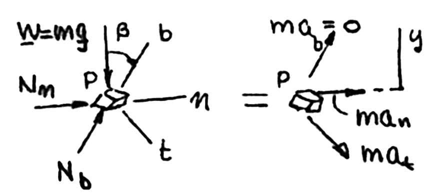
$$0 + mgh = \frac{1}{2}mv^2 + mgy$$

$$v^2 = 2g(h-y)$$

$\mathbf{e}_n$ along principal normal, horizontal and directed toward y axis

$\mathbf{e}_t$ tangent to centerline of the chute

$\mathbf{e}_D$ along binormal



$$\beta = \tan^{-1}\frac{h}{2\pi R} = \tan^{-1}\frac{(12\text{ ft})}{2\pi(8\text{ ft})} = 13.427°$$

$$ma_b = 0$$

since $\quad a_b = 0$

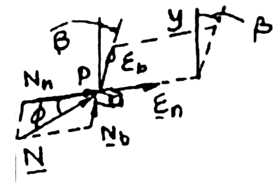
Note: Friction is zero,

$$\Sigma F_t = ma_t: \quad mg\sin\beta = ma_t \quad a_t = g\sin\beta$$

$$\Sigma F_b = ma_b: \quad N_b - W\cos\beta = 0 \quad N_B = W\cos\beta$$

$$\Sigma F_n = ma_n: \quad N_n = \frac{mv^2}{e} = \frac{m2g(h-y)}{e} = 2W\frac{(h-y)}{e}$$

The total normal force is the resultant of $\mathbf{N}_b$ and $\mathbf{N}_m$, it lies in the b–m plane, and forms angle ϕ with m axis.



$$\tan\phi = N_b/N_n$$

$$\tan\phi = W\cos\beta \Big/ \frac{2(w(h-y)}{e}$$

$$\tan\phi = (e/2(h-y))\cos\beta$$

Given: $$e = R\left[1+\left(\frac{h}{2\pi R}\right)^2\right] = R(1+\tan^2\beta) = \frac{R}{\cos^2\beta}$$

Thus, $$\tan\phi = \frac{e}{2(h-y)}\cos\beta = \frac{R}{2(h-y)\cos\beta}$$

$$\tan\phi = \frac{8\text{ ft}}{2(12-y)\cos 13.427°} = \frac{4.112}{12-y}$$

or $$\cot\phi = 0.243(12-y) \blacktriangleleft$$

(*b*) At Point *B*: $y = 0$ for x, y, z axes, we write, with $W = 20$ lb,

$$N_x = N_b\sin\beta = W\cos\beta\sin\beta = (20\text{ lb})\cos 13.427°\sin 13.427° \quad N_x = 4.517\text{ lb}$$

$$N_y = N_b\cos\beta = W\cos^2\beta = (20\text{ lb})\cos^2 13.427° \quad N_y = 18.922\text{ lb}$$

$$N_z = -N_n = -2w\frac{h-y}{e} = -2W\frac{h-y}{R/\cos^2\beta}$$

$$N_z = -2(20\text{ lb})\frac{(12\text{ ft-}0)}{8\text{ ft}}\cos^2 13.427° \quad N_z = -56.765\text{ lb}$$

$$N = \sqrt{(4.517)^2 + (18.922)^2 + (-56.765)^2} \qquad N = 60.0\text{ lb} \blacktriangleleft$$

$$\cos\theta_x = \frac{N_x}{N} = \frac{4.517}{60} \qquad \theta_x = 85.7° \blacktriangleleft$$

$$\cos\theta_y = \frac{N_y}{N} = \frac{18.922}{60} \qquad \theta_y = 71.6° \blacktriangleleft$$

$$\cos\theta_z = \frac{N_z}{N} = -\frac{56.742}{60} \qquad \theta_z = 161.1° \blacktriangleleft$$

PROBLEM 13.79*

Prove that a force $F(x, y, z)$ is conservative if, and only if, the following relations are satisfied:

$$\frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x} \quad \frac{\partial F_y}{\partial z} = \frac{\partial F_z}{\partial y} \quad \frac{\partial F_z}{\partial x} = \frac{\partial F_x}{\partial z}$$

SOLUTION

For a conservative force, Equation (13.22) must be satisfied.

$$F_x = -\frac{\partial V}{\partial x} \quad F_y = -\frac{\partial V}{\partial y} \quad F_z = -\frac{\partial V}{\partial z}$$

We now write
$$\frac{\partial F_x}{\partial y} = -\frac{\partial^2 V}{\partial x \partial y} \quad \frac{\partial F_y}{\partial x} = -\frac{\partial^2 V}{\partial y \partial x}$$

Since $\dfrac{\partial^2 V}{\partial x \partial y} = \dfrac{\partial^2 V}{\partial y \partial x}$:

$$\frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x} \blacktriangleleft$$

We obtain in a similar way

$$\frac{\partial F_y}{\partial z} = \frac{\partial F_z}{\partial y} \quad \frac{\partial F_z}{\partial x} = \frac{\partial F_x}{\partial z} \blacktriangleleft$$

PROBLEM 13.80

The force $\mathbf{F} = (yz\mathbf{i} + zx\mathbf{j} + xy\mathbf{k})/xyz$ acts on the particle $P(x, y, z)$ which moves in space. (*a*) Using the relation derived in Problem 13.79, show that this force is a conservative force. (*b*) Determine the potential function associated with **F**.

SOLUTION

(*a*)

$$F_x = \frac{yz}{xyz} \quad F_y = \frac{zx}{xyz} \quad F_z = \frac{xy}{xyz}$$

$$\frac{\partial F_x}{\partial y} = \frac{\partial\left(\frac{1}{x}\right)}{\partial y} = 0 \quad \frac{\partial F_y}{\partial x} = \frac{\partial\left(\frac{1}{y}\right)}{\partial x} = 0$$

Thus,

$$\frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x}$$

The other two equations derived in Problem 13.79 are checked in a similar way.

(*b*) Recall that

$$F_x = -\frac{\partial V}{\partial x}, \qquad F_y = -\frac{\partial V}{\partial y}, \qquad F_z = -\frac{\partial V}{\partial z}$$

$$F_x = \frac{1}{x} = -\frac{\partial V}{\partial x} \qquad V = -\ln x + f(y, z) \tag{1}$$

$$F_y = \frac{1}{y} = -\frac{\partial V}{\partial y} \qquad V = -\ln y + g(z, x) \tag{2}$$

$$F_z = \frac{1}{z} = -\frac{\partial V}{\partial z} \qquad V = -\ln z + h(x, y) \tag{3}$$

Equating (1) and (2)

$$-\ln x + f(y, z) = -\ln y + g(z, x)$$

Thus,

$$f(y, z) = -\ln y + k(z) \tag{4}$$

$$g(z, x) = -\ln x + k(z) \tag{5}$$

Equating (2) and (3)

$$-\ln z + h(x, y) = -\ln y + g(z, x)$$

$$g(z, x) = -\ln z + l(x)$$

From (5),

$$g(z, x) = -\ln x + k(z)$$

PROBLEM 13.80 (Continued)

Thus,

$$k(z) = -\ln z$$

$$l(x) = -\ln x$$

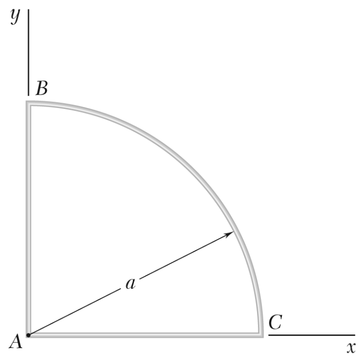
From (4),

$$f(y,z) = -\ln y - \ln z$$

Substitute for $f(y,z)$ in (1)

$$V = -\ln x - \ln y - \ln z$$

$V = -\ln xyz$ ◀



PROBLEM 13.81*

A force $\mathbf{F}$ acts on a particle $P(x, y)$ which moves in the xy plane. Determine whether $\mathbf{F}$ is a conservative force and compute the work of $\mathbf{F}$ when P describes in a clockwise sense the path A, B, C, A including the quarter circle $x^2 + y^2 = a^2$, if (a) $\mathbf{F} = ky\mathbf{i}$, ($b$) $\mathbf{F} = k(y\mathbf{i} + x\mathbf{j})$.

SOLUTION

(a) $\quad F_x = ky \quad F_y = 0 \quad \dfrac{\partial F_x}{\partial y} = k \quad \dfrac{\partial F_y}{\partial x} = 0$

Thus, $\quad \dfrac{\partial F_x}{\partial y} \neq \dfrac{\partial F_y}{\partial x} \qquad$ $\mathbf{F}$ is not conservative.

$$U_{ABCA} = \int_{ABCA} \mathbf{F} \cdot d\mathbf{r} = \int_A^B ky\mathbf{i} \cdot dy\mathbf{j} + \int_B^C ky\mathbf{i} \cdot (dx\mathbf{i} + dy\mathbf{j}) + \int_B^A ky\mathbf{i} \cdot dx\mathbf{j}$$

$$\int_A^B = 0, \ \mathbf{F} \text{ is perpendicular to the path.}$$

$$\int_B^C ky\mathbf{i} \cdot (dx\mathbf{i} + dy\mathbf{j}) = \int_B^C ky\,dx$$

From B to C, the path is a quarter circle with origin at A.

Thus, $\quad x^2 + y^2 = a^2$

$$y = \sqrt{a^2 - x^2}$$

Along BC, $\quad \displaystyle\int_B^C kydx = \int_0^a k\sqrt{a^2 - x^2}\,dx = \frac{\pi ka^2}{4}$

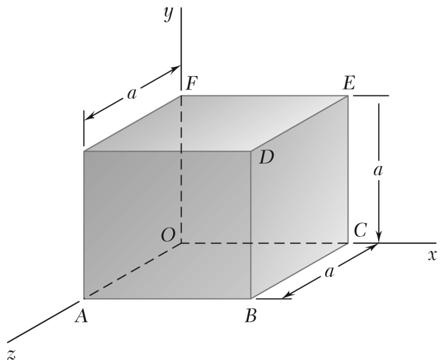
$$\int_C^A ky\mathbf{i} \cdot dx\mathbf{j} = 0 \quad (y = 0 \text{ on } CA)$$

$$U_{ABCA} = \int_A^B + \int_B^C + \int_C^A = 0 + \frac{\pi ka^2}{4} + 0 \qquad U_{ABCA} = \frac{\pi ka^2}{4} \blacktriangleleft$$

(b) $\quad F_x = ky \quad F_y = kx \quad \dfrac{\partial F_x}{\partial y} = k \quad \dfrac{\partial F_y}{\partial x} = k$

$\dfrac{\partial F_x}{\partial y} = \dfrac{\partial F_y}{\partial x}, \qquad$ $\mathbf{F}$ is conservative.

Since $ABCA$ is a closed loop and $\mathbf{F}$ is conservative, $\qquad U_{ABCA} = 0 \blacktriangleleft$



PROBLEM 13.82*

The potential function associated with a force **P** in space is known to be $V(x, y, z) = -(x^2 + y^2 + z^2)^{1/2}$. (*a*) Determine the *x*, *y*, and *z* components of **P**. (*b*) Calculate the work done by **P** from *O* to *D* by integrating along the path *OABD*, and show that it is equal to the negative of the change in potential from *O* to *D*.

SOLUTION

(*a*) $$P_x = -\frac{\partial V}{\partial x} = -\frac{\partial[-(x^2 + y^2 + z^2)^{1/2}]}{\partial x} = x(x^2 + y^2 + z^2)^{-1/2} \blacktriangleleft$$

$$P_y = -\frac{\partial V}{\partial y} = -\frac{\partial[-(x^2 + y^2 + z^2)^{1/2}]}{\partial y} = y(x^2 + y^2 + z^2)^{-1/2} \blacktriangleleft$$

$$P_z = -\frac{\partial V}{\partial z} = -\frac{\partial[-(x^2 + y^2 + z^2)^{1/2}]}{\partial z} = z(x^2 + y^2 + z^2)^{-1/2} \blacktriangleleft$$

(*b*) $$U_{OABD} = U_{OA} + U_{AB} + U_{BD}$$

O–A: P_y and P_x are perpendicular to *O–A* and do no work.

Also, on *O–A* $\quad x = y = 0 \quad \text{and} \quad P_z = 1$

Thus, $$U_{O-A} = \int_0^a P_z dz = \int_0^a dz = a$$

A–B: P_z and P_y are perpendicular to *A–B* and do no work.

Also, on *A–B* $\quad y = 0, \quad z = a \quad \text{and} \quad P_x = \dfrac{x}{(x^2 + a^2)^{1/2}}$

Thus, $$U_{A-B} = \int_0^a \frac{xdx}{(x^2 + a^2)^{1/2}}$$
$$= a(\sqrt{2} - 1)$$

B–D: P_x and P_z are perpendicular to *B–D* and do no work.

On *B–D*, $\quad k = a$

$z = a$

$$P_y = \frac{y}{(y^2 + 2a^2)^{1/2}}$$

PROBLEM 13.82* (Continued)

Thus, $$U_{BD} = \int_0^a \frac{y}{(y^2 + 2a^2)^{1/2}} dy = (y^2 + 2a^2)^{1/2} \Big|_0^a$$

$$U_{BD} = (a^2 + 2a^2)^{1/2} - (2a^2)^{1/2} = a(\sqrt{3} - \sqrt{2})$$

$$U_{OABD} = U_{O-A} + U_{A-B} + U_{B-D}$$

$$= a + a(\sqrt{2} - 1) + a(\sqrt{3} - \sqrt{2})$$

$U_{OABD} = a\sqrt{3}$ ◀

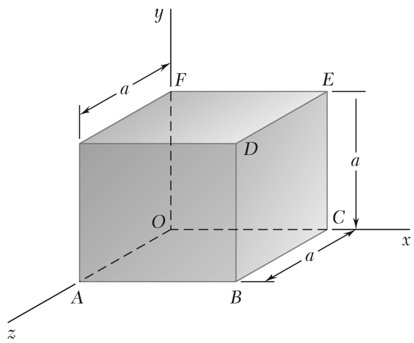
$$\Delta V_{OD} = V(a,a,a) - V(0,0,0)$$

$$= -(a^2 + a^2 + a^2)^{1/2} - 0$$

$\Delta V_{OD} = -a\sqrt{3}$ ◀

Thus, $$U_{OABD} = -\Delta V_{OD}$$

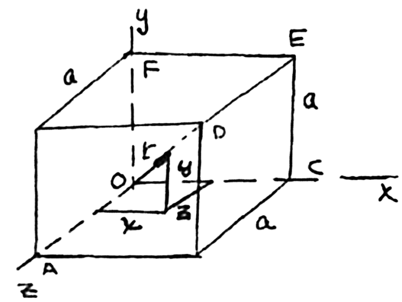
PROBLEM 13.83*



(*a*) Calculate the work done from D to O by the force **P** of Problem 13.82 by integrating along the diagonal of the cube. (*b*) Using the result obtained and the answer to part *b* of Problem 13.82, verify that the work done by a conservative force around the closed path *OABDO* is zero.

PROBLEM 13.82 The potential function associated with a force **P** in space is known to be $V(x, y, z) = -(x^2 + y^2 + z^2)^{1/2}$. (*a*) Determine the x, y, and z components of **P**. (*b*) Calculate the work done by **P** from O to D by integrating along the path *OABD*, and show that it is equal to the negative of the change in potential from O to D.

SOLUTION



From solution to (*a*) of Problem 13.82

$$\mathbf{P} = \frac{x\mathbf{i} + y\mathbf{j} + z\mathbf{k}}{(x^2 + y^2 + z^2)^{1/2}}$$

(*a*) $U_{OD} = \int_O^D \mathbf{P} \cdot d\mathbf{r}$

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

$$d\mathbf{r} = dx\mathbf{i} + dy\mathbf{j} + dz\mathbf{k}$$

$$\mathbf{P} = \frac{x\mathbf{i} + y\mathbf{j} + z\mathbf{k}}{(x^2 + y^2 + z^2)^{1/2}}$$

Along the diagonal. $x = y = z$

Thus, $\mathbf{P} \cdot dr = \dfrac{3x}{(3x^2)^{1/2}} = \sqrt{3}$

$$U_{O-D} = \int_0^a \sqrt{3}\, dx = \sqrt{3}a$$

$U_{OD} = \sqrt{3}a$ ◀

(*b*) $U_{OABDO} = U_{OABD} + U_{DO}$

From Problem 13.82

$U_{OABD} = \sqrt{3}a$ at left

The work done from D to O along the diagonal is the negative of the work done from O to D.

$U_{DO} = -U_{OD} = -\sqrt{3}a$ [see part (*a*)]

Thus,

$$U_{OABDO} = \sqrt{3}a - \sqrt{3}a = 0$$ ◀

PROBLEM 13.84*

The force $\mathbf{F} = (x\mathbf{i} + y\mathbf{j} + z\mathbf{k})/(x^2 + y^2 + z^2)^{3/2}$ acts on the particle $P(x, y, z)$ which moves in space. (*a*) Using the relations derived in Problem 13.79, prove that $\mathbf{F}$ is a conservative force. (*b*) Determine the potential function $V(x, y, z)$ associated with $\mathbf{F}$.

SOLUTION

(*a*)

$$F_x = \frac{x}{(x^2 + y^2 + z^2)^{3/2}}$$

$$F_y = \frac{y}{(x^2 + y^2 + z^2)^{1/2}}$$

$$\frac{\partial F_x}{\partial y} = \frac{x\left(-\frac{3}{2}\right)(2y)}{(x^2 + y^2 + z^2)^{5/2}}$$

$$\frac{\partial F_y}{\partial x} = \frac{y\left(-\frac{3}{2}\right)2y}{(x^2 + y^2 + z^2)^{5/2}}$$

Thus, $$\frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x}$$

The other two equations derived in Problem 13.79 are checked in a similar fashion.

(*b*) Recalling that $$F_x = -\frac{\partial V}{\partial x}, \quad F_y = -\frac{\partial V}{\partial y}, \quad F_z = -\frac{\partial V}{\partial z}$$

$$F_x = -\frac{\partial V}{\partial x} \quad V = -\int \frac{x}{(x^2 + y^2 + z^2)^{3/2}} dx$$

$$V = (x^2 + y^2 + z^2)^{-1/2} + f(y, z)$$

Similarly integrating $\partial V/\partial y$ and $\partial V/\partial z$ shows that the unknown function $f(x, y)$ is a constant.

$$V = \frac{1}{(x^2 + y^2 + z^2)^{1/2}} \blacktriangleleft$$

PROBLEM 13.85

(*a*) Determine the kinetic energy per unit mass which a missile must have after being fired from the surface of the earth if it is to to reach an infinite distance from the earth. (*b*) What is the initial velocity of the missile (called the *escape velocity*)? Give your answers in SI units and show that the answer to part *b* is independent of the firing angle.

SOLUTION

At the surface of the earth, $g = 9.81 \text{ m/s}^2$

$$r_1 = R = 6370 \text{ km} = 6.37 \times 10^6 \text{ m}$$

Centric force at the surface of the earth,

$$F = mg = \frac{GMm}{R^2}$$

$$GM = gR^2 = (9.81)(6.37 \times 10^6)^2 = 398.06 \times 10^{12} \text{ m}^3/\text{s}^2$$

Let position 1 be on the surface of the earth $(r_1 = R)$ and position 2 be at $r_2 = OD.$ Apply the conservation of energy principle.

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}mv_1^2 - \frac{GMm}{r_1} = \frac{1}{2}mv_2^2 + \frac{GMm}{r_2}$$

$$T_1 = T_2 + \frac{GMm}{R} - \frac{GMm}{\infty}$$

$$\frac{T_1}{m} = \frac{T_2}{m} + \frac{GM}{R} = \frac{T_2}{m} + gR$$

For the escape condition set $\frac{T_2}{m} = 0$

$$\frac{T_1}{m} = gR = (9.81 \text{ m/s}^2)(6.37 \times 10^6 \text{ m}) = 62.49 \times 10^6 \text{ m}^2/\text{s}^2$$

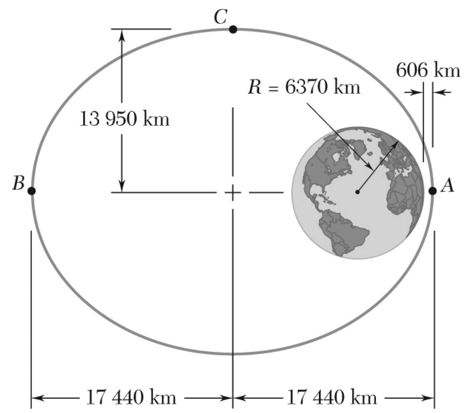
(*a*) $\frac{T_1}{m} = 62.5 \text{ MJ/kg}$ ◀

$$\frac{1}{2}mv_{\text{esc}}^2 = gr$$

$$v_{\text{esc}} = \sqrt{2gR}$$

(*b*) $v_{\text{esc}} = \sqrt{(2)(9.81)(6.37 \times 10^6)} = 11.18 \times 10^3 \text{ m/s}$ $v_{\text{esc}} = 11.18 \text{ km/s}$ ◀

Note that the escape condition depends only on the speed in position 1 and is independent of the direction of the velocity (firing angle).



PROBLEM 13.86

A satellite describes an elliptic orbit of minimum altitude 606 km above the surface of the earth. The semimajor and semiminor axes are 17,440 km and 13,950 km, respectively. Knowing that the speed of the satellite at Point C is 4.78 km/s, determine (*a*) the speed at Point A, the perigee, (*b*) the speed at Point B, the apogee.

SOLUTION

$$r_A = 6370 + 606 = 6976 \text{ km} = 6.976\times 10^6 \text{ m}$$

$$r_C = \sqrt{(17440-6976)^2 + (13950)^2} = 17438.4 \text{ km} = 17.4384\times 10^6 \text{ m}$$

$$r_B = (2)(17440) - 6976 = 27904 \text{ km} = 27.904\times 10^6 \text{ m}$$

For earth, $R = 6370 \text{ km} = 6.37\times 10^6 \text{ m}$

$$GM = gR^2 = (9.81 \text{ m/s}^2)(6.370\times 10^6)^2 = 398.06\times 10^{12} \text{ m}^3/\text{s}^2$$

$$v_C = 4.78 \text{ km/s} = 4780 \text{ m/s}$$

(*a*) *Speed at Point A*: Use conservation of energy.

$$T_A + V_A = T_C + V_C$$

$$\frac{1}{2}mv_A^2 - \frac{GMm}{r_A} = \frac{1}{2}mv_C^2 - \frac{GMm}{r_C}$$

$$v_A^2 = v_C^2 + 2GM\left(\frac{1}{r_A} - \frac{1}{r_C}\right)$$

$$= (4780)^2 + (2)(398.06\times 10^{12})\left[\frac{1}{6.976\times 10^6} - \frac{1}{17.4384\times 10^6}\right]$$

$$= 91.318\times 10^6 \text{ m}^2/\text{s}^2$$

$$V_A = 9.556\times 10^3 \text{ m/s}$$

$v_A = 9.56 \text{ km/s}$ ◀

PROBLEM 13.86 (Continued)

(*b*) *Speed at Point B*: Use conservation of energy.

$$T_B + V_B = T_C + V_C$$

$$\frac{1}{2}mv_B^2 - \frac{GMm}{r_B} = \frac{1}{2}mv_C^2 - \frac{GMm}{r_C}$$

$$v_B^2 = v_C^2 + 2GM\left(\frac{1}{r_B} - \frac{1}{r_C}\right)$$

$$= (4780)^2 + (2)(398.06\times10^{12})\left[\frac{1}{27.904\times10^6} - \frac{1}{17.4384\times10^6}\right]$$

$$= 5.7258\times10^6 \text{ m}^2/\text{s}^2$$

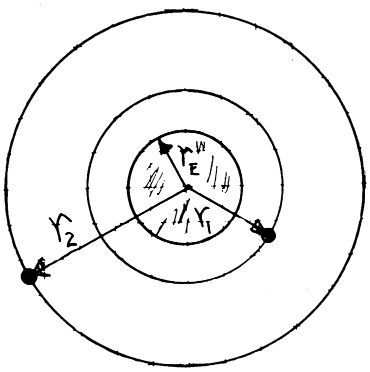
$$v_B = 2.39\times10^3 \text{ m/s}$$

$v_B = 2.39$ km/s ◀

PROBLEM 13.87

While describing a circular orbit 200 mi above the earth a space vehicle launches a 6000-lb communications satellite. Determine (*a*) the additional energy required to place the satellite in a geosynchronous orbit at an altitude of 22,000 mi above the surface of the earth, (*b*) the energy required to place the satellite in the same orbit by launching it from the surface of the earth, excluding the energy needed to overcome air resistance. (A *geosynchronous orbit* is a circular orbit in which the satellite appears stationary with respect to the ground).

SOLUTION



Geosynchronous orbit

$$r_1 = 3960 + 200 = 4160 \text{ mi} = 21.965 \times 10^6 \text{ ft}$$

$$r_2 = 3960 + 22{,}000 = 25{,}960 \text{ mi} = 137.07 \times 10^6 \text{ ft}$$

Total energy $\quad E = T + V = \dfrac{1}{2}mv^2 - \dfrac{GMm}{r}$

$$M = \text{mass of earth}$$

$$m = \text{mass of satellite}$$

Newton's second law $\quad F = ma_n: \quad \dfrac{GMm}{r^2} = \dfrac{mv^2}{r} \Rightarrow v^2 = \dfrac{GM}{r}$

$$T = \frac{1}{2}mv^2 = m\frac{GM}{2r} \qquad V = -\frac{GMm}{r}$$

$$E = T + V = \frac{1}{2}\frac{GMm}{r} - \frac{GMm}{r} = -\frac{1}{2}\frac{GMm}{r}$$

$$GM = gR_E^2 \qquad E = -\frac{1}{2}\frac{gR_E^2 m}{r} = -\frac{1}{2}\frac{R_E^2 W}{r} \text{ where } (W = mg)$$

$$E = -\frac{1}{2}\frac{(6000)(20.9088 \times 10^6 \text{ ft})^2}{r} = -\frac{1.3115 \times 10^{18}}{r} \text{ ft} \cdot \text{lb}$$

Geosynchronous orbit at $r_2 = 137.07 \times 10^6$ ft

$$E_{Gs} = \frac{-1.3115 \times 10^{18}}{137.07 \times 10^6} = -9.5681 \times 10^9 \text{ ft} \cdot \text{lb}$$

(*a*) At 200 mi, $\quad r_1 = 21.965 \times 10^6$ ft

$$E_{200} = -\frac{1.3115 \times 10^{18}}{21.965 \times 10^6} = -5.9709 \times 10^{10}$$

$$\Delta E_{300} = E_{Gs} - E_{200} = 5.0141 \times 10^{10}$$

$$\Delta E_{300} = 50.1 \times 10^9 \text{ ft} \cdot \text{lb} \blacktriangleleft$$

PROBLEM 13.87 (Continued)

(*b*) Launch from earth

At launch pad $$E_E = -\frac{GMm}{R_E} = \frac{-gR_E^2 m}{R_E} = -WR_E$$

$$E_E = -6000(3960 \times 5280) = -1.25453 \times 10^{11}$$

$$\Delta E_E = E_{Gs} - E_E = -9.5681 \times 10^9 + 125.453 \times 10^9$$

$$\Delta E_E = 115.9 \times 10^9 \text{ ft} \cdot \text{lb} \blacktriangleleft$$

PROBLEM 13.88

A lunar excursion module (LEM) was used in the Apollo moon-landing missions to save fuel by making it unnecessary to launch the entire Apollo spacecraft from the moon's surface on its return trip to earth. Check the effectiveness of this approach by computing the energy per pound required for a spacecraft (as weighed on the earth) to escape the moon's gravitational field if the spacecraft starts from (*a*) the moon's surface, (*b*) a circular orbit 50 mi above the moon's surface. Neglect the effect of the earth's gravitational field. (The radius of the moon is 1081 mi and its mass is 0.0123 times the mass of the earth.)

SOLUTION

Note: $GM_{\text{moon}} = 0.0123\, GM_{\text{earth}}$

By Eq. 12.30, $GM_{\text{moon}} = 0.0123\, gR_E^2$

At ∞ distance from moon: $r_2 = \infty$, Assume $v_2 = 0$

$$\begin{aligned} E_2 &= T_2 + V_2 \\ &= 0 - \frac{GM_M m}{\infty} \\ &= 0 - 0 \\ &= 0 \end{aligned}$$

(*a*) On surface of moon: $R_M = 1081 \text{ mi} = 5.7077 \times 10^6 \text{ ft}$

$v_1 = 0 \quad T_1 = 0$ $\qquad R_E = 3960 \text{ mi} = 20.909 \times 10^6 \text{ ft}$

$V_1 = -\dfrac{GM_M m}{R_M}$ $\qquad E_1 = T_1 + V_1 = 0 - \dfrac{0.0123\, gR_E^2 m}{R_M}$

$$E_1 = -\frac{(0.0123)(32.2 \text{ ft/s}^2)(20.909 \times 10^6 \text{ ft})^2 m}{(5.7077 \times 10^6 \text{ ft})}$$

W_E = Weight of LEM on the earth

$$E_1 = (-30.336 \times 10^6 \text{ ft}^2/\text{s}^2)m \qquad m = \frac{W_E}{g}$$

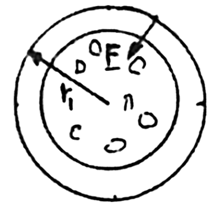
$$E_1 = \left(-\frac{30.336 \times 10^6}{32.2 \text{ ft/s}^2} \text{ft}^2/\text{s}^2\right) W_E$$

$$\begin{aligned} \Delta E &= E_2 - E_1 \\ &= 0 + (942.1 \times 10^3 \text{ ft} \cdot \text{lb/lb}) W_E \end{aligned}$$

Energy per pound: $\dfrac{\Delta E}{W_E} = 942 \times 10^3 \text{ ft} \cdot \text{lb/lb}$ ◀

(*b*) $r_1 = R_M + 50 \text{ mi}$

$r_1 = (1081 \text{ mi} + 50 \text{ mi}) = 1131 \text{ mi} = 5.9717 \times 10^6 \text{ ft}$



Newton's second law:

$$F = ma_n: \quad \frac{GM_M m}{r_1^2} = m\frac{v_1^2}{r_1}$$

$$v_1^2 = \frac{GMm}{r_1} \qquad T_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}m\frac{GM_M}{r_1}$$

$$V_1 = -\frac{GM_M m}{r_1}$$

$$E_1 = T_1 + V_1 = \frac{1}{2}\frac{GM_M m}{r_1} - \frac{GM_M m}{r_1}$$

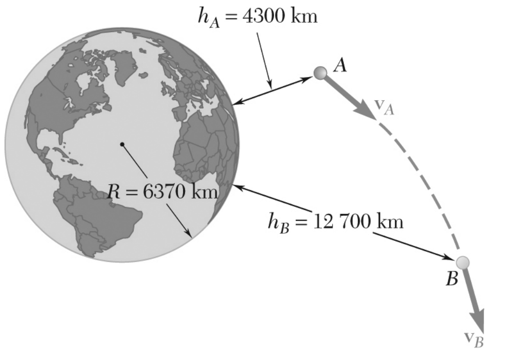
$$E_1 = -\frac{1}{2}\frac{GM_M m}{r_1} = -\frac{1}{2}\frac{0.0123\, gR_E^2 m}{r_1}$$

$$E_1 = -\frac{1}{2}\frac{(0.0123)(32.2 \text{ ft/s}^2)(20.909 \times 10^6 \text{ ft})^2 m}{5.9717 \times 10^6 \text{ ft}}$$

$$E_1 = \frac{(14.498 \times 10^6 \text{ ft}^2/\text{s}^2)W_E}{(32.2 \text{ ft/s}^2)} = 450.2 \times 10^3 \text{ ft} \cdot \text{lb/lb}\, W_E$$

$$\Delta E = E_2 - E_1 = 0 + 450.2 \times 10^3 \text{ ft} \cdot \text{lb/lb}\, W_E$$

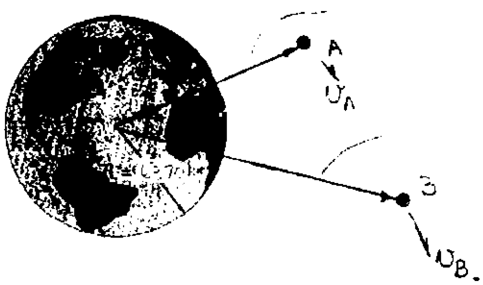
Energy per pound: $\dfrac{\Delta E}{W_E} = 450 \times 10^3 \text{ ft} \cdot \text{lb/lb}$ ◀



PROBLEM 13.89

Knowing that the velocity of an experimental space probe fired from the earth has a magnitude $v_A = 32.5$ Mm/h at Point A, determine the speed of the probe as it passes through Point B.

SOLUTION



$$r_A = R + h_A = 6370 + 4300 = 10740 \text{ km} = 10.670 \times 10^6 \text{ m}$$

$$r_B = 6370 + 12700 = 19070 \text{ km} = 19.070 \times 10^6 \text{ m}$$

$$GM = gR^2 = (9.81)(6.370 \times 10^6)^2 = 398.06 \times 10^{12} \text{ m}^3/\text{s}^2$$

$$v_A = 32.5 \text{ Mm/h} = 9.0278 \times 10^3 \text{ m/s}$$

Use conservation of energy.

$$T_B + V_B = T_A + V_A$$

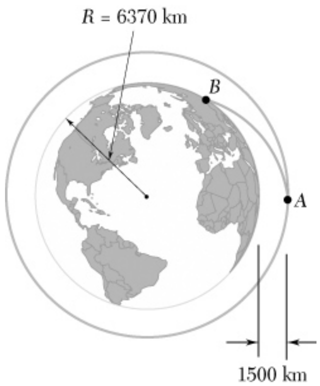
$$\frac{1}{2}mv_B^2 - \frac{GMm}{r_B} = \frac{1}{2}mv_A^2 - \frac{GMm}{r_A}$$

$$v_B^2 = v_A^2 + 2GM\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

$$= (9.0278 \times 10^3)^2 + (2)(398.06 \times 10^{12})\left[\frac{1}{19.070 \times 10^6} - \frac{1}{10.670 \times 10^6}\right]$$

$$= 48.635 \times 10^6 \text{ m}^2/\text{s}^2$$

$v_B = 6.97 \times 10^3$ m/s $\qquad\qquad v_B = 25.1$ Mm/h ◀



PROBLEM 13.90

A spacecraft is describing a circular orbit at an altitude of 1500 km above the surface of the earth. As it passes through Point A, its speed is reduced by 40 percent and it enters an elliptic crash trajectory with the apogee at Point A. Neglecting air resistance, determine the speed of the spacecraft when it reaches the earth's surface at Point B.

SOLUTION

Circular orbit velocity

$$\frac{v_C^2}{r} = \frac{GM}{r^2}, \ GM = gR^2$$

$$v_C^2 = \frac{GM}{r} = \frac{gR^2}{r} = \frac{(9.81 \text{ m/s}^2)(6.370 \times 10^6 \text{ m})^2}{(6.370 \times 10^6 \text{ m} + 1.500 \times 10^6 \text{ m})}$$

$$v_C^2 = 50.579 \times 10^6 \text{ m}^2/\text{s}^2$$

$$v_C = 7112 \text{ m/s}$$

Velocity reduced to 60% of v_C gives $v_A = 4267$ m/s.

Conservation of energy:

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}\cancel{m}v_A^2 - \frac{GM\cancel{m}}{r_A} = \frac{1}{2}\cancel{m}v_B^2 - \frac{GM\cancel{m}}{r_B}$$

$$\frac{1}{2}(4.267 \times 10^3)^2 - \frac{9.81(6.370 \times 10^6)^2}{(7.870 \times 10^6)} = \frac{v_B^2}{2} - \frac{9.81(6.370 \times 10^6)^2}{(6.370 \times 10^6)}$$

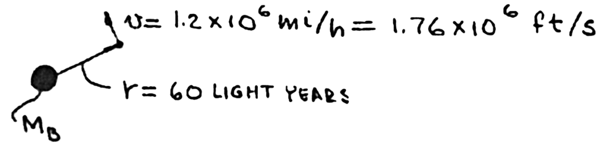
$$v_B = 6.48 \times 10^3 \text{ m/s}$$

$v_B = 6.48$ km/s ◀

PROBLEM 13.91

Observations show that a celestial body traveling at 1.2×10^6 mi/h appears to be describing about Point B a circle of radius equal to 60 light years. Point B is suspected of being a very dense concentration of mass called a black hole. Determine the ratio M_B/M_S of the mass at B to the mass of the sun. (The mass of the sun is 330,000 times the mass of the earth, and a light year is the distance traveled by light in one year at a velocity of 186,300 mi/s.)

SOLUTION



One light year is the distance traveled by light in one year.

Speed of light = 186,300 mi/s

$$r = (60\text{ yr})(186{,}300\text{ mi/s})(5280\text{ ft/mi})(365\text{ days/yr})(24\text{ h/day})(3600\text{ s/h})$$

$$r = 1.8612\times10^{18}\text{ ft}$$

Newton's second law

$$F = \frac{GM_Bm}{r^2} = m\frac{v^2}{r}$$

$$M_B = \frac{rv^2}{G}$$

$$GM_{\text{earth}} = gR_{\text{earth}}^2$$

$$= (32.2\text{ ft/s}^2)(3960\text{ mi}\times5280\text{ ft/mi})^2$$

$$= 14.077\times10^{15}(\text{ft}^3/\text{s}^2)$$

$$M_{\text{sun}} = 330{,}000M_E: \quad GM_{\text{sun}} = 330{,}000\,GM_{\text{earth}}$$

$$GM_{\text{sun}} = (330{,}000)(14.077\times10^{15})$$

$$= 4.645\times10^{21}\text{ ft}^3/\text{s}^2$$

$$G = \frac{4.645\times10^{21}}{M_{\text{sun}}}$$

$$M_B = \frac{rv^2}{G} = \frac{rv^2M_{\text{sun}}}{4.645\times10^{21}}$$

$$\frac{M_B}{M_{\text{sun}}} = \frac{(1.8612\times10^{18})(1.76\times10^6)^2}{4.645\times10^{21}}$$

$$\frac{M_B}{M_{\text{sun}}} = 1.241\times10^9 \blacktriangleleft$$

PROBLEM 13.92

(*a*) Show that, by setting $r = R + y$ in the right-hand member of Eq. (13.17′) and expanding that member in a power series in y/R, the expression in Eq. (13.16) for the potential energy V_g due to gravity is a first-order approximation for the expression given in Eq. (13.17′). (*b*) Using the same expansion, derive a second-order approximation for V_g.

SOLUTION

$$V_g = -\frac{WR^2}{r} \quad \text{setting } r = R + y\text{:} \quad V_g = -\frac{WR^2}{R+y} = -\frac{WR}{1+\frac{y}{R}}$$

$$V_g = -WR\left(1+\frac{y}{R}\right)^{-1} = -WR\left[1+\frac{(-1)}{1}\frac{y}{R}+\frac{(-1)(-2)}{1\cdot 2}\left(\frac{y}{R}\right)^2+\cdots\right]$$

We add the constant WR, which is equivalent to changing the datum from $r = \infty$ to $r = R$:

$$V_g = WR\left[\frac{y}{R}-\left(\frac{y}{R}\right)^2+\cdots\right]$$

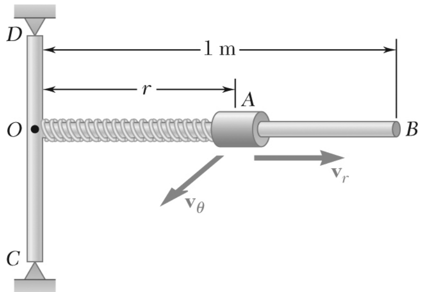
(*a*) First order approximation:

$$V_g = WR\left(\frac{y}{R}\right) = Wy \quad \blacktriangleleft$$

[Equation 13.16]

(*b*) Second order approximation: $$V_g = WR\left[\frac{y}{R}-\left(\frac{y}{R}\right)^2\right]$$

$$V_g = Wy - \frac{Wy^2}{R} \quad \blacktriangleleft$$



PROBLEM 13.93

Collar A has a mass of 3 kg and is attached to a spring of constant 1200 N/m and of undeformed length equal to 0.5 m. The system is set in motion with $r = 0.3$ m, $v_\theta = 2$ m/s, and $v_r = 0$. Neglecting the mass of the rod and the effect of friction, determine the radial and transverse components of the velocity of the collar when $r = 0.6$ m.

SOLUTION

Let position 1 be the initial position.

$$r_1 = 0.3 \text{ m}$$

$$(v_r)_1 = 0, \qquad (v_\theta)_1 = 2 \text{ m/s}, \qquad v_1 = 2 \text{ m/s}$$

$$x_1 = r_1 - l_0 = (0.3 - 0.5) = -0.2 \text{ m}$$

Let position 2 be when $r = 0.6$ m.

$$r_2 = 0.6 \text{ m}$$

$$(v_r)_2 = ?, \qquad (v_\theta)_2 = ?, \qquad v_2 = ?$$

$$x_2 = r_2 - l_0 = (0.6 - 0.5) = 0.1 \text{ m}$$

Conservation of angular momentum: $\quad r_1 m(v_\theta)_1 = v_2 m(v_\theta)_2$

$$(v_\theta)_2 = \frac{r_1 (v_\theta)_1}{r_2} = \frac{(0.3)(2)}{0.6} = 1.000 \text{ m/s}$$

Conservation of energy: $\quad T_1 + V_1 = T_2 + V_2$

$$\frac{1}{2} m v_1^2 + \frac{1}{2} k x_1^2 = \frac{1}{2} m v_2^2 + \frac{1}{2} k x_2^2$$

$$v_2^2 = v_1^2 + \frac{k}{m}\left(x_1^2 - x_2^2\right)$$

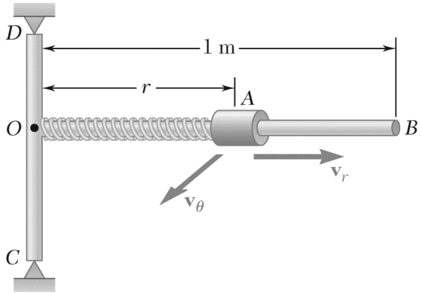
$$= (2)^2 + \frac{1200}{3}\left[(0.2)^2 - (0.1)^2\right] = 16 \text{ m}^2/\text{s}^2$$

$$(v_r)_2^2 = v_2^2 - (v_\theta)^2 = 16 - 1 = 15 \text{ m}^2/\text{s}^2$$

$$v_r = \pm 3.87 \text{ m/s}$$

$v_r = \pm 3.87$ m/s ◀

$v_\theta = 1.000$ m/s ◀



PROBLEM 13.94

Collar A has a mass of 3 kg and is attached to a spring of constant 1200 N/m and of undeformed length equal to 0.5 m. The system is set in motion with $r = 0.3$ m, $v_\theta = 2$ m/s, and $v_r = 0$. Neglecting the mass of the rod and the effect of friction, determine (*a*) the maximum distance between the origin and the collar, (*b*) the corresponding speed. (*Hint:* Solve the equation obtained for r by trial and error.)

SOLUTION

Let position 1 be the initial position.

$$r_1 = 0.3 \text{ m}$$

$$(v_r)_1 = 0, \qquad (v_\theta)_1 = 2 \text{ m/s}, \qquad v_1 = 2 \text{ m/s}$$

$$x_1 = r_1 - l_0 = 0.3 - 0.5 = -0.2 \text{ m}$$

$$T_1 = \frac{1}{2}mv^2 = \frac{1}{2}(3)(2)^2 = 6 \text{ J}$$

$$V_1 = \frac{1}{2}kx_1^2 = \frac{1}{2}(1200)(-0.2)^2 = 24 \text{ J}$$

Let position 2 be when r is maximum. $(v_r)_2 = 0$

$$r_2 = r_m$$

$$x_2 = (r_m - 0.5)$$

$$T_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}(3)(v_\theta)_2^2 = 1.5(v_\theta)_2^2$$

$$V_2 = \frac{1}{2}kx_2^2 = \frac{1}{2}(1200)(r_m - 0.5)^2$$

$$= 600(r_m - 0.5)^2$$

Conservation of angular momentum: $\quad r_1 m(v_\theta)_1 = v_2 m(v_\theta)_2$

$$(v_\theta)_2 = \frac{r_1}{r_2}(v_\theta), = \frac{(0.3)}{r_m}(2) = \frac{0.6}{r_m}$$

Conservation of energy: $\quad T_1 + V_1 = T_2 + V_2$

$$6 + 24 = 1.5(v_\theta)_2^2 + 600(r_m - 0.5)^2$$

$$30 = (1.5)\left(\frac{0.6}{r_m}\right)^2 + 600(r_m - 0.5)^2$$

$$f(r_m) = \frac{0.54}{r_m^2} + 600(r_m - 0.5)^2 - 30 = 0$$

PROBLEM 13.94 (Continued)

Solve for r_m by trial and error.

r_m (m)	0.5	1.0	0.8	0.7	0.72	0.71
$f(r_m)$	–27.8	120.5	24.8	–4.9	0.080	–2.469

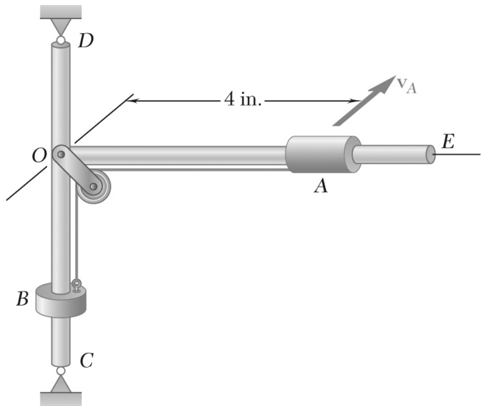
$$r_m = 0.72 - \frac{(0.01)(0.08)}{2.467 + 0.08} = 0.7197 \text{ m}$$

(*a*) *Maximum distance.* $r_m = 0.720 \text{ m}$ ◀

(*b*) *Corresponding speed.*

$$(v_\theta)_2 = \frac{0.6}{0.7197} = 0.8337 \text{ m/s}$$

$$(v_r)_2 = 0$$ $v_2 = 0.834 \text{ m/s}$ ◀



PROBLEM 13.95

A 4-lb collar A and a 1.5-lb collar B can slide without friction on a frame, consisting of the horizontal rod OE and the vertical rod CD, which is free to rotate about CD. The two collars are connected by a cord running over a pulley that is attached to the frame at O. At the instant shown, the velocity $\mathbf{v}_A$ of collar A has a magnitude of 6 ft/s and a stop prevents collar B from moving. If the stop is suddenly removed, determine (a) the velocity of collar A when it is 8 in. from O, (b) the velocity of collar A when collar B comes to rest. (Assume that collar B does not hit O, that collar A does not come off rod OE, and that the mass of the frame is negligible.)

SOLUTION

Masses:
$$m_A = \frac{4}{32.2} = 0.12422 \text{ lb}\cdot\text{s}^2/\text{ft}$$
$$m_B = \frac{1.5}{32.2} = 0.04658 \text{ lb}\cdot\text{s}^2/\text{ft}$$

Constraint of the cord. Let r be the radial distance to the center of collar A and y be the distance that collar B moves up from its initial level. $y = \Delta r;\quad \dot{y} = v_r$

(a) Let position 1 be the initial position just after the stop at B is removed and position 2 be when the collar is 8 in. (0.66667 ft) from O.

$$r_1 = 4 \text{ in.} = 0.33333 \text{ ft} \qquad (v_r)_1 = 0$$
$$r_2 = 8 \text{ in.} = 0.66667 \text{ ft}$$
$$\Delta r = y_2 = 8 - 4 = 4 \text{ in.} = 0.33333 \text{ ft}$$

Potential energy: $V_1 = 0,$

$$V_2 = W_B y_2 = (1.5)(0.33333) = 0.5 \text{ ft}\cdot\text{lb}$$

Conservation of angular momentum of collar A:

$$m_A r_1 (v_\theta)_1 = m_A r_2 (v_\theta)_2$$
$$(v_\theta)_2 = \frac{r_1 (v_\theta)_1}{r_2} = \frac{(0.33333)(6)}{0.66667} = 3 \text{ ft/s}$$

Conservation of energy: $T_1 + V_1 = T_2 + V_2$

$$\frac{1}{2} m_A [(v_r)_1^2 + (v_\theta)_1^2] + \frac{1}{2} m_B \dot{y}_1^2 = \frac{1}{2} m_A [(v_r)_2^2 + (v_\theta)_2^2] + \frac{1}{2} m_B \dot{y}^2 + V_2$$
$$\frac{1}{2} m_A [0 + (v_\theta)_1^2] + 0 + 0 = \frac{1}{2} m_A [(v_r)_2^2 + (v_\theta)_2^2] + \frac{1}{2} m_B (v_r)_2^2 + 0.5$$

$$\frac{1}{2}(0.12422)(6)^2 = \frac{1}{2}(0.12422)[(v_r)_2^2 + (3)^2] + \frac{1}{2}(0.04658)(v_r)_2^2 + 0.5$$

$$2.236 = 0.06211(v_r)_2^2 + 0.559 + 0.02329(v_r)_2^2 + 0.5$$

$$0.0854(v_r)_2^2 = 1.177$$

$$(v_r)_2 = 13.78 \text{ ft}^2/\text{s}^2$$

$(v_r)_2 = 3.71$ ft/s ◀

$(v_\theta)_2 = 3.00$ ft/s ◀

$v = 4.77$ ft/s ◀

(*b*) Let position 3 be when collar *B* comes to rest.

$$y_3 = r_3 - 0.33333, \quad (v_r)_3 = 0, \quad \dot{y}_3 = 0$$

Conservation of angular momentum of collar *A*.

$$m_A r_1 (v_\theta)_1 = m_A r_3 (v_\theta)_3$$

$$(v_\theta)_3 = \frac{r_1 (v_\theta)_1}{r_3} = \frac{(0.33333)(6)}{r_3} = \frac{2}{r_3}$$

Conservation of energy: $T_1 + V_1 = T_3 + V_3$

$$\frac{1}{2}m_A[(v_r)_1^2 + (v_\theta)_1^2] + \frac{1}{2}m_B \dot{y}_1^2 = \frac{1}{2}m_A[(v_r)_3^2 + (v_\theta)_3^2] + \frac{1}{2}m_B \dot{y}_3^2 + w_B y_3$$

$$\frac{1}{2}(0.12422)[0 + (6)^2] + 0 = \frac{1}{2}(0.12422)\left[0 + \left(\frac{2}{r_3}\right)^2\right] + 0 + (1.5)(r_3 - 0.33333)$$

$$2.236 = \frac{0.24844}{r_3^2} + 1.5r_3 - 0.5$$

$$1.5r_3^3 - 2.736r_3^2 + 0.24844 = 0$$

Solving the cubic equation for r_3,

$$r_3 = 1.7712 \text{ ft}, \quad -0.2805 \text{ ft}, \quad 0.33333 \text{ ft}$$

Since $r_3 > r_1 = 0.33333$ ft, the required root is

$$r_3 = 1.7712 \text{ ft}$$

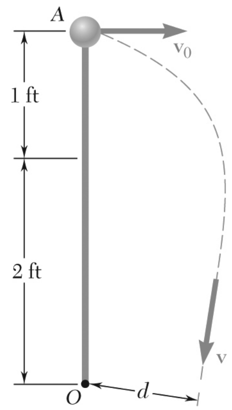
Corresponding velocity of collar *A*:

$(v_r)_3 = 0$ ◀

$$(v_\theta)_3 = \frac{2}{r_3} = \frac{2}{1.7712}$$

$(v_\theta)_3 = 1.129$ ft/s ◀

$v_3 = 1.129$ ft/s ◀



PROBLEM 13.96

A 1.5-lb ball that can slide on a *horizontal* frictionless surface is attached to a fixed Point O by means of an elastic cord of constant $k = 1$ lb/in. and undeformed length 2 ft. The ball is placed at Point A, 3 ft from O, and given an initial velocity $\mathbf{v}_0$ perpendicular to OA. Determine (a) the smallest allowable value of the initial speed v_0 if the cord is not to become slack, (b) the closest distance d that the ball will come to Point O if it is given half the initial speed found in part a.

SOLUTION

Let L_1 be the initial stretched length of the cord and L_2 the length of the closest approach to Point O if the cord does not become slack. Let position 1 be the initial state and position 2 be that of closest approach to Point O. The only horizontal force acting on the ball is the conservative central force due to the elastic cord. At the point of closest approach the velocity of the ball is perpendicular to the cord.

Conservation of angular momentum: $$r_1 m v_1 = r_2 m v_2$$

$$L_1 m v_0 = L_2 m v_2 \quad \text{or} \quad v_2 = \frac{L_1 v_0}{L_2}$$

Conservation of energy: $$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2} m v_1^2 + \frac{1}{2} k (L_1 - L_0)^2 = \frac{1}{2} m v_2^2 + \frac{1}{2} k (L_2 - L_0)^2$$

$$v_1^2 - v_2^2 = -\frac{k}{m}[(L_1 - L_0)^2 + (L_2 - L_0)^2]$$

$$v_0^2 - \frac{L_1^2}{L_2^2} v_0^2 = -\frac{k}{m}[(L_1 - L_0)^2 + (L_2 - L_0)^2]$$

Data: $$L_0 = 2 \text{ ft}, \quad L_1 = 3 \text{ ft}$$

$L_2 = L_0 = 2$ ft for zero tension in the cord at the point of closest approach.

$$k = 1 \text{ lb/in.} = 12 \text{ lb/ft}$$

$$m = W/g = 1.5/32.2 = 0.04658 \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$v_0^2 - \frac{(3)^2}{(2)^2} v_0 = -\frac{12}{0.04658}[(3 - 2)^2 + (2 - 2)^2]$$

$$-1.25\, v_0^2 = -257.6$$

(a) $$v_0^2 = 206.1 \text{ ft}^2/\text{s}^2 \qquad v_0 = 14.36 \text{ ft/s} \blacktriangleleft$$

(*b*) Let $v_0 = \frac{1}{2}(14.36 \text{ ft/s}) = 7.18 \text{ ft/s}^2$ so that the cord is slack in the position of closest approach to Point *O*.

Let position 1 be the initial position and position 2 be position of closest approach with the cord being slack.

Conservation of energy: $T_1 + V_1 = T_2 + V_2$

$$\frac{1}{2}mv_0^2 + \frac{1}{2}k(L_1 - L_0)^2 = \frac{1}{2}mv_2^2$$

$$v_2^2 = v_0^2 + \frac{k}{m}(L - L_0)^2$$

$$= (7.18)^2 + \frac{12}{0.04658}(3-2)^2 = 309.17 \text{ ft}^2/\text{s}^2$$

$$v_2 = 17.583 \text{ ft/s}$$

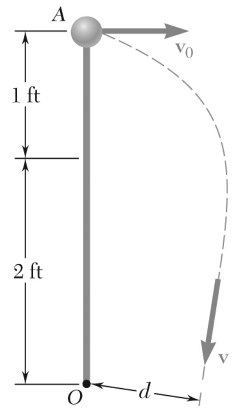
Conservation of angular momentum:

$$r_1 m v_1 = r_2 m v_2 \sin\phi$$

$$r_2 \sin\phi = d = \frac{r_1 v_1}{v_2} = \frac{L_1 v_0}{v_2}$$

$$d = \frac{(3)(7.18)}{17.583}$$

$d = 1.225$ ft ◀



PROBLEM 13.97

A 1.5-lb ball that can slide on a *horizontal* frictionless surface is attached to a fixed Point O by means of an elastic cord of constant $k = 1$ lb/in. and undeformed length 2 ft. The ball is placed at Point A, 3 ft from O, and given an initial velocity $\mathbf{v}_0$ perpendicular to OA, allowing the ball to come within a distance $d = 9$ in. of Point O after the cord has become slack. Determine (a) the initial speed v_0 of the ball, (b) its maximum speed.

SOLUTION

Let L_1 be the initial stretched length of the cord. Let position 1 be the initial position. Let position 2 be the position of closest approach to point after the cord has become slack. While the cord is slack there are no horizontal forces acting on the ball, so the velocity remains constant. While the cord is stretched, the only horizontal force acting on the ball is the conservative central force due to the elastic cord. At the point of closest approach the velocity of the ball is perpendicular to the radius vector.

Conservation of angular momentum: $r_1 m v_1 = r_2 m v_2$

$$L_1 v_0 = d\,v_2 \qquad \text{or} \qquad v_2 = \frac{L_1}{d} v_0$$

Conservation of energy: $T_1 + V_1 = T_2 + V_2$

$$\frac{1}{2} m v_0^2 + \frac{1}{2} k (L_1 - L_0)^2 = \frac{1}{2} m v_2^2 + 0$$

$$v_0^2 - v_2^2 = -\frac{k}{m}(L_1 - L_0)^2$$

$$v_0^2 - \left(\frac{L_1}{d^2} v_0\right)^2 = -\frac{k}{m}(L_1 - L_0)$$

Data: $L_0 = 2$ ft, $L_1 = 3$ ft, $d = 9$ in. $= 0.75$ ft

$k = 1$ lb/in. $= 12$ lb/ft

$m = W/g = 1.5/32.2 = 0.04658$ lb $\cdot$ s^2/ft

$$v_0^2 - \left(\frac{3\, v_0}{0.75}\right)^2 = -\frac{12}{0.04658}(3-2)^2$$

$$-15 v_0^2 = -257.6$$

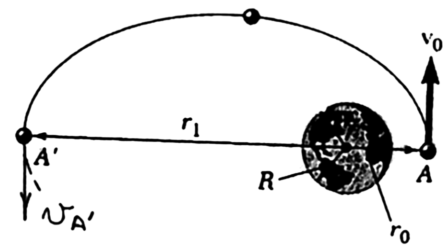
(a) $v_0^2 = 17.17$ ft^2/s^2 $\qquad v_0 = 4.14$ ft/s ◀

(b) Maximum speed. $v_m = v_2 = \dfrac{3v_0}{0.75}$ $\qquad v_m = 16.58$ ft/s ◀

PROBLEM 13.98

Using the principles of conservation of energy and conservation of angular momentum, solve part *a* of Sample Problem 12.9.

SOLUTION



$$R = 6370 \text{ km}$$
$$r_0 = 500 \text{ km} + 6370 \text{ km}$$
$$r_0 = 6870 \text{ km} = 6.87 \times 10^6 \text{ m}$$
$$v_0 = 36,900 \text{ km/h}$$
$$= \frac{36.9 \times 10^6 \text{ m}}{3.6 \times 10^3 \text{ s}}$$
$$= 10.25 \times 10^3 \text{ m/s}$$

Conservation of angular momentum:

$$r_0 m v_0 = r_1 m v_A, \qquad r_0 = r_{\min}, \;\; r_1 = r_{\max}$$
$$v_{A'} = \left(\frac{r_0}{r_1}\right) v_0 = \left(\frac{6.870 \times 10^6}{r_1}\right)(10.25 \times 10^3)$$
$$v_{A'} = \frac{70.418 \times 10^9}{r_1} \qquad (1)$$

Conservation of energy:

Point A:

$$v_0 = 10.25 \times 10^3 \text{ m/s}$$
$$T_A = \frac{1}{2} m v_0^2 = \frac{1}{2} m (10.25 \times 10^3)^2$$
$$T_A = (m)(52.53 \times 10^6)(\text{J})$$
$$V_A = -\frac{GMm}{r_0}$$
$$GM = gR^2 = (9.81 \text{ m/s}^2)(6.37 \times 10^6 \text{ m})^2$$
$$GM = 398 \times 10^{12} \text{ m}^3/\text{s}^2$$
$$r_0 = 6.87 \times 10^6 \text{ m}$$
$$V_A = -\frac{(398 \times 10^{12} \text{ m}^3/\text{s}^2) m}{(6.87 \times 10^6 \text{ m})}$$
$$= -57.93 \times 10^6 \, m \text{ (J)}$$

Point A':

$$T_{A'} = \frac{1}{2} m v_{A'}^2$$

$$V_{A'} = -\frac{GMm}{r_1}$$

$$= -\frac{398\times 10^{12}\ m}{r_1} \text{(J)}$$

$$T_A + V_A = T_{A'} + V_{A'}$$

$$52.53\times 10^6\ \cancel{m} - 57.93\times 10^6\ \cancel{m} = \frac{1}{2}\cancel{m} v_{A'}^2 - \frac{398\times 10^{12}\ \cancel{m}}{r_1}$$

Substituting for $v_{A'}$ from (1)

$$-5.402\times 10^6 = \frac{(70.418\times 10^9)^2}{(2)(r_1)^2} - \frac{398\times 10^{12}}{r_1}$$

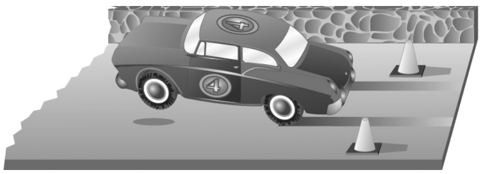
$$-5.402\times 10^6 = \frac{(2.4793\times 10^{21})}{r_1^2} - \frac{398\times 10^{12}}{r_1}$$

$$(5.402\times 10^6) r_1^2 - (398\times 10^{12}) r_1 + 2.4793\times 10^{21} = 0$$

$$r_1 = 66.7\times 10^6 \text{ m},\ 6.87\times 10^6 \text{ m}$$

$r_{max} = 66{,}700$ km ◀

PROBLEM 13.99



Solve sample Problem 13.8, assuming that the elastic cord is replaced by a central force **F** of magnitude $(80/r^2)$ N directed toward O.

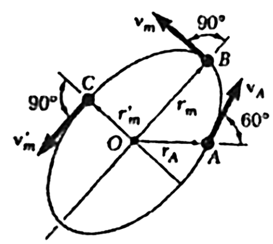
PROBLEM 13.8 Skid marks on a drag racetrack indicate that the rear (drive) wheels of a car slip for the first 20 m of the 400-m track.(*a*) Knowing that the coefficient of kinetic friction is 0.60, determine the speed of the car at the end of the first 20-m portion of the track if it starts from rest and the front wheels are just off the ground. (*b*) What is the maximum theoretical speed for the car at the finish line if, after skidding for 20 m, it is driven without the wheels slipping for the remainder of the race? Assume that while the car is rolling without slipping, 60 percent of the weight of the car is on the rear wheels and the coefficient of static friction is 0.75. Ignore air resistance and rolling resistance.

SOLUTION

(*a*) The force exerted on the sphere passes through O. Angular momentum about O is conserved.

Minimum velocity is at B, where the distance from O is maximum.

Maximum velocity is at C, where distance from O is minimum.



$$r_A m v_A \sin 60° = r_m m v_m$$

$$(0.5 \text{ m})(0.6 \text{ kg})(20 \text{ m/s})\sin 60° = r_m (0.6 \text{ kg}) v_m$$

$$v_m = \frac{8.66}{r_m} \qquad (1)$$

Conservation of energy:

At Point A,

$$T_A = \frac{1}{2} m v_A^2 = \frac{1}{2}(0.6 \text{ kg})(20 \text{ m/s})^2 = 120 \text{ J}$$

$$V = \int F dr = \int \frac{80}{r^2} dr = \frac{-80}{r},$$

$$V_A = \frac{-80}{0.5} = -160 \text{ J}$$

At Point B,

$$T_B = \frac{1}{2} m v_m^2 = \frac{1}{2}(0.6 \text{ kg}) v_m^2 = 0.3 v_m^2$$

and Point C:
$$V_B = \frac{-80}{r_m}$$

$$T_A + V_A = T_B + V_B$$

$$120 - 160 = 0.3 v_m^2 - \frac{80}{r_m} \qquad (2)$$

Substitute (1) into (2)

$$-40 = (0.3)\left(\frac{8.66}{r_m}\right)^2 - \frac{80}{r_m}$$

$$r_m^2 - 2\,r_m + 0.5625 = 0$$

$$r_m' = 0.339 \text{ m} \quad \text{and} \quad r_m = 1.661 \text{ m}$$

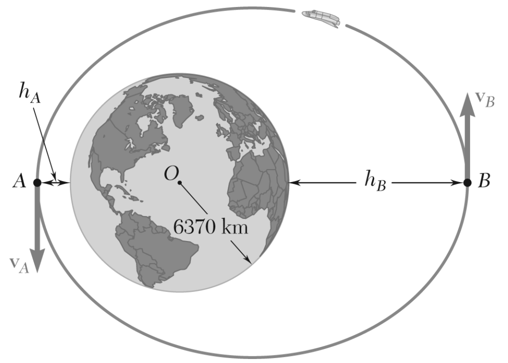
$r_{\max} = 1.661$ m ◀

$r_{\min} = 0.339$ m ◀

(*b*) Substitute r_m' and r_m from results of part (*a*) into (1) to get corresponding maximum and minimum values of the speed.

$$v_m' = \frac{8.66}{0.339} = 25.6 \text{ m/s}$$ $v_{\max} = 25.6$ m/s ◀

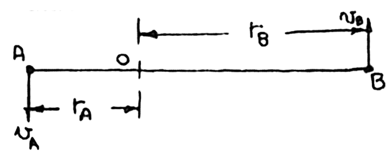
$$v_m = \frac{8.66}{1.661} = 5.21 \text{ m/s}$$ $v_{\min} = 5.21$ m/s ◀



PROBLEM 13.100

A spacecraft is describing an elliptic orbit of minimum altitude $h_A = 2400$ km and maximum altitude $h_B = 9600$ km above the surface of the earth. Determine the speed of the spacecraft at A.

SOLUTION



$$r_A = 6370 \text{ km} + 2400 \text{ km}$$
$$r_A = 8770 \text{ km}$$
$$r_B = 6370 \text{ km} + 9600 \text{ km}$$
$$= 15{,}970 \text{ km}$$

Conservation of momentum: $r_A m v_A = r_B m v_B$

$$v_B = \frac{r_A}{r_B} v_A = \frac{8770}{15{,}970} v_A = 0.5492 v_A \quad (1)$$

Conservation of energy: $T_A = \frac{1}{2} m v_A^2 \quad V_A = \frac{-GMm}{r_A} \quad T_B = \frac{1}{2} m v_B^2 \quad V_B = \frac{-GMm}{r_B}$

$$GM = gR^2 = (9.81 \text{ m/s}^2)(6370 \times 10^3 \text{ m})^2 = 398.1 \times 10^{12} \text{ m}^3/\text{s}^2$$

$$V_A = \frac{-(398.1 \times 10^{12})\, m}{8770 \times 10^3} = -45.39 \times 10^6\, m$$

$$V_B = \frac{-(398.1 \times 10^{12})\, m}{(15{,}970 \times 10^3)} = -24.93\, m$$

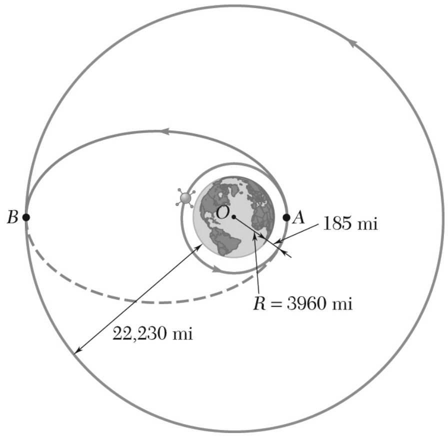
$$T_A + V_A = T_B + V_B:$$

$$\frac{1}{2} \cancel{m} v_A^2 - 45.39 \times 10^6 \cancel{m} = \frac{1}{2} \cancel{m} v_B^2 - 24.93 \times 10^6\, m \quad (2)$$

Substituting for v_B in (2) from (1)

$$v_A^2 [1 - (0.5492)^2] = 40.92 \times 10^6$$
$$v_A^2 = 58.59 \times 10^6 \text{ m}^2/\text{s}^2$$
$$v_A = 7.65 \times 10^3 \text{ m/s}$$

$v_A = 27.6 \times 10^3$ km/h ◀



PROBLEM 13.101

While describing a circular orbit, 185 mi above the surface of the earth, a space shuttle ejects at Point A an inertial upper stage (IUS) carrying a communication satellite to be placed in a geosynchronous orbit (see Problem 13.87) at an altitude of 22,230 mi above the surface of the earth. Determine (a) the velocity of the IUS relative to the shuttle after its engine has been fired at A, (b) the increase in velocity required at B to plae the satellite in its final orbit.

SOLUTION

For earth,
$$R = 3960 \text{ mi} = 20.909\times 10^6 \text{ ft}$$
$$g = 32.2 \text{ ft}^2$$
$$GM = gR^2 = (32.2)(20.909\times 10^6)^2 = 14.077\times 10^{15} \text{ ft}^3/\text{s}^2$$

Speed on a circular orbits of radius r, r_A, and r_B.
$$F = ma_n$$
$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$
$$v^2 = \frac{GM}{r} \qquad v = \sqrt{\frac{GM}{r}}$$
$$r_A = 3960 + 185 = 4145 \text{ mi} = 21.886\times 10^6 \text{ ft}$$
$$(v_A)_{\text{circ}} = \sqrt{\frac{14.077\times 10^{15}}{21.886\times 10^6}} = 25.362\times 10^3 \text{ ft/s}$$
$$r_B = 3960 + 22230 = 26190 \text{ mi} = 138.283\times 10^6 \text{ ft}$$
$$(v_B)_{\text{circ}} = \sqrt{\frac{14.077\times 10^{15}}{138.283\times 10^6}} = 10.089\times 10^3 \text{ ft/s}$$

Calculate speeds at A and B for path AB.

Conservation of angular momentum: $mr_A v_A \sin\phi_A = mr_B v_B \sin\phi_A$
$$v_B = \frac{r_A v_A \sin 90°}{r_B \sin 90°} = \frac{21.886\times 10^6 v_A}{138.283\times 10^6} = 0.15816 v_A$$

PROBLEM 13.101 (Continued)

Conservation of energy: $$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}mv_A^2 + \frac{GMm}{r_A} = \frac{1}{2}mv_B^2 - \frac{GMm}{r_B}$$

$$v_A^2 - v_B^2 = 2GM\left(\frac{1}{r_A} - \frac{1}{r_B}\right) = \frac{2GM\,(r_B - r_A)}{r_A r_B}$$

$$v_A^2 - (0.15816v_A)^2 = \frac{(2)(14.077\times10^{15})(116.397\times10^6)}{(21.886\times10^6)(138.283\times10^6)}$$

$$0.97499v_A^2 = 1.082796\times10^9$$

$$v_A = 33.325\times10^3 \text{ ft/s}$$

$$v_B = (0.15816)(33.325\times10^6) = 5.271\times10^3 \text{ ft/s}$$

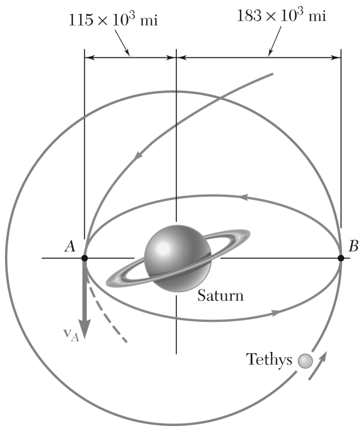
(*a*) Increase in speed at *A*:

$$\Delta v_A = 33.325\times10^3 - 25.362\times10^3 = 7.963\times10^3 \text{ ft/s}$$ $\Delta v_A = 7960$ ft/s ◀

(*b*) Increase in speed at *B*:

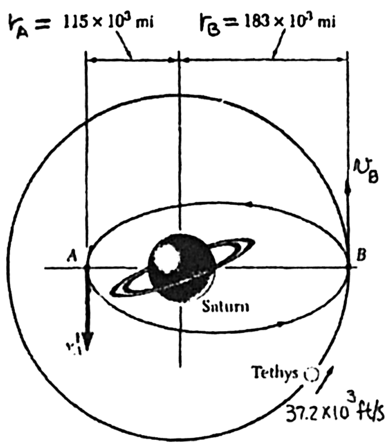
$$\Delta v_B = 10.089\times10^3 - 5.271\times10^3 = 4.818\times10^3 \text{ ft/s}$$ $\Delta v_B = 4820$ ft/s ◀

PROBLEM 13.102



A spacecraft approaching the planet Saturn reaches Point A with a velocity $\mathbf{v}_A$ of magnitude 68.8×10^3 ft/s. It is to be placed in an elliptic orbit about Saturn so that it will be able to periodically examine Tethys, one of Saturn's moons. Tethys is in a circular orbit of radius 183×10^3 mi about the center of Saturn, traveling at a speed of 37.2×10^3 ft/s. Determine (a) the decrease in speed required by the spacecraft at A to achieve the desired orbit, (b) the speed of the spacecraft when it reaches the orbit of Tethys at B.

SOLUTION



(a)

$$r_A = 607.2\times10^6 \text{ ft}$$
$$r_B = 966.2\times10^6 \text{ ft}$$

$v_A' =$ speed of spacecraft in the elliptical orbit after its speed has been decreased.

Elliptical orbit between A and B.

Conservation of energy

Point A: $$T_A = \frac{1}{2}mv_A'^2$$

$$V_A = \frac{-GM_{\text{sat}}m}{r_A}$$

$M_{\text{sa}} =$ Mass of Saturn, determine GM_{sa} from the speed of Tethys in its circular orbit.

(Eq. 12.44) $$v_{\text{circ}} = \sqrt{\frac{GM_{\text{sat}}}{r}} \quad GM_{\text{sat}} = r_B v_{\text{circ}}^2$$

$$GM_{\text{sat}} = (966.2\times10^6 \text{ ft}^2)(37.2\times10^3 \text{ ft/s})^2$$
$$= 1.337\times10^{18} \text{ ft}^3/\text{s}^2$$
$$V_A = -\frac{(1.337\times10^{18} \text{ ft}^3/\text{s}^2)\, m}{(607.2\times10^6 \text{ ft})}$$
$$= -2.202\times10^9\, m$$

PROBLEM 13.102 (Continued)

Point B: $$T_B = \frac{1}{2}mv_B^2 \qquad V_B = \frac{-GM_{\text{sat}}m}{r_B} = -\frac{(1.337\times 10^{18}\ \text{ft}^3/\text{s}^2)\,m}{(966.2\times 10^6\ \text{ft})}$$

$$V_B = 1.384\times 10^9$$

$$T_A + V_A = T_B + V_B;$$

$$\frac{1}{2}\cancel{m}v_A'^2 - 2.202\times 10^9\,\cancel{m} = \frac{1}{2}\cancel{m}v_B^2 - 1.384\times 10^9\,\cancel{m}$$

$$v_A'^2 - v_B^2 = 1.636\times 10^9$$

Conservation of angular momentum:

$$r_A m v_A' = r_B m v_B \qquad v_B = \frac{r_A}{r_B}v_A' = \frac{607.2\times 10^6}{966.2\times 10^6}v_A' = 0.6284v_A'$$

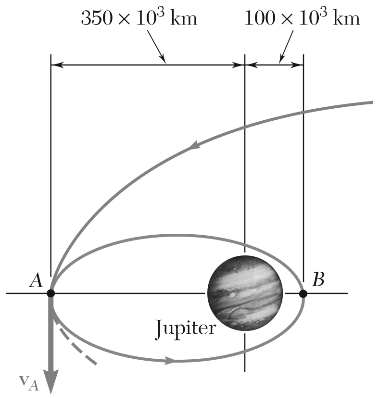
$$v_A'^2[1-(0.6284)^2] = 1.636\times 10^9$$

$$v_A' = 52,005\ \text{ft/s}$$

(a) $\Delta v_A = v_A - v_A' = 68,800 - 52,005$ $\qquad \Delta v_A = 16,795\ \text{ft/s}$ ◀

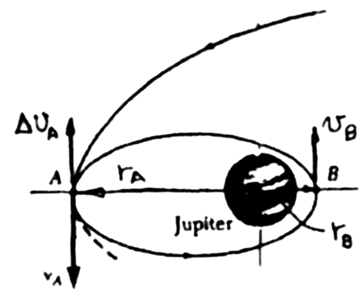
(b) $v_B = \dfrac{r_A}{r_B}v_A' = (0.6284)(52,005)$ $\qquad v_B = 32,700\ \text{ft/s}$ ◀

PROBLEM 13.103



A spacecraft traveling along a parabolic path toward the planet Jupiter is expected to reach Point A with a velocity $\mathbf{v}_A$ of magnitude 26.9 km/s. Its engines will then be fired to slow it down, placing it into an elliptic orbit which will bring it to within 100×10^3 km of Jupiter. Determine the decrease in speed Δv at Point A which will place the spacecraft into the required orbit. The mass of Jupiter is 319 times the mass of the earth.

SOLUTION



Conservation of energy.

Point A:

$$T_A = \frac{1}{2}m(v_A - \Delta v_A)^2$$

$$V_A = \frac{-GM_J m}{r_A}$$

$$GM_J = 319GM_E = 319 gR_E^2$$

$$R_E = 6.37 \times 10^6 \text{ m}$$

$$GM_J = (319)(9.81 \text{ m/s}^2)(6.37 \times 10^6 \text{ m})^2$$

$$GM_J = 126.98 \times 10^{15} \text{ m}^3/\text{s}^2$$

$$r_A = 350 \times 10^6 \text{ m}$$

$$V_A = \frac{-(126.98 \times 10^{15} \text{ m}^3/\text{s}^2)m}{(350 \times 10^6 \text{ m})}$$

$$V_A = -(362.8 \times 10^6)m$$

Point B:

$$T_B = \frac{1}{2}mv_B^2$$

$$V_B = \frac{-GM_J m}{r_B} = \frac{-(126.98 \times 10^{15} \text{ m}^3/\text{s}^2)m}{(100 \times 10^6 m)}$$

$$V_B = -(1269.8 \times 10^6)m$$

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}m(v_A - \Delta v_A)^2 - 362.8 \times 10^6 m = \frac{1}{2}mv_B^2 - 1269.8 \times 10^6 \text{m}$$

$$(v_A - \Delta v_A)^2 - v_B^2 = -1814 \times 10^6 \qquad (1)$$

PROBLEM 13.103 (Continued)

Conservation of angular momentum.

$$r_A = 350\times10^6 \text{ m}$$
$$r_B = 100\times10^6 \text{ m}$$
$$r_A m(v_A - \Delta v_A) = r_B m v_B$$
$$v_B = \left(\frac{r_A}{r_B}\right)(v_A - \Delta v_A)$$
$$= \left(\frac{350}{100}\right)(v_A - \Delta v_A) \qquad (2)$$

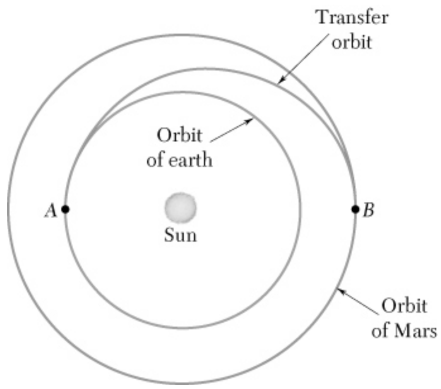
Substitute v_B in (2) into (1)

$$(v_A - \Delta v_A)^2[1-(3.5)^2] = -1814\times10^6$$
$$(v_A - \Delta v_A)^2 = 161.24\times10^6$$
$$(v_A - \Delta v_A) = \mp 12.698\times10^3 \text{ m/s}$$

(Take positive root; negative root reverses flight direction.)

$$v_A = 26.9\times10^3 \text{ m/s} \qquad \text{(given)}$$
$$\Delta v_A = (26.9\times10^3 \text{ m/s} - 12.698\times10^3 \text{ m/s})$$

$$\Delta v_A = 14.20 \text{ km/s} \blacktriangleleft$$



PROBLEM 13.104

As a first approximation to the analysis of a space flight from the earth to Mars, it is assumed that the orbits of the earth and Mars are circular and coplanar. The mean distances from the sun to the earth and to Mars are 149.6×10^6 km and 227.8×10^6 km, respectively. To place the spacecraft into an elliptical transfer orbit at Point *A*, its speed is increased over a short interval of time to v_A which is faster than the earth's orbital speed. When the spacecraft reaches Point *B* on the elliptical transfer orbit, its speed v_B is increased to the orbital speed of Mars. Knowing that the mass of the sun is 332.8×10^3 times the mass of the earth, determine the increase in velocity required (*a*) at *A*, (*b*) at *B*.

SOLUTION

M = mass of the sun

$$GM = 332.8(10)^3(9.81\text{ m/s}^2)(6.37 \times 10^6\text{ m})^2 = 1.3247(10)^{20}\text{ m}^3/\text{s}^2$$

Circular orbits

$$\text{Earth}\quad v_E = \sqrt{\frac{GM}{149.6(10)^9}} = 29.758\text{ m/s}$$

$$\text{Mars}\quad v_M = \sqrt{\frac{GM}{227.8(10)^9}} = 24.115\text{ m/s}$$

Conservation of angular momentum

Elliptical orbit

$$v_A(149.6) = v_B(227.8)$$

Conservation of energy

$$\frac{1}{2}v_A^2 - \frac{GM}{149.6(10)^9} = \frac{1}{2}v_B^2 - \frac{GM}{227.8(10)^9}$$

$$v_A = v_B\frac{(227.8)}{(149.6)} = 1.52273v_B$$

$$\frac{1}{2}(1.52273)^2 v_B^2 - \frac{1.3247(10)^{20}}{149.6(10)^9} = \frac{1}{2}v_B^2 - \frac{1.3247(10)^{20}}{227.8(10)^9}$$

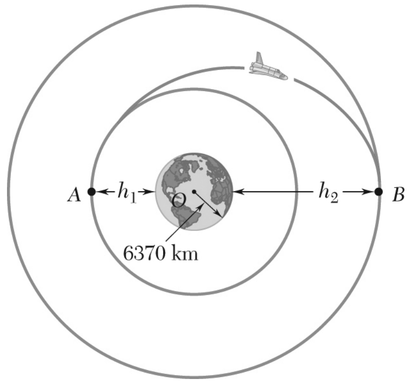
$$0.65935v_B^2 = 3.0398(10)^8$$

$$v_B^2 = 4.6102(10)^8$$

$$v_B = 21{,}471\text{ m/s},\quad v_A = 32{,}695\text{ m/s}$$

(*a*) Increase at *A*, $v_A - v_E = 32.695 - 29.758 = 2.94$ km/s ◀

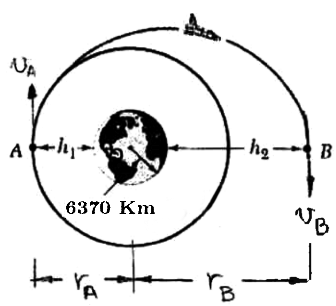
(*b*) Increase at *B*, $v_B - v_M = 24.115 - 21.471 = 2.64$ km/s ◀



PROBLEM 13.105

The optimal way of transferring a space vehicle from an inner circular orbit to an outer coplanar circular orbit is to fire its engines as it passes through A to increase its speed and place it in an elliptic transfer orbit. Another increase in speed as it passes through B will place it in the desired circular orbit. For a vehicle in a circular orbit about the earth at an altitude $h_1 = 200$ mi, which is to be transferred to a circular orbit at an altitude $h_2 = 500$ mi, determine (*a*) the required increases in speed at A and at B, (*b*) the total energy per unit mass required to execute the transfer.

SOLUTION



Elliptical orbit between A and B

Conservation of angular momentum

$$mr_Av_A = mr_Bv_B$$

$$v_A = \frac{r_B}{r_A}v_B = \frac{7.170}{6.690}v_B$$

$$r_A = 6370 \text{ km} + 320 \text{ km} = 6690 \text{ km}, \qquad r_A = 6.690 \times 10^6 \text{ m}$$

$$v_A = 1.0718 v_B \tag{1}$$

$$r_B = 6370 \text{ km} + 800 \text{ km} = 7170 \text{ km}, \qquad r_B = 7.170 \times 10^6 \text{ m}$$

$$R = (6370 \text{ km}) = 6.37 \times 10^6 \text{ m}$$

Conservation of energy

$$GM = gR^2 = (9.81 \text{ m/s}^2)(6.37 \times 10^6 \text{ m})^2 = 398.060 \times 10^{12} \text{ m}^3/\text{s}^2$$

Point A:

$$T_A = \frac{1}{2}mv_A^2 \qquad V_A = -\frac{GMm}{r_A} = -\frac{(398.060 \times 10^{12})m}{(6.690 \times 10^6)}$$

$$V_A = 59.501 \times 10^6 \text{ m}$$

Point B:

$$T_B = \frac{1}{2}mv_B^2 \qquad V_B = -\frac{GMm}{r_B} = -\frac{(398.060 \times 10^{12})m}{(7.170 \times 10^6)}$$

$$V_B = 55.5 \times 10^6 \text{ m}$$

PROBLEM 13.105 (Continued)

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}mv_A^2 - 59.501\times 10^6 \text{ m} = \frac{1}{2}mv_B^2 - 55.5\times 10^6 \text{ m}$$

$$v_A^2 - v_B^2 = 8.002\times 10^6$$

From (1) $\quad v_A = 1.0718 v_B \qquad v_B^2[(1.0718)^2 - 1] = 8.002\times 10^6$

$$v_B^2 = 53.79\times 10^6 \text{ m}^2/\text{s}^2, \qquad v_B = 7334 \text{ m/s}$$

$$v_A = (1.0718)(7334 \text{ m/s}) = 7861 \text{ m/s}$$

Circular orbit at A and B

(Equation 12.44)

$$(v_A)_C = \sqrt{\frac{GM}{r_A}} = \sqrt{\frac{398.060\times 10^{12}}{6.690\times 10^6}} = 7714 \text{ m/s}$$

$$(v_B)_C = \sqrt{\frac{GM}{r_B}} = \sqrt{\frac{398.060\times 10^{12}}{7.170\times 10^6}} = 7451 \text{ m/s}$$

(*a*) Increases in speed at A and B

$$\Delta v_A = v_A - (v_A)_C = 7861 - 7714 = 147 \text{ m/s} \blacktriangleleft$$

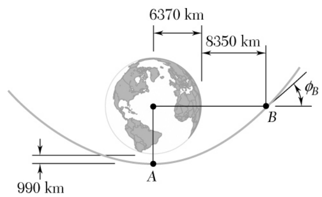
$$\Delta v_B = (v_B)_C - v_B = 7451 - 7334 = 117 \text{ m/s} \blacktriangleleft$$

(*b*) Total energy per unit mass

$$E/m = \frac{1}{2}[(v_A)^2 - (v_A)_C^2 + (v_B)_C^2 - (v_B)^2]$$

$$E/m = \frac{1}{2}[(7861)^2 - (7714)^2 + (7451)^2 - (7334)^2]$$

$$E/m = 2.01\times 10^6 \text{ J/kg} \blacktriangleleft$$



PROBLEM 13.106

During a flyby of the earth, the velocity of a spacecraft is 10.4 km/s as it reaches its minimum altitude of 990 km above the surface at Point A. At Point B the spacecraft is observed to have an altitude of 8350 km. Determine (a) the magnitude of the velocity at Point B, (b) the angle ϕ_B.

SOLUTION

At A:

$$h_A = vr = [1.04(10)^4 \text{ m/s}][6.37(10)^6 \text{ m} + 0.990(10)^6 \text{ m}]$$

$$h_A = 76.544(10)^9 \text{ m}^2/\text{s}$$

$$\frac{1}{m}(T_A + V_A) = \frac{1}{2}v^2 - \frac{GM}{r}$$

$$= \frac{1}{2}[1.04(10)^4]^2 - \frac{(9.81)[6.37(10)^6]^2}{[6.37(10)^6 + 0.990(10)^6]} \cong 0$$

(Parabolic orbit)

At B:

$$\frac{1}{m}(T_B + V_B) = \frac{1}{2}v_B^2 - \frac{GM}{r_B} = 0$$

$$\frac{1}{2}v_B^2 = \frac{(9.81)[6.37(10^6)]^2}{[6.37(10)^6 + 8.35(10)^6]}$$

$$v_B^2 = 54.084(10)^6$$

(a) $v_B = 7.35$ km/s ◀

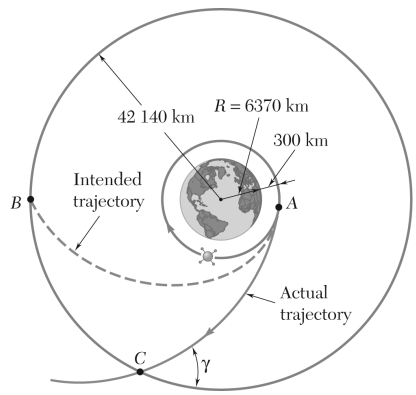
$$h_B = v_B \sin\phi_B r_B = 76.544(10)^9$$

$$\sin\phi_B = \frac{76.544(10)^9}{7.35(10^6)[6.37(10)^6 + 8.35(10)^6]}$$

$$= 0.707483$$

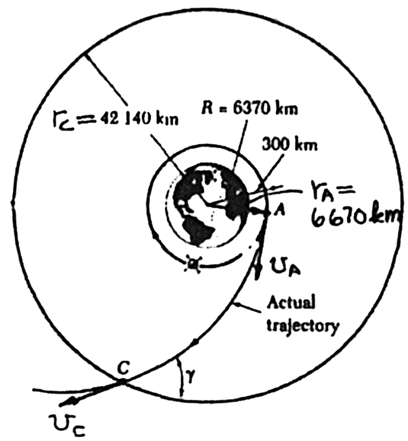
(b) $\phi_B = 45.0°$ ◀

PROBLEM 13.107



A space platform is in a circular orbit about the earth at an altitude of 300 km. As the platform passes through A, a rocket carrying a communications satellite is launched from the platform with a relative velocity of magnitude 3.44 km/s in a direction tangent to the orbit of the platform. This was intended to place the rocket in an elliptic transfer orbit bringing it to Point B, where the rocket would again be fired to place the satellite in a geosynchronous orbit of radius 42,140 km. After launching, it was discovered that the relative velocity imparted to the rocket was too large. Determine the angle γ at which the rocket will cross the intended orbit at Point C.

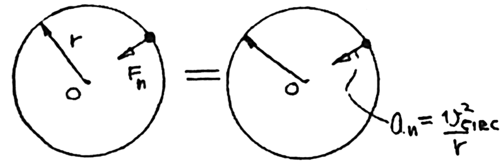
SOLUTION



$$R = 6370 \text{ km}$$
$$r_A = 6370 \text{ km} + 300 \text{ km}$$
$$r_A = 6.67 \times 10^6 \text{ m}$$
$$r_C = 42.14 \times 10^6 \text{ m}$$
$$GM = gR^2$$
$$GM = (9.81 \text{ m/s})(6.37 \times 10^6 \text{ m})^2$$
$$GM = 398.1 \times 10^{12} \text{ m}^3/\text{s}^2$$

For any circular orbit:

$$F_n = ma_n = \frac{mv^2_{\text{circ}}}{r}$$



$$F_n = \frac{GMm}{r^2} = m\frac{v^2_{\text{circ}}}{r}$$
$$v_{\text{circ}} = \sqrt{\frac{GM}{r}}$$

Velocity at A:

$$(v_A)_{\text{circ}} = \sqrt{\frac{GM}{r_A}} = \sqrt{\frac{(398.1 \times 10^{12} \text{ m}^3/\text{s}^3)}{(6.67 \times 10^6 \text{ m})}} = 7.726 \times 10^3 \text{ m/s}$$
$$v_A = (v_A)_{\text{circ}} + (v_A)_R = 7.726 \times 10^3 + 3.44 \times 10^3 = 11.165 \times 10^3 \text{ m/s}$$

PROBLEM 13.107 (Continued)

Velocity at C:

Conservation of energy: $T_A + V_A = T_C + V_C$

$$\frac{1}{2}\cancel{m}v_A^2 - \frac{GM\cancel{m}}{r_A} = \frac{1}{2}\cancel{m}v_C^2 - \frac{GM\cancel{m}}{r_C}$$

$$v_C^2 = v_A^2 + 2GM\left(\frac{1}{r_C} - \frac{1}{r_A}\right)$$

$$= (11.165\times10^3)^2 + 2(398.1\times10^{12})\left(\frac{1}{42.14\times10^6} - \frac{1}{6.67\times10^6}\right)$$

$$v_C^2 = 124.67\times10^6 - 100.48\times10^6$$

$$= 24.19\times10^6 \text{ m}^2/\text{s}^2$$

$$v_C = 4.919\times10^3 \text{ m/s}$$

Conservation of angular momentum:

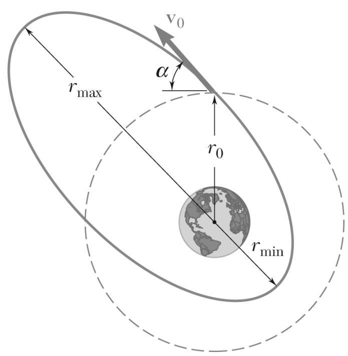
$$r_A m v_A = r_C m v_C \cos\gamma$$

$$\cos\gamma = \frac{r_A v_A}{r_C v_C}$$

$$= \frac{(6.67\times10^6)(11.165\times10^3)}{(42.14\times10^6)(4.919\times10^3)}$$

$$\cos\gamma = 0.35926$$

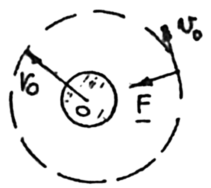
$\gamma = 68.9°$ ◀



PROBLEM 13.108

A satellite is projected into space with a velocity $\mathbf{v}_0$ at a distance r_0 from the center of the earth by the last stage of its launching rocket. The velocity $\mathbf{v}_0$ was designed to send the satellite into a circular orbit of radius r_0. However, owing to a malfunction of control, the satellite is not projected horizontally but at an angle α with the horizontal and, as a result, is propelled into an elliptic orbit. Determine the maximum and minimum values of the distance from the center of the earth to the satellite.

SOLUTION



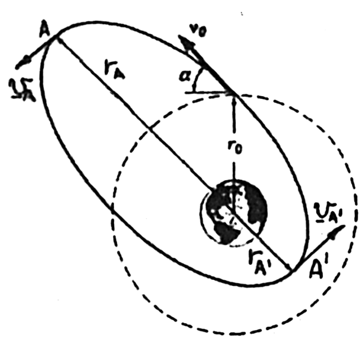
For circular orbit of radius r_0

$$F = ma_n \qquad \frac{GMm}{r_0^2} = m\frac{v_0^2}{r_0}$$

$$v_0^2 = \frac{GM}{r_0}$$

But v_0 forms an angle α with the intended circular path.

For elliptic orbit.



Conservation of angular momentum:

$$r_0 m v_0 \cos\alpha = r_A m v_A$$

$$v_A = \left(\frac{r_0}{r_A}\cos\alpha\right)v_0 \tag{1}$$

Conservation of energy:

$$\frac{1}{2}mv_0^2 - \frac{GMm}{r_0} = \frac{1}{2}mv_A^2 - \frac{GMm}{r_A}$$

$$v_0^2 - v_A^2 = \frac{2GM}{r_0}\left(1 - \frac{r_0}{r_A}\right)$$

Substitute for v_A from (1)

$$v_0^2\left[1 - \left(\frac{r_0}{r_A}\right)^2\cos^2\alpha\right] = \frac{2GM}{r_0}\left(1 - \frac{r_0}{r_A}\right)$$

But $\quad v_0^2 = \dfrac{GM}{r_0}, \qquad$ thus $\quad 1 - \left(\dfrac{r_0}{r_A}\right)^2\cos^2\alpha = 2\left(1 - \dfrac{r_0}{r_A}\right)$

$$\cos^2\alpha\left(\frac{r_0}{r_A}\right)^2 - 2\left(\frac{r_0}{r_A}\right) + 1 = 0$$

PROBLEM 13.108 (Continued)

Solving for $\dfrac{r_0}{r_A}$

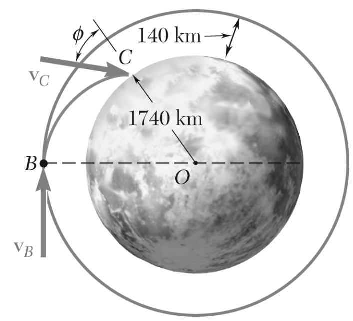
$$\frac{r_0}{r_A} = \frac{+2 \pm \sqrt{4 - 4\cos^2\alpha}}{2\cos^2\alpha} = \frac{1 \pm \sin\alpha}{1 - \sin^2\alpha}$$

$$r_A = \frac{(1 + \sin\alpha)(1 - \sin\alpha)}{1 \pm \sin\alpha} r_0 = (1 \mp \sin\alpha) r_0$$

also valid for Point A'

Thus,

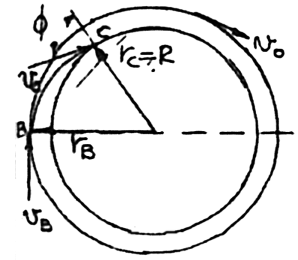
$$r_{\max} = (1 + \sin\alpha) r_0 \qquad r_{\min} = (1 - \sin\alpha) r_0 \blacktriangleleft$$



PROBLEM 13.109

Upon the LEM's return to the command module, the Apollo spacecraft of Problem 13.88 was turned around so that the LEM faced to the rear. The LEM was then cast adrift with a velocity of 200 m/s relative to the command module. Determine the magnitude and direction (angle ϕ formed with the vertical OC) of the velocity $\mathbf{v}_C$ of the LEM just before it crashed at C on the moon's surface.

SOLUTION



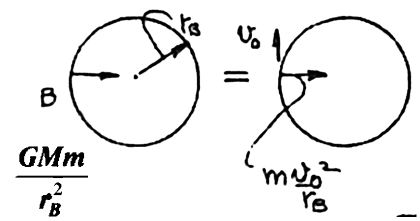
$R = 1740$ km

Command module in circular orbit

$$r_B = 1740 + 140 = 1880 \text{ km} = 1.88 \times 10^6 \text{ m}$$

$$GM_{\text{moon}} = 0.0123\, GM_{\text{earth}} = 0.0123\, gR^2$$

$$= 0.0123(9.81)(6.37 \times 10^6)^2 = 4.896 \times 10^{12} \text{ m}^3/\text{s}^2$$



$$\Sigma F = ma_n \quad \frac{GM_m m}{r_B^2} = \frac{mv_0^2}{r_B}$$

$$v_0 = \sqrt{\frac{GMm}{r_B}} = \sqrt{\frac{4.896 \times 10^{12}}{1.88 \times 10^6}}$$

$$v_0 = 1614 \text{ m/s} \qquad v_B = 1614 - 200 = 1414 \text{ m/s}$$

Conservation of energy between B and C:

$$\frac{1}{2}mv_B^2 - \frac{GM_m m}{r_B} = \frac{1}{2}mv_C^2 - \frac{GM_m m}{r_C} \qquad r_C = R$$

$$v_C^2 = v_B^2 + \frac{2GMm}{r_B}\left(\frac{r_B}{R} - 1\right)$$

$$v_C^2 = (1414 \text{ m/s})^2 + 2\frac{(4.896 \times 10^{12} \text{ m}^3/\text{s}^2)}{(1.88 \times 10^6 \text{ m})}\left(\frac{1.88 \times 10^6}{1.74 \times 10^6} - 1\right)$$

$$v_C^2 = 1.999 \times 10^6 + 0.4191 \times 10^6 = 2.418 \times 10^6 \text{ m}^2/\text{s}^2$$

$v_C = 1555$ m/s ◀

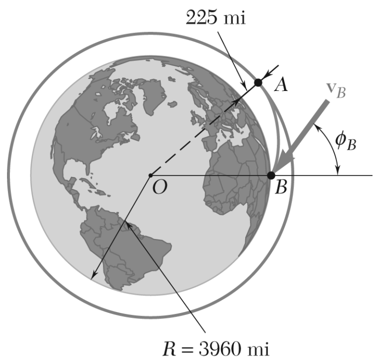
Conservation of angular momentum:

$$r_B m v_B = R m v_C \sin\phi$$

$$\sin\phi = \frac{r_B v_B}{r_C v_C} = \frac{(1.88 \times 10^6 \text{ m})(1414 \text{ m/s})}{(1.74 \times 10^6 \text{ m})(1555 \text{ m/s})} = 0.98249$$

$$\phi = 79.26°$$

$\phi = 79.3°$ ◀



PROBLEM 13.110

A space vehicle is in a circular orbit at an altitude of 225 mi above the earth. To return to earth, it decreases its speed as it passes through *A* by firing its engine for a short interval of time in a direction opposite to the direction of its motion. Knowing that the velocity of the space vehicle should form an angle $\phi_B = 60°$ with the vertical as it reaches Point *B* at an altitude of 40 mi, determine (*a*) the required speed of the vehicle as it leaves its circular orbit at *A*, (*b*) its speed at Point *B*.

SOLUTION

(*a*)

$$r_A = 3960 \text{ mi} + 225 \text{ mi} = 4185 \text{ mi}$$

$$r_A = 4185 \text{ mi} \times 5280 \text{ ft/mi} = 22,097 \times 10^3 \text{ ft}$$

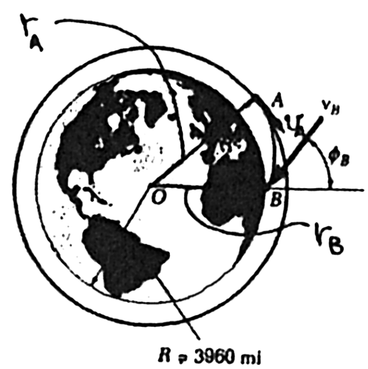
$$r_B = 3960 \text{ mi} + 40 \text{ mi} = 4000 \text{ mi}$$

$$r_B = 4000 \times 5280 = 21,120 \times 10^3 \text{ ft}$$

$$R = 3960 \text{ mi} = 20,909 \times 10^3 \text{ ft}$$

$$GM = gR^2 = (32.2 \text{ ft/s}^2)(20,909 \times 10^3 \text{ ft})^2$$

$$GM = 14.077 \times 10^{15} \text{ ft}^3/\text{s}^2$$



Conservation of energy:

$$T_A = \frac{1}{2} m v_A^2$$

$$V_A = \frac{-GMm}{r_A} = \frac{-14.077 \times 10^{15} m}{22,097 \times 10^3} = -637.1 \times 10^6 m$$

$$T_B = \frac{1}{2} m v_B^2$$

$$V_B = \frac{-GMm}{r_B} = \frac{-14.077 \times 10^{15} m}{21,120 \times 10^3} = -666.5 \times 10^6 m$$

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2} \cancel{m} v_A^2 - 637.1 \times 10^6 \cancel{m} = \frac{1}{2} \cancel{m} v_B^2 - 666.5 \times 10^6 \cancel{m}$$

$$v_A^2 = v_B^2 - 58.94 \times 10^6 \qquad (1)$$

Conservation of angular momentum:

$$r_A m v_A = r_B m v_B \sin\phi_B$$

$$v_B = \frac{(r_A)v_A}{(r_B)(\sin\phi_B)} = \frac{4185}{4000}\left(\frac{1}{\sin 60°}\right)v_A$$

$$v_B = 1.208\, v_A \qquad (2)$$

Substitute v_B from (2) in (1)

$$v_A^2 = (1.208 v_A)^2 - 58.94\times 10^6$$

$$v_A^2[(1.208)^2 - 1] = 58.94\times 10^6$$

$$v_A^2 = 128.27\times 10^6 \text{ ft}^2/\text{s}^2$$

(*a*) $v_A = 11.32\times 10^3$ ft/s ◀

(*b*) From (2)

$$v_B = 1.208 v_A$$

$$= 1.208(11.32\times 10^6)$$

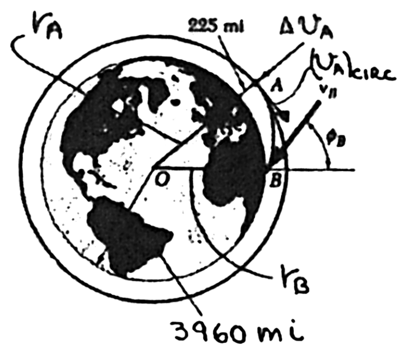
$$= 13.68\times 10^3 \text{ ft/s}$$

$v_B = 13.68\times 10^3$ ft/s ◀

PROBLEM 13.111*

In Problem 13.110, the speed of the space vehicle was decreased as it passed through A by firing its engine in a direction opposite to the direction of motion. An alternative strategy for taking the space vehicle out of its circular orbit would be to turn it around so that its engine would point away from the earth and then give it an incremental velocity $\Delta \mathbf{v}_A$ toward the center O of the earth. This would likely require a smaller expenditure of energy when firing the engine at A, but might result in too fast a descent at B. Assuming this strategy is used with only 50 percent of the energy expenditure used in Problem 13.109, determine the resulting values of ϕ_B and v_B.

SOLUTION



$$r_A = 3960 \text{ mi} + 225 \text{ mi}$$

$$r_A = 4185 \text{ mi} = 22.097\times 10^6 \text{ ft}$$

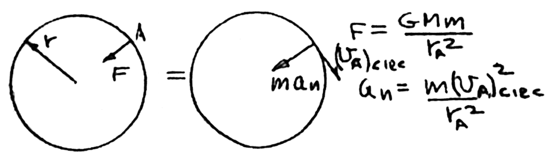
$$r_B = 3960 \text{ mi} + 40 \text{ mi} = 4000 \text{ mi}$$

$$r_B = 21.120\times 10^6 \text{ ft}$$

$$GM = gR^2 = (32.2 \text{ ft/s}^2)[(3960)(5280) \text{ ft}^2]$$

$$GM = 14.077\times 10^{15} \text{ ft}^3/\text{s}^2$$

Velocity in circular orbit at 225 m altitude:



Newton's second law

$$F = ma_n: \quad \frac{GMm}{r_A^2} = \frac{m(v_A)_{\text{circ}}^2}{r_A}$$

$$(v_A)_{\text{circ}} = \sqrt{\frac{GM}{r_A}} = \sqrt{\frac{14.077\times 10^{15}}{22.097\times 10^6}}$$

$$= 25.24\times 10^3 \text{ ft/s}$$

Energy expenditure:

From Problem 13.110, $\qquad v_A = 11.32\times 10^3 \text{ ft/s}$

Energy, $\qquad \Delta E_{109} = \frac{1}{2}m(v_A)_{\text{circ}}^2 - \frac{1}{2}mv_A^2$

$$\Delta E_{109} = \frac{1}{2}m(25.24\times 10^3)^2 - \frac{1}{2}m(11.32\times 10^3)^2$$

$$\Delta E_{109} = 254.46\times 10^6\, m \text{ ft}\cdot\text{lb}$$

$$\Delta E_{110} = (0.50)\Delta E_{109} = \frac{(254.46\times 10^6\, m)}{2} \text{ft}\cdot\text{lb}$$

PROBLEM 13.111* (Continued)

Thus, additional kinetic energy at A is

$$\frac{1}{2}m(\Delta v_A)^2 = \Delta E_{110} = \frac{(254.46\times 10^6 m)}{2}\ \text{ft}\cdot\text{lb} \qquad (1)$$

Conservation of energy between A and B:

$$T_A = \frac{1}{2}m[(v_A)^2_{\text{circ}} + (\Delta v_A)^2] \qquad V_A = \frac{-GMm}{r_A}$$

$$T_B = \frac{1}{2}mv_B^2 \qquad V_B = \frac{-GMm}{r_A}$$

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}m(25.24\times 10^3)^2 + \frac{254.46\times 10^6 m}{2} - \frac{14.077\times 10^{15} m}{22.097\times 10^6} = \frac{1}{2}mv_B^2 - \frac{14.077\times 10^{15} m}{21.120\times 10^6}$$

$$v_B^2 = 637.06\times 10^6 + 254.46\times 10^6 - 1274.1\times 10^6 + 1333\times 10^6$$

$$v_B^2 = 950.4\times 10^3$$

$$v_B = 30.88\times 10^3\ \text{ft/s} \blacktriangleleft$$

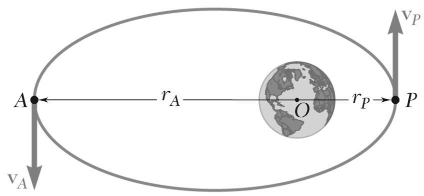
Conservation of angular momentum between A and B:

$$r_A m(v_A)_{\text{circ}} = r_B m v_B \sin\phi_B$$

$$\sin\phi_B = \left(\frac{r_A}{r_B}\right)\frac{(v_A)_{\text{circ}}}{(v_B)} = \frac{(4185)}{(4000)}\frac{(25.24\times 10^3)}{(30.88\times 10^3)} = 0.8565$$

$$\phi_B = 58.9° \blacktriangleleft$$

PROBLEM 13.112



Show that the values v_A and v_P of the speed of an earth satellite at the apogee A and the perigee P of an elliptic orbit are defined by the relations

$$v_A^2 = \frac{2GM}{r_A + r_P}\frac{r_P}{r_A} \qquad v_P^2 = \frac{2GM}{r_A + r_P}\frac{r_A}{r_P}$$

where M is the mass of the earth, and r_A and r_P represent, respectively, the maximum and minimum distances of the orbit to the center of the earth.

SOLUTION

Conservation of angular momentum:

$$r_A m v_A = r_P m v_P$$

$$v_A = \frac{r_P}{r_A} v_P \qquad (1)$$

Conservation of energy:

$$\frac{1}{2} m v_P^2 - \frac{GMm}{r_P} = \frac{1}{2} m v_A^2 - \frac{GMm}{r_A} \qquad (2)$$

Substituting for v_A from (1) into (2)

$$v_P^2 - \frac{2GM}{r_P} = \left(\frac{r_P}{r_A}\right)^2 v_P^2 - \frac{2GM}{r_A}$$

$$\left(1 - \left(\frac{r_P}{r_A}\right)^2\right) v_P^2 = 2GM\left(\frac{1}{r_P} - \frac{1}{r_A}\right)$$

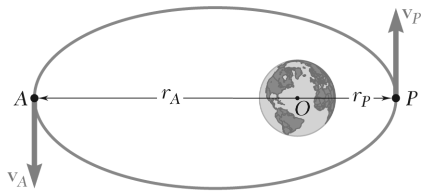
$$\frac{r_A^2 - r_P^2}{r_A^2} v_P^2 = 2GM \frac{r_A - r_P}{r_A r_P}$$

with $$r_A^2 - r_P^2 = (r_A - r_P)(r_A + r_P)$$

$$v_P^2 = \frac{2GM}{r_A + r_P}\left(\frac{r_A}{r_P}\right) \quad (3) \blacktriangleleft$$

Exchanging subscripts P and A

$$v_A^2 = \frac{2GM}{r_A + r_P}\left(\frac{r_P}{r_A}\right) \qquad \text{Q.E.D.} \quad \blacktriangleleft$$

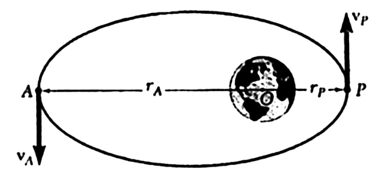


PROBLEM 13.113

Show that the total energy E of an earth satellite of mass m describing an elliptic orbit is $E = -GMm/(r_A + r_P)$, where M is the mass of the earth, and r_A and r_P represent, respectively, the maximum and minimum distances of the orbit to the center of the earth. (Recall that the gravitational potential energy of a satellite was defined as being zero at an infinite distance from the earth.)

SOLUTION

See solution to Problem 13.112 (above) for derivation of Equation (3).



$$v_P^2 = \frac{2GM}{(r_A + r_P)} \frac{r_A}{r_P}$$

Total energy at Point P is

$$E = T_P + V_P = \frac{1}{2} m v_P^2 - \frac{GMm}{r_P}$$

$$= \frac{1}{2}\left[\frac{2GMm}{(r_A + r_0)} \frac{r_A}{r_P}\right] - \frac{GMm}{r_P}$$

$$= GMm\left[\frac{r_A}{r_P(r_A + r_P)} - \frac{1}{r_P}\right]$$

$$= GMm \frac{(r_A - r_A - r_P)}{r_P(r_A + r_P)}$$

$$E = -\frac{GMm}{r_A + r_P} \blacktriangleleft$$

Note: Recall that gravitational potential of a satellite is defined as being zero at an infinite distance from the earth.

PROBLEM 13.114*

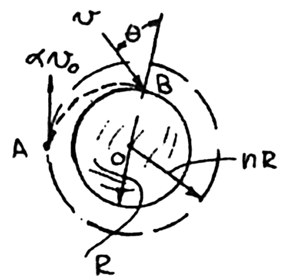
A space probe describes a circular orbit of radius nR with a velocity $\mathbf{v}_0$ about a planet of radius R and center O. Show that (a) in order for the probe to leave its orbit and hit the planet at an angle θ with the vertical, its velocity must be reduced to $\alpha\mathbf{v}_0$, where

$$\alpha = \sin\theta\sqrt{\frac{2(n-1)}{n^2 - \sin^2\theta}}$$

(b) the probe will not hit the planet if α is larger than $\sqrt{2/(1+n)}$.

SOLUTION

(a) Conservation of energy:



At A:
$$T_A = \frac{1}{2}m(\alpha v_0)^2$$
$$V_A = -\frac{GMm}{nR}$$

At B:
$$T_B = \frac{1}{2}mv^2$$
$$V_B = -\frac{GMm}{R}$$

M = mass of planet

m = mass of probe

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}\cancel{m}(\alpha v_0)^2 - \frac{GM\cancel{m}}{nR} = \frac{1}{2}\cancel{m}v^2 - \frac{GM\cancel{m}}{R} \qquad (1)$$

Conservation of angular momentum:

$$nRm\alpha v_0 = Rmv\sin\theta$$

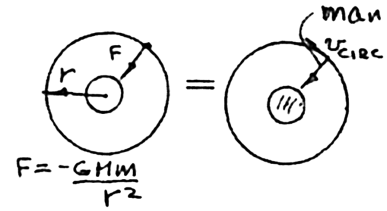
$$v = \frac{n\alpha v_0}{\sin\theta} \qquad (2)$$

Replacing v in (1) by (2)

$$(\alpha v_0)^2 - \frac{2GM}{nR} = \left(\frac{n\alpha v_0}{\sin\theta}\right)^2 - \frac{2GM}{R} \qquad (3)$$

PROBLEM 13.114* (Continued)

For any circular orbit.



$$a_n = \frac{v^2}{r}$$

Newton's second law

$$\frac{-GMm}{r^2} = \frac{m(v)^2_{\text{circ}}}{r}$$

$$v_{\text{circ}} = \sqrt{\frac{GM}{r}}$$

For $r = nR$, $$v_0 = v_{\text{circ}} = \sqrt{\frac{GM}{nR}}$$

Substitute for v_0 in (3)

$$\alpha^2 \frac{G\cancel{M}}{n\cancel{R}} - \frac{2G\cancel{M}}{n\cancel{R}} = \frac{n^2\alpha^2}{\sin^2\theta}\left(\frac{G\cancel{M}}{n\cancel{R}}\right) - \frac{2G\cancel{M}}{\cancel{R}}$$

$$\alpha^2\left[1 - \frac{n^2}{\sin^2\theta}\right] = 2(1-n)$$

$$\alpha^2 = \frac{2(1-n)(\sin^2\theta)}{(\sin^2\theta - n^2)} = \frac{2(n-1)\sin^2\theta}{(n^2 - \sin^2\theta)}$$

$$\alpha = \sin\theta\sqrt{\frac{2(n-1)}{n^2 - \sin^2\theta}} \qquad \text{Q.E.D.} \qquad \blacktriangleleft$$

(*b*) Probe will just miss the planet if $\theta > 90°$,

$$\alpha = \sin 90°\sqrt{\frac{2(n-1)}{n^2 - \sin^2 90°}} = \sqrt{\frac{2}{n+1}} \qquad \blacktriangleleft$$

Note: $$n^2 - 1 = (n-1)(n+1)$$

PROBLEM 13.115

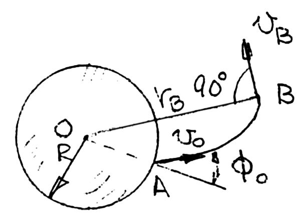
A missile is fired from the ground with an initial velocity $\mathbf{v}_0$ forming an angle ϕ_0 with the vertical. If the missile is to reach a maximum altitude equal to αR, where R is the radius of the earth, (*a*) show that the required angle ϕ_0 is defined by the relation

$$\sin\phi_0 = (1+\alpha)\sqrt{1-\frac{\alpha}{1+\alpha}\left(\frac{v_{\text{esc}}}{v_0}\right)^2}$$

where v_{esc} is the escape velocity, (*b*) determine the range of allowable values of v_0.

SOLUTION

(*a*) $$r_A = R$$



Conservation of angular momentum:

$$Rmv_0\sin\phi_0 = r_B m v_B$$

$$r_B = R + \alpha R = (1+\alpha)R$$

$$v_B = \frac{Rv_0\sin\phi_0}{(1+\alpha)R} = \frac{v_0\sin\phi_0}{(1+\alpha)} \tag{1}$$

Conservation of energy:

$$T_A + V_A = T_B + V_B \qquad \frac{1}{2}mv_0^2 - \frac{GMm}{R} = \frac{1}{2}mv_B^2 - \frac{GMm}{(1+\alpha)R}$$

$$v_0^2 - v_B^2 = \frac{2GMm}{R}\left(1-\frac{1}{1+\alpha}\right) = \frac{2GMm}{R}\left(\frac{\alpha}{1+\alpha}\right)$$

Substitute for v_B from (1)

$$v_0^2\left(1-\frac{\sin^2\phi_0}{(1+\alpha)^2}\right) = \frac{2GMm}{R}\left(\frac{\alpha}{1+\alpha}\right)$$

From Equation (12.43): $$v_{\text{esc}}^2 = \frac{2GM}{R}$$

$$v_0^2\left(1-\frac{\sin^2\phi_0}{(1+\alpha)^2}\right) = v_{\text{esc}}^2\left(\frac{\alpha}{1+\alpha}\right)$$

$$\frac{\sin^2\phi_0}{(1+\alpha)^2} = 1-\left(\frac{v_{\text{esc}}}{v_0}\right)^2\frac{\alpha}{1+\alpha} \tag{2}$$

$$\sin\phi_0 = (1+\alpha)\sqrt{1-\frac{\alpha}{1+\alpha}\left(\frac{v_{\text{esc}}}{v_0}\right)^2} \qquad \text{Q.E.D.}$$

PROBLEM 13.115 (Continued)

(*b*) Allowable values of v_0 (for which maximum altitude = αR)

$$0 < \sin^2 \phi_0 < 1$$

For $\sin\phi_0 = 0$, from (2)

$$0 = 1 - \left(\frac{v_{\text{esc}}}{v_0}\right)^2 \frac{\alpha}{1+\alpha}$$

$$v_0 = v_{\text{esc}}\sqrt{\frac{\alpha}{1+\alpha}}$$

For $\sin\phi_0 = 1$, from (2)

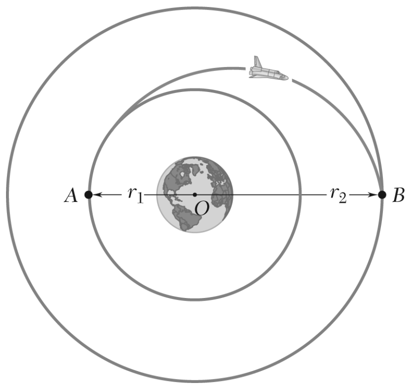
$$\frac{1}{(1+\alpha)^2} = 1 - \left(\frac{v_{\text{esc}}}{v_0}\right)^2 \frac{\alpha}{1+\alpha}$$

$$\left(\frac{v_{\text{esc}}}{v_0}\right)^2 = \frac{1}{\alpha}\left(1+\alpha-\frac{1}{1+\alpha}\right) = \frac{1+2\alpha+\alpha^2-1}{\alpha(1+\alpha)} = \frac{2+\alpha}{1+\alpha}$$

$$v_0 = v_{\text{esc}}\sqrt{\frac{1+\alpha}{2+\alpha}}$$

$$v_{\text{esc}}\sqrt{\frac{\alpha}{1+\alpha}} < v_0 < v_{\text{esc}}\sqrt{\frac{1+\alpha}{2+\alpha}} \quad \blacktriangleleft$$

PROBLEM 13.116



A spacecraft of mass m describes a circular orbit of radius r_1 around the earth. (a) Show that the additional energy ΔE which must be imparted to the spacecraft to transfer it to a circular orbit of larger radius r_2 is

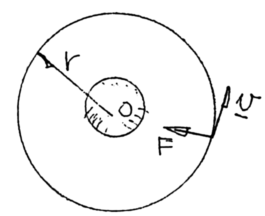
$$\Delta E = \frac{GMm(r_2 - r_1)}{2r_1 r_2}$$

where M is the mass of the earth. (b) Further show that if the transfer from one circular orbit to the other is executed by placing the spacecraft on a transitional semielliptic path AB, the amounts of energy ΔE_A and ΔE_B which must be imparted at A and B are, respectively, proportional to r_2 and r_1:

$$\Delta E_A = \frac{r_2}{r_1 + r_2}\Delta E \qquad \Delta E_B = \frac{r_1}{r_1 + r_2}\Delta E$$

SOLUTION

(a) For a *circular* orbit of radius r



$$F = ma_n: \quad \frac{GMm}{r^2} = m\frac{v^2}{r}$$

$$v^2 = \frac{GM}{r}$$

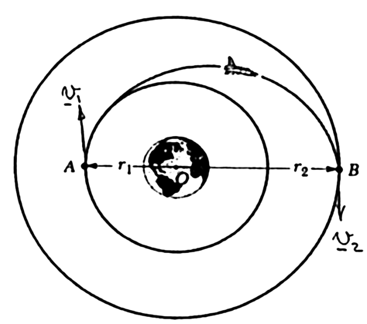
$$E = T + V = \frac{1}{2}mv^2 - \frac{GMm}{r} = -\frac{1}{2}\frac{GMm}{r} \tag{1}$$

Thus ΔE required to pass from circular orbit of radius r_1 to circular orbit of radius r_2 is

$$\Delta E = E_1 - E_2 = -\frac{1}{2}\frac{GMm}{r_1} + \frac{1}{2}\frac{GMm}{r_2}$$

$$\Delta E = \frac{GMm(r_2 - r_1)}{2r_1 r_2} \qquad \text{Q.E.D.} \tag{2}$$

(b) For an elliptic orbit, we recall Equation (3) derived in Problem 13.113 (with $v_P = v_1$)



$$v_1^2 = \frac{2Gm}{(r_1 + r_2)}\frac{r_2}{r_1}$$

At Point A: Initially spacecraft is in a circular orbit of radius r_1.

$$v_{\text{circ}}^2 = \frac{GM}{r_1}$$

$$T_{\text{circ}} = \frac{1}{2}mv_{\text{circ}}^2 = \frac{1}{2}m\frac{GM}{r_1}$$

After the spacecraft engines are fired and it is placed on a semi-elliptic path AB, we recall

$$v_1^2 = \frac{2GM}{(r_1 + r_2)} \cdot \frac{r_2}{r_1}$$

and

$$T_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}m\frac{2GMr_2}{r_1(r_1 + r_2)}$$

At Point A, the increase in energy is

$$\Delta E_A = T_1 - T_{\text{circ}} = \frac{1}{2}m\frac{2GMr_2}{r_1(r_1 + r_2)} - \frac{1}{2}m\frac{GM}{r_1}$$

$$\Delta E_A = \frac{GMm(2r_2 - r_1 - r_2)}{2r_1(r_1 + r_2)} = \frac{GMm(r_2 - r_1)}{2r_1(r_1 + r_2)}$$

$$\Delta E_A = \frac{r_2}{r_1 + r_2}\left[\frac{GMm(r_2 - r_1)}{2r_1r_2}\right]$$

Recall Equation (2):

$$\Delta E_A = \frac{r_2}{(r_1 + r_2)}\Delta E \qquad \text{Q.E.D.}$$

A similar derivation at Point B yields,

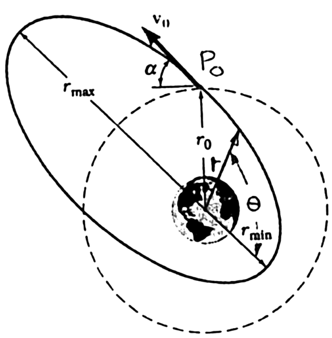
$$\Delta E_B = \frac{r_1}{(r_1 + r_2)}\Delta E \qquad \text{Q.E.D.}$$

PROBLEM 13.117*

Using the answers obtained in Problem 13.108, show that the intended circular orbit and the resulting elliptic orbit intersect at the ends of the minor axis of the elliptic orbit.

PROBLEM 13.108 A satellite is projected into space with a velocity $\mathbf{v}_0$ at a distance r_0 from the center of the earth by the last stage of its launching rocket. The velocity $\mathbf{v}_0$ was designed to send the satellite into a circular orbit of radius r_0. However, owing to a malfunction of control, the satellite is not projected horizontally but at an angle α with the horizontal and, as a result, is propelled into an elliptic orbit. Determine the maximum and minimum values of the distance from the center of the earth to the satellite.

SOLUTION



If the point of intersection P_0 of the circular and elliptic orbits is at an end of the minor axis, then v_0 is parallel to the major axis. This will be the case only if $\alpha + 90° = \theta_0$, that is if $\cos\theta_0 = -\sin\alpha$. We must therefore prove that

$$\cos\theta_0 = -\sin\alpha \tag{1}$$

We recall from Equation (12.39):

$$\frac{1}{r} = \frac{GM}{h^2} + C\cos\theta \tag{2}$$

When $\theta = 0$, $\quad r = r_{\min}$ and $r_{\min} = r_0(1-\sin\alpha)$

$$\frac{1}{r_0(1-\sin\alpha)} = \frac{GM}{h^2} + C \tag{3}$$

For $\theta = 180°$, $\quad r = r_{\max} = r_0(1+\sin\alpha)$

$$\frac{1}{r_0(1+\sin\alpha)} = \frac{GM}{h^2} - C \tag{4}$$

Adding (3) and (4) and dividing by 2:

$$\frac{GM}{h^2} = \frac{1}{2r_0}\left(\frac{1}{1-\sin\alpha} + \frac{1}{1+\sin\alpha}\right)$$
$$= \frac{1}{r_0\cos^2\alpha}$$

Subtracting (4) from (3) and dividing by 2:

$$C = \frac{1}{2r_0}\left(\frac{1}{1-\sin\alpha} - \frac{1}{1+\sin\alpha}\right) = \left(\frac{1}{2r_0}\right)\frac{2\sin\alpha}{1-\sin^2\alpha}$$
$$C = \frac{\sin\alpha}{r_0\cos^2\alpha}$$

PROBLEM 13.117* (Continued)

Substituting for $\frac{GM}{h^2}$ and C into Equation (2)

$$\frac{1}{r} = \frac{1}{r_0 \cos^2 \alpha}(1 + \sin \alpha \cos \theta) \tag{5}$$

Letting $r = r_0$ and $\theta = \theta_0$ in Equation (5), we have

$$\cos^2 \alpha = 1 + \sin \alpha \cos \theta_0$$

$$\cos \theta_0 = \frac{\cos^2 \alpha - 1}{\sin \alpha}$$

$$= -\frac{\sin^2 \alpha}{\sin \alpha}$$

$$= -\sin \alpha$$

This proves the validity of Equation (1) and thus P_0 is an end of the minor axis of the elliptic orbit.

PROBLEM 13.118*

(*a*) Express in terms of r_{min} and v_{max} the angular momentum per unit mass, *h*, and the total energy per unit mass, *E*/*m*, of a space vehicle moving under the gravitational attraction of a planet of mass *M* (Figure 13.15). (*b*) Eliminating v_{max} between the equations obtained, derive the formula

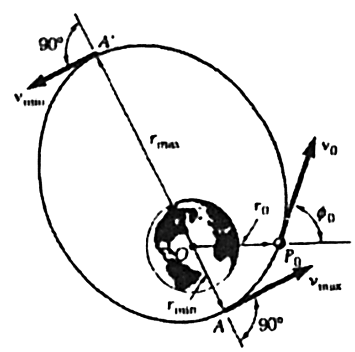
$$\frac{1}{r_{min}} = \frac{GM}{h^2}\left[1+\sqrt{1+\frac{2E}{m}\left(\frac{h}{GM}\right)^2}\right]$$

(*c*) Show that the eccentricity ε of the trajectory of the vehicle can be expressed as

$$\varepsilon = \sqrt{1+\frac{2E}{m}\left(\frac{h}{GM}\right)^2}$$

(*d*) Further show that the trajectory of the vehicle is a hyperbola, an ellipse, or a parabola, depending on whether *E* is positive, negative, or zero.

SOLUTION



(*a*) Point *A*:

Angular momentum per unit mass.

$$h = \frac{H_0}{m} = \frac{r_{min} m v_{max}}{m}$$

$$h = r_{min} v_{max} \quad (1) \blacktriangleleft$$

Energy per unit mass

$$\frac{E}{m} = \frac{1}{m}(T+V)$$

$$\frac{E}{m} = \frac{1}{m}\left(\frac{1}{2}mv_{max}^2 - \frac{GMm}{r_{min}}\right) = \frac{1}{2}v_{max}^2 - \frac{GM}{r_{min}} \quad (2) \blacktriangleleft$$

(*b*) From Eq. (1): $v_{max} = h/r_{min}$ substituting into (2)

$$\frac{E}{m} = \frac{1}{2}\frac{h^2}{r_{min}^2} - \frac{GM}{r_{min}}$$

$$\left(\frac{1}{r_{min}}\right)^2 - \frac{2GM}{h^2}\cdot\frac{1}{r_{min}} - \frac{2\left(\frac{E}{m}\right)}{h^2} = 0$$

PROBLEM 13.118* (Continued)

Solving the quadratic: $$\frac{1}{r_{\min}} = \frac{GM}{h^2} + \sqrt{\left(\frac{GM}{h^2}\right)^2 + \frac{2\left(\frac{E}{m}\right)}{h^2}}$$

Rearranging

$$\frac{1}{r_{\min}} = \frac{GM}{h^2}\left[1 + \sqrt{1 + \frac{2E}{m}\left(\frac{h}{GM}\right)^2}\right] \qquad (3) \blacktriangleleft$$

(*c*) Eccentricity of the trajectory:

Eq. (12.39′) $\dfrac{1}{r} = \dfrac{GM}{h^2}(1 + \varepsilon\cos\theta)$

When $\theta = 0$, $\cos\theta = 1$ and $r = r_{\min}$

Thus,

$$\frac{1}{r_{\min}} = \frac{GM}{h^2}(1 + \varepsilon) \qquad (4)$$

Comparing (3) and (4), $$\varepsilon = \sqrt{1 + \frac{2E}{m}\left(\frac{h}{GM}\right)^2} \qquad (5)$$

(*d*) Recalling discussion in section 12.12 and in view of Eq. (5)

1. Hyperbola if $\varepsilon > 1$, that is, if $E > 0$ ◀
2. Parabola if $\varepsilon = 1$, that is, if $E = 0$ ◀
3. Ellipse if $\varepsilon < 1$, that is, if $E < 0$ ◀

Note: For circular orbit $\varepsilon = 0$ and

$$1 + \frac{2E}{m}\left(\frac{h}{GM}\right)^2 = 0 \qquad \text{or} \qquad E = -\left(\frac{GM}{h}\right)^2 \frac{m}{2},$$

but for circular orbit $v^2 = \dfrac{GM}{r}$ and $h^2 = v^2 r^2 = GMr$,

thus $$E = -\frac{1}{2}m\frac{(GM)^2}{GMr} = -\frac{1}{2}\frac{GMm}{r}$$

PROBLEM 13.CQ4

A large insect impacts the front windshield of a sports car traveling down a road. Which of the following statements is true during the collision?

(*a*) The car exerts a greater force on the insect than the insect exerts on the car.

(*b*) The insect exerts a greater force on the car than the car exerts on the insect.

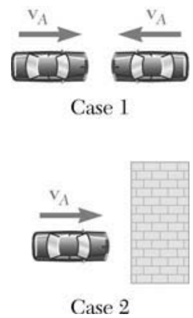
(*c*) The car exerts a force on the insect, but the insect does not exert a force on the car.

(*d*) The car exerts the same force on the insect as the insect exerts on the car.

(*e*) Neither exerts a force on the other; the insect gets smashed simply because it gets in the way of the car.

SOLUTION

Answer: (*d*) This is Newton's 3rd Law.



PROBLEM 13.CQ5

The expected damages associated with two types of perfectly plastic collisions are to be compared. In the first case, two identical cars traveling at the same speed impact each other head on. In the second case, the car impacts a massive concrete wall. In which case would you expect the car to be more damaged?

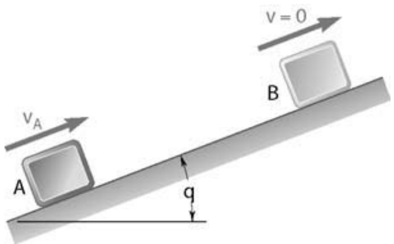
(*a*) Case 1

(*b*) Case 2

(*c*) The same damage in each case

SOLUTION

Answer: (*c*) In both cases the car will come to a complete stop, so the applied impulse will be the same.

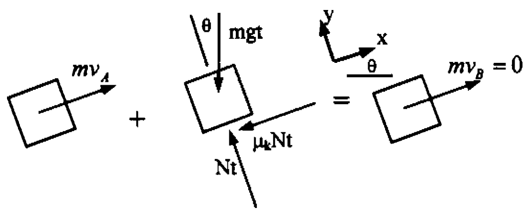


PROBLEM 13.F1

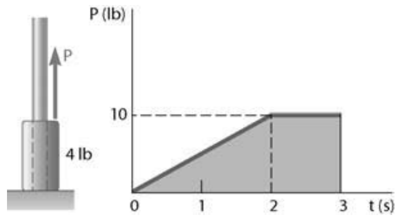
The initial velocity of the block in position A is 30 ft/s. The coefficient of kinetic friction between the block and the plane is $\mu_k = 0.30$. Draw impulse-momentum diagrams that could be used to determine the time it takes for the block to reach B with zero velocity, if $\theta = 20°$.

SOLUTION

Answer:



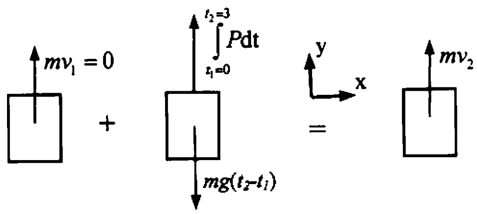
PROBLEM 13.F2



A 4-lb collar which can slide on a frictionless vertical rod is acted upon by a force **P** which varies in magnitude as shown. Knowing that the collar is initially at rest, draw impulse-momentum diagrams that could be used to determine its velocity at $t = 3$ s.

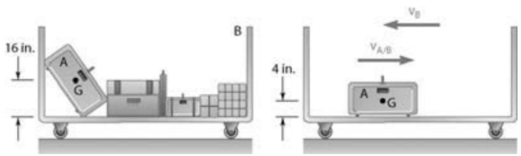
SOLUTION

Answer:



Where $\int_{t_1=0}^{t_2=3} Pdt$ is the area under the curve.

PROBLEM 13.F3

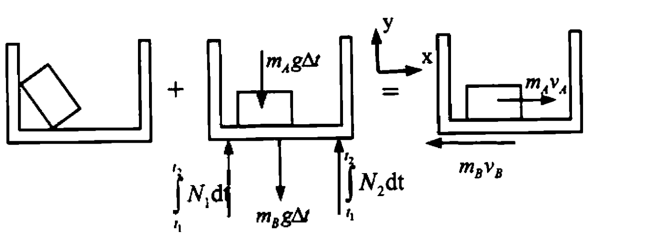


The 15-kg suitcase *A* has been propped up against one end of a 40-kg luggage carrier *B* and is prevented from sliding down by other luggage. When the luggage is unloaded and the last heavy trunk is removed from the carrier, the suitcase is free to slide down, causing the 40-kg carrier to move to the left with a velocity v_B of magnitude 0.8 m/s. Neglecting friction, draw impulse-momentum diagrams that could be used to determine (*a*) the velocity of *A* as it rolls on the carrier and (*b*) the velocity of the carrier after the suitcase hits the right side of the carrier without bouncing back.

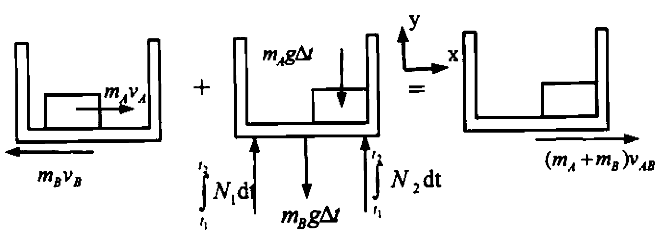
SOLUTION

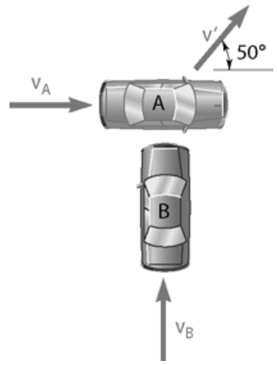
Answer:

(*a*)



(*b*)



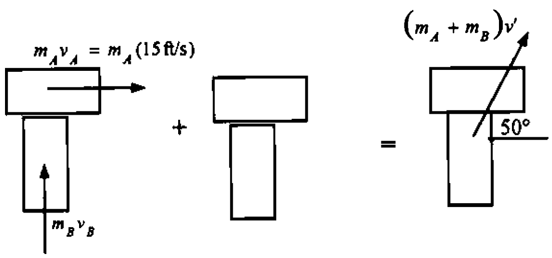


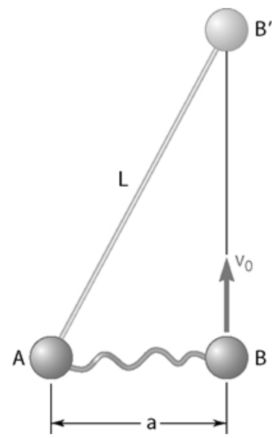
PROBLEM 13.F4

Car A was traveling west at a speed of 15 m/s and car B was traveling north at an unknown speed when they slammed into each other at an intersection. Upon investigation it was found that after the crash the two cars got stuck and skidded off at an angle of 50° north of east. Knowing the masses of A and B are m_A and m_B respectively, draw impulse-momentum diagrams that could be used to determine the velocity of B before impact.

SOLUTION

Answer:



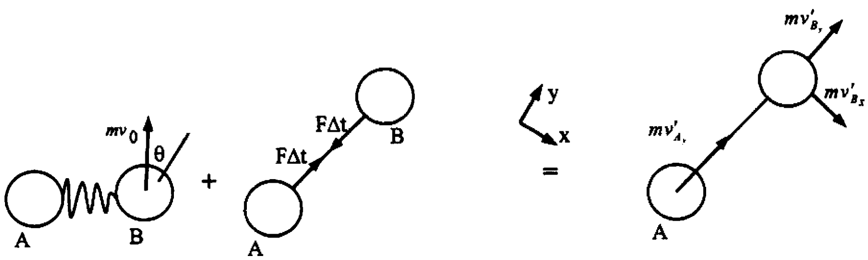


PROBLEM 13.F5

Two identical spheres A and B, each of mass m, are attached to an inextensible inelastic cord of length L and are resting at a distance a from each other on a frictionless horizontal surface. Sphere B is given a velocity $\mathbf{v}_0$ in a direction perpendicular to line AB and moves it without friction until it reaches B' where the cord becomes taut. Draw impulse-momentum diagrams that could be used to determine the magnitude of the velocity of each sphere immediately after the cord has become taut.

SOLUTION

Answer:



Where $v'_{A_y} = v'_{B_y}$ since the cord is inextensible.

PROBLEM 13.119

A 35,000 Mg ocean liner has an initial velocity of 4 km/h. Neglecting the frictional resistance of the water, determine the time required to bring the liner to rest by using a single tugboat which exerts a constant force of 150 kN.

SOLUTION

$$m = 35,000 \ \text{Mg} = 35 \times 10^6 \text{ kg}$$

$$F = 150 \times 10^3 \text{ N}$$

$$v_1 = 4 \text{ km/hr} = 1.1111 \text{ m/s}$$

$$mv_1 - Ft = 0$$

$$(35 \times 10^6 \text{ kg})(1.1111 \text{ m/s}) - (150 \times 10^3 \text{ N})t = 0$$

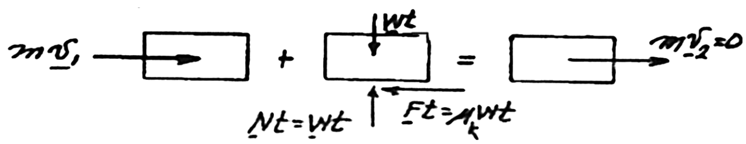
$$t = 259.26 \text{ s}$$

$$t = 4 \text{ min} 19 \text{ s} \blacktriangleleft$$

PROBLEM 13.120

A 2500-lb automobile is moving at a speed of 60 mi/h when the brakes are fully applied, causing all four wheels to skid. Determine the time required to stop the automobile (*a*) on dry pavement ($\mu_k = 0.75$), (*b*) on an icy road ($\mu_k = 0.10$).

SOLUTION



$$v_1 = 60 \text{ mph} = 88 \text{ ft/s}$$

$$mv_1 - \mu_k W t = 0$$

$$t = \frac{mv_1}{\mu_k W} = \frac{mv_1}{\mu_k mg} = \frac{v_1}{\mu_k g}$$

(*a*) For $\mu_k = 0.75$

$$t = \frac{88 \text{ ft/s}}{(0.75)(32.2 \text{ ft/s}^2)}$$

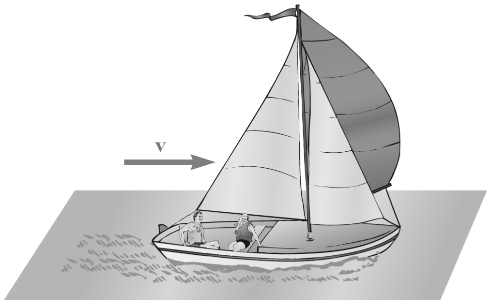
$t = 3.64$ s ◀

(*b*) For $\mu_k = 0.10$

$$t = \frac{88 \text{ ft/s}}{(0.10)(32.2 \text{ ft/s}^2)}$$

$t = 27.3$ s ◀

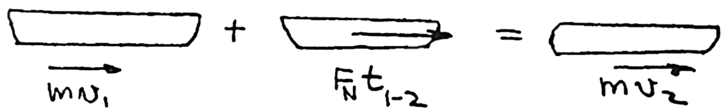
PROBLEM 13.121



A sailboat weighing 980 lb with its occupants is running down wind at 8 mi/h when its spinnaker is raised to increase its speed. Determine the net force provided by the spinnaker over the 10-s interval that it takes for the boat to reach a speed of 12 mi/h.

SOLUTION

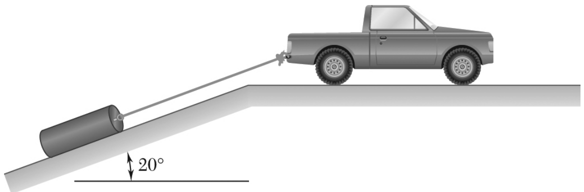
$$v_1 = 8 \text{ mi/h} = 11.73 \text{ ft/s} \quad t_{1-2} = 10 \text{ sec}$$
$$v_2 = 12 \text{ mi/h} = 17.60 \text{ ft/s}$$



$$m \cdot v_1 + \text{imp}_{1-2} = mv_2$$
$$m(11.73 \text{ ft/s}) + F_n(10 \text{ s}) = m(17.60 \text{ ft/s})$$

$$F_n = \frac{(980 \text{ lb})(17.60 \text{ ft/s} - 11.73 \text{ ft/s})}{(32.2 \text{ ft/s}^2)(10 \text{ s})} \qquad F_n = 178.6 \text{ lb} ◀$$

Note: F_n is the *net* force *provided* by the sails. The force on the sails is actually greater and includes the force needed to overcome the water resistance on the hull.

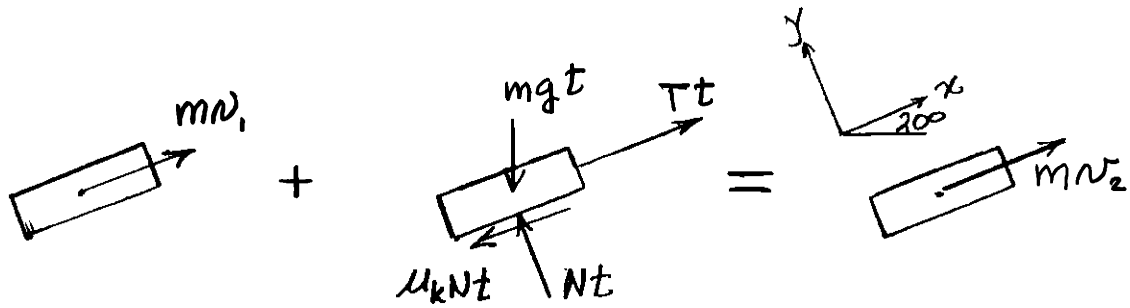


PROBLEM 13.122

A truck is hauling a 300-kg log out of a ditch using a winch attached to the back of the truck. Knowing the winch applies a constant force of 2500 N and the coefficient of kinetic friction between the ground and the log is 0.45, determine the time for the log to reach a speed of 0.5m/s.

SOLUTION

Apply the principle of impulse and momentum to the log.



$$m\mathbf{v}_1 + \Sigma \mathbf{Imp}_{1\to 2} = m\mathbf{v}_2$$

Components in y-direction:

$$0 + Nt - mgt\cos 20° = 0$$

$$N = mg\cos 20°$$

Components in x-direction:

$$0 + Tt - mgt\sin 20° - \mu_k Nt = mv_2$$

$$(T - mg\sin 20° - \mu_k mg\cos 20°)t = mv_2$$

$$[T - mg(\sin 20° + \mu_k \cos 20°)]t = mv_2$$

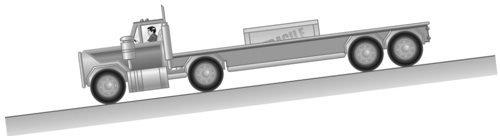
Data: $T = 2500$ N, $m = 300$ kg, $\mu_k = 0.45$,

$g = 9.81$ m/s^2, $v_2 = 0.5$ m/s

$$[2500 - (300)(9.81)(\sin 20° + 0.45\cos 20°)]t = (300)(0.5)$$

$$248.95\, t = 150$$

$t = 0.603$ s ◀



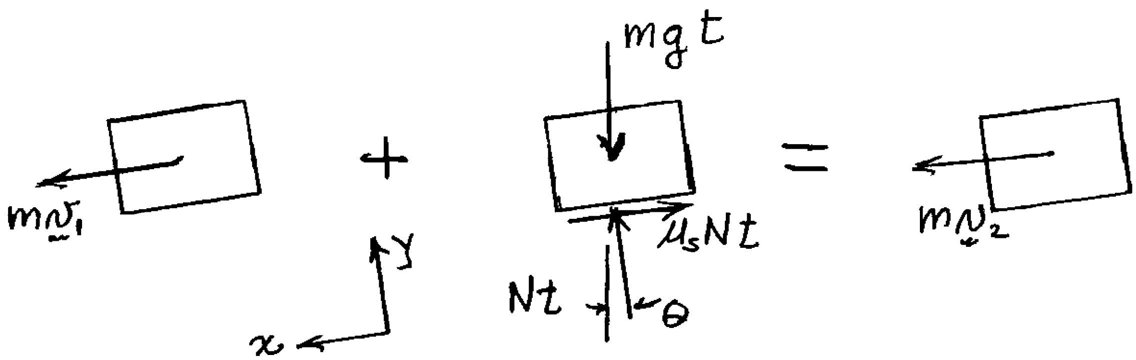
PROBLEM 13.123

A truck is traveling down a road with a 3-percent grade at a speed of 55 mi/h when the brakes are applied. Knowing the coefficients of friction between the load and the flatbed trailer shown are $\mu_s = 0.40$ and $\mu_k = 0.35$, determine the shortest time in which the rig can be brought to a stop if the load is not to shift.

SOLUTION

Apply the principle impulse-momentum to the crate, knowing that, if the crate does not shift, the velocity of the crate matches that of the truck. For impending slip the friction and normal components of the contact force between the crate and the flatbed trailer satisfy the following equation:

$$F_f = \mu_s N$$



$$m\mathbf{v}_1 + \Sigma\mathbf{Imp}_{1\to2} = m\mathbf{v}_2$$

Components in y-direction: $\quad 0 + Nt - mgt\cos\theta = 0$

$$N = mg\cos\theta$$

Components in x-direction: $\quad mv_1 + mgt\sin\theta - \mu_s Nt = mv_2$

$$mv_1 + mgt(\sin\theta - \mu_s\cos\theta) = 0$$

$$t = \frac{v_1}{g(\mu_s\cos\theta - \sin\theta)}$$

Data: $\quad v_1 = 55 \text{ mi/h} = 80.667 \text{ ft/s}, \quad v_2 = 0,$

$$g = 32.2 \text{ ft/s}^2, \quad \mu_s = 0.40, \quad \tan\theta = 3/100$$

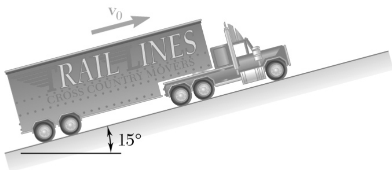
$$\theta = 1.71835°$$

$$\mu_s\cos\theta - \sin\theta = 0.36983$$

$$t = \frac{80.667}{(32.2)(0.36983)}$$

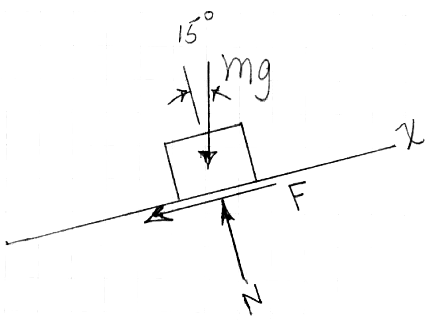
$t = 6.77$ s ◀

PROBLEM 13.124



Steep safety ramps are built beside mountain highways to enable vehicles with defective brakes to stop. A 10-ton truck enters a 15° ramp at a high speed v_0 = 108 ft/s and travels for 6 s before its speed is reduced to 36 ft/s. Assuming constant deceleration, determine (*a*) the magnitude of the braking force, (*b*) the additional time required for the truck to stop. Neglect air resistance and rolling resistance.

SOLUTION



$$W = 20{,}000 \text{ lb}$$

$$m = \frac{20{,}000}{32.2} = 621.118 \text{ lb}\cdot\text{s}^2/\text{ft}$$

Momentum in the x direction

$$x\!: mv_0 - (F + mg\sin 15°)t = mv_1$$

$$621.118(108) - (F + mg\sin 15°)6 = (621.118)(36)$$

$$F + mg\sin 15° = 7453.4$$

(*a*) $F = 7453.4 - 20{,}000\sin 15° = 2277$ lb

$F = 2280$ lb ◀

(*b*) $mv_0 - (F + mg\sin 15°)t = 0$ $\quad t$ = total time

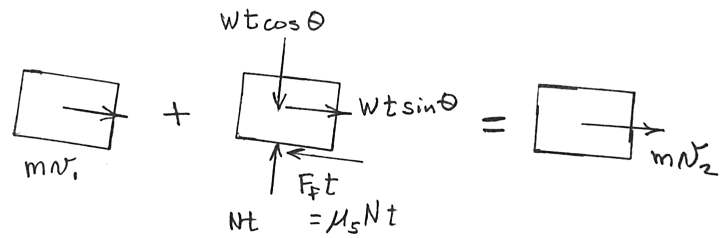
$621.118(108) - 7453.4t = 0;$ $\quad t = 9.00$ s

Additional time = 9 – 6 $\quad t = 3.00$ s ◀

PROBLEM 13.125

Baggage on the floor of the baggage car of a high-speed train is not prevented from moving other than by friction. The train is travelling down a 5 percent grade when it decreases its speed at a constant rate from 120 mi/h to 60 mi/h in a time interval of 12 s. Determine the smallest allowable value of the coefficient of static friction between a trunk and the floor of the baggage car if the trunk is not to slide.

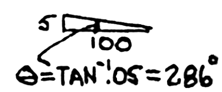
SOLUTION



$$v_1 = 120 \text{ mi/h} = 176 \text{ ft/s}$$

$$v_2 = 60 \text{ mi/h} = 88 \text{ ft/s}$$

$$t_{1-2} = 12 \text{ s}$$



$$Nt_{1-2} = Wt_{1-2}\cos\theta$$

$$\searrow^{+}\ \cancel{m}v_1 - \mu_s \cancel{m}gt_{1-2}\cos\theta + \cancel{m}gt_{1-2}\sin\theta = \cancel{m}v_2$$

$$(176 \text{ ft/s}) - \mu_s(32.2 \text{ ft/s}^2)(12 \text{ s})(\cos 2.86°) + (32.2 \text{ ft/s}^2)(12 \text{ s})(\sin 2.86°) = 88 \text{ ft/s}$$

$$\mu_s = \frac{176 - 88 + (32.2)(12)(\sin 2.86°)}{(32.2)(12)(\cos 2.86°)}$$

$\mu_s = 0.278$ ◀

PROBLEM 13.126

A 2-kg particle is acted upon by the force, expressed in newtons, $\mathbf{F} = (8-6t)\mathbf{i} + (4-t^2)\mathbf{j} + (4+t)\mathbf{k}$. Knowing that the velocity of the particle is $\mathbf{v} = (150\text{ m/s})\mathbf{i} + (100\text{ m/s})\mathbf{j} - (250\text{ m/s})\mathbf{k}$ at $t = 0$, determine (*a*) the time at which the velocity of the particle is parallel to the *yz* plane, (*b*) the corresponding velocity of the particle.

SOLUTION

$$m\mathbf{v}_0 + \int \mathbf{F}dt = m\mathbf{v} \qquad (1)$$

Where

$$\int \mathbf{F}dt = \int_0^t [(8-6t)\mathbf{i} + (4-t^2)\mathbf{j} + (4+t)\mathbf{k}]dt$$

$$= (8t - 3t^2)\mathbf{i} + \left(4t - \frac{1}{3}t^3\right)\mathbf{j} + \left(4t + \frac{1}{2}t^2\right)\mathbf{k}$$

Substituting $m = 2$ kg, $v_0 = 150\mathbf{i} + 100\mathbf{j} - 250\mathbf{k}$ into (1):

$$(2\text{ kg})(150\mathbf{i} + 100\mathbf{j} - 250\mathbf{k}) + (8t - 3t^2)\mathbf{i} + \left(4t - \frac{1}{3}t^3\right)\mathbf{j} + \left(4t + \frac{1}{2}t^2\right)\mathbf{k} = (2\text{ kg})\mathbf{v}$$

$$\mathbf{v} = \left(150 + 4t - \frac{3}{2}t^2\right)\mathbf{i} + \left(100 + 2t - \frac{1}{6}t^3\right)\mathbf{j} + \left(-250 + 2t + \frac{1}{4}t^2\right)\mathbf{k}$$

(*a*) $\mathbf{v}$ is parallel to *yz* plane when $v_x = 0$, that is, when

$$150 + 4t - \frac{3}{2}t^2 = 0 \quad t = 11.422\text{ s}$$

$t = 11.42$ s ◀

(*b*)

$$\mathbf{v} = \left[100 + 2(11.422) - \frac{1}{6}(11.422)^3\right]\mathbf{j}$$

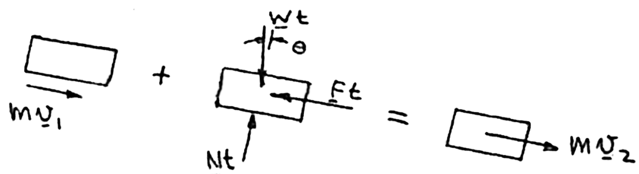
$$+\left[-250 + 2(11.422) + \frac{1}{4}(11.422)^2\right]\mathbf{k}$$

$\mathbf{v} = -(125.5\text{ m/s})\mathbf{j} - (194.5\text{ m/s})\mathbf{k}$ ◀

PROBLEM 13.127

A truck is traveling down a road with a 4-percent grade at a speed of 60 mi/h when its brakes are applied to slow it down to 20 mi/h. An antiskid braking system limits the braking force to a value at which the wheels of the truck are just about to slide. Knowing that the coefficient of static friction between the road and the wheels is 0.60, determine the shortest time needed for the truck to slow down.

SOLUTION



$$\theta = \tan^{-1}\frac{4}{100} = 2.29°$$

$$m\mathbf{v}_1 + \Sigma\,\mathbf{imp}_{1-2} = m\mathbf{v}_2$$

$$+\searrow \quad mv_1 + Wt\sin\theta - Ft = mv_2$$

$$v_1 = 60 \text{ mi/h} = 88 \text{ ft/s} \qquad N = W\cos\theta \quad W = mg$$

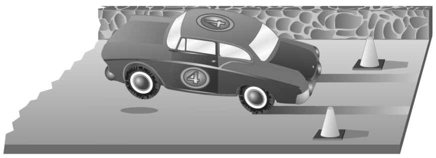
$$v_2 = 20 \text{ mi/h} = 29.33 \text{ ft/s} \qquad F = \mu_s N = \mu_s W\cos\theta$$

$$(\cancel{m})(88 \text{ ft/s}) + (\cancel{m})(32.2 \text{ ft/s}^2)(t)(\sin 2.29°) - (0.60)(\cancel{m})(32.2 \text{ ft/s}^2)(\cos 2.29°)(t) = (\cancel{m})(29.33 \text{ ft/s})$$

$$t = \frac{88 - 29.33}{32.2[(0.60)\cos 2.29° - \sin 2.29°]}$$

$t = 3.26$ s ◀

PROBLEM 13.128



Skid marks on a drag race track indicate that the rear (drive) wheels of a car slip for the first 20 m of the 400-m track. (*a*) Knowing that the coefficient of kinetic friction is 0.60, determine the shortest possible time for the car to travel the initial 20-m portion of the track if it starts from rest with its front wheels just off the ground. (*b*) Determine the minimum time for the car to run the whole race if, after skidding for 20 m, the wheels roll without sliding for the remainder of the race. Assume for the rolling portion of the race that 65 percent of the weight is on the rear wheels and that the coefficient of static friction is 0.85. Ignore air resistance and rolling resistance.

SOLUTION

(*a*) First 20 m

Velocity at 20 m. Rear wheels skid to generate the maximum force resulting in maximum velocity and minimum time since all the weight is on the rear wheel: This force is $F = \mu_k N = 0.60W$.

Work and energy. $T_0 + U_{0-20} = T_{20}$

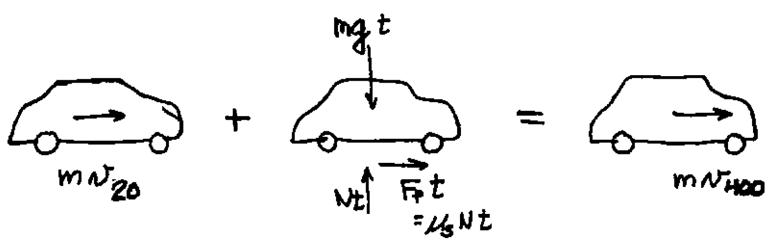
$$T_0 = 0 \qquad U_{0-20} = (F)(20) \qquad T_{20} = \frac{1}{2}mv_{20}^2$$

$$0 + \mu_k mg(20) = \frac{1}{2}mv_{20}^2$$

$$v_{20}^2 = (2)(0.60)(20\text{ m})(9.81\text{ m/s}^2)$$

$$v_{20} = 15.344\text{ m/s}$$

Impulse-momentum.



$$\xrightarrow{+}\; 0 + \mu_k mgt_{0-20} = mv_{20} \qquad v_{20} = 15.344\text{ m/s}$$

$$t_{0-20} = \frac{15.344\text{ m/s}}{(0.60)(9.81\text{ m/s}^2)}$$

$t_{0-20} = 2.61\text{ s}$ ◀

(*b*) For the whole race:

The maximum force on the wheels for the first 20 m is $F = \mu_k mg = 0.60\,mg$. For remaining 360 m, the maximum force, if there is no sliding and 65 percent of the weight is on the rear (drive) wheels, is

$$F = \mu_s (0.65)\, mg = (0.85)(0.65)\, mg = 0.5525\, mg$$

Velocity at 400 m.

Work and energy. $$T_0 + U_{0-20} + U_{20-400} = T_{400}$$

$$T_0 = 0 \qquad U_{0-20} = (0.60mg)(20 \text{ m}), \qquad U_{60-400} = (0.5525mg)(380 \text{ m})$$

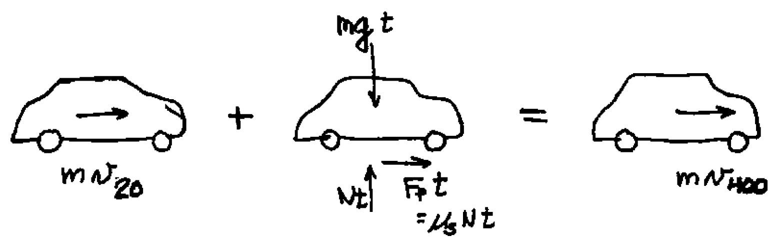
$$T_{400} = \frac{1}{2} m v_{400}^2$$

$$0 + 12\,mg + (0.5525)(380)mg = \frac{1}{2} m v_{400}^2$$

$$v_{400} = 65.990 \text{ m/s}$$

Impulse–momentum.

From 20 m to 400 m



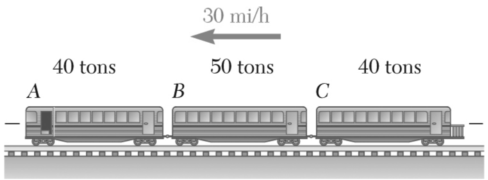
$$F = \mu_s N = 0.510mg$$

$$v_{20} = 15.344 \text{ m/s}$$

$$v_{400} = 65.990 \text{ m/s}$$

$$m(15.344) + 0.5525\, mg t_{20-400} = m(65.990); \quad t_{20-400} = 9.3442 \text{ s}$$

$$t_{0-400} = t_{0-20} + t_{20-400} = 2.61 + 9.34 \qquad t_{0-400} = 11.95 \text{ s} \blacktriangleleft$$



PROBLEM 13.129

The subway train shown is traveling at a speed of 30 mi/h when the brakes are fully applied on the wheels of cars B and C, causing them to slide on the track, but are not applied on the wheels of car A. Knowing that the coefficient of kinetic friction is 0.35 between the wheels and the track, determine (a) the time required to bring the train to a stop, (b) the force in each coupling.

SOLUTION

Weights of cars: $W_A = W_C = 80{,}000 \text{ lb}, \ W_B = 100{,}000 \text{ lb}$

Masses of cars: $m_A = m_C = 2484 \text{ lb} \cdot \text{s}^2/\text{ft}, \ m_B = 3106 \text{ lb} \cdot \text{s}^2/\text{ft}$

For each car the normal force (upward) is equal in magnitude to the weight of the car.

$$N_A = N_C = 80{,}000 \text{ lb} \qquad N_B = 100{,}000 \text{ lb}$$

Friction forces:

$$F_A = 0 \qquad \text{(brakes not applied)}$$

$$F_B = (0.35)(100{,}000) = 35000 \text{ lb}$$

$$F_C = (0.35)(80{,}000) = 28{,}000 \text{ lb}$$

Stopping data: $v_1 = 30 \text{ mi/h} = 44 \text{ ft/s}, \ v_2 = 0.$

(a) Apply the principle of impulse-momentum to the entire train.

$$m = m_A + m_B + m_C = 8074 \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$F = F_A + F_B + F_C = 63{,}000 \text{ lb}$$

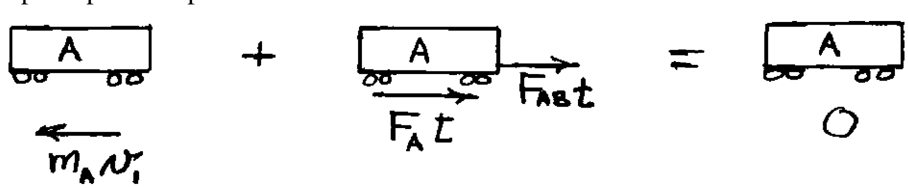
$$-mv_1 + Ft = mv_2$$

$$t = \frac{m(v_1 - v_2)}{F} = \frac{(8074)(44)}{63{,}000} = 5.639 \text{ s}$$

$t = 5.64 \text{ s}$ ◀

(b) Coupling force F_{AB}:

Apply the principle of impulse-momentum to car A alone.



$$-m_A v_1 + F_A t + F_{AB} t = 0$$

$$-(2484)(44) + 0 + F_{AB}(5.639) = 0$$

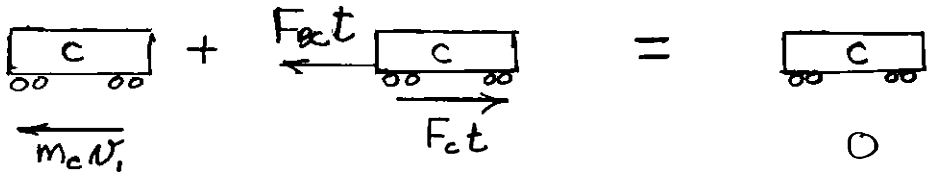
$$F_{AB} = 19{,}390 \text{ lb}$$

$F_{AB} = 19{,}390 \text{ lb (tension)}$ ◀

PROBLEM 13.129 (Continued)

Coupling force F_{BC}:

Apply the principle of impulse-momentum to car C alone.



$$-m_Cv_1 + F_Ct - F_{BC}t = 0$$

$$-(2484)(44) + (28000)(5.639) - F_{BC}(5.639) = 0$$

$$F_{BC} = 8620 \text{ lb} \qquad F_{BC} = 8620 \text{ lb (tension)} \blacktriangleleft$$

PROBLEM 13.130

Solve Problem 13.129 assuming that the brakes are applied only on the wheels of car *A*.

PROBLEM 13.129 The subway train shown is traveling at a speed of 30 mi/h when the brakes are fully applied on the wheels of cars *B* and *C*, causing them to slide on the track, but are not applied on the wheels of car *A*. Knowing that the coefficient of kinetic friction is 0.35 between the wheels and the track, determine (*a*) the time required to bring the train to a stop, (*b*) the force in each coupling.

SOLUTION

Weights of cars: $W_A = W_C = 80{,}000 \text{ lb}, \ W_B = 100{,}000 \text{ lb}$

Masses of cars: $m_A = m_C = 2484 \text{ lb}\cdot\text{s}^2/\text{ft}, \ m_B = 3106 \text{ lb}\cdot\text{s}^2/\text{ft}$

For each car the normal force (upward) is equal in magnitude to the weight of the car.

$$N_A = N_C = 80{,}000 \text{ lb} \qquad N_B = 100{,}000 \text{ lb}$$

Friction forces: $F_A = (0.35)(80{,}000) = 28{,}000 \text{ lb}$

$$\left.\begin{aligned} F_B &= 0 \\ F_C &= 0 \end{aligned}\right\} \text{(brakes not applied)}$$

Stopping data: $v_1 = 30 \text{ mi/h} = 44 \text{ ft/s}, \quad v_2 = 0.$

(*a*) Apply the principle of impulse-momentum to the entire train.

$$m = m_A + m_B + m_C = 8074 \text{ lb}\cdot\text{s}^2/\text{ft}$$

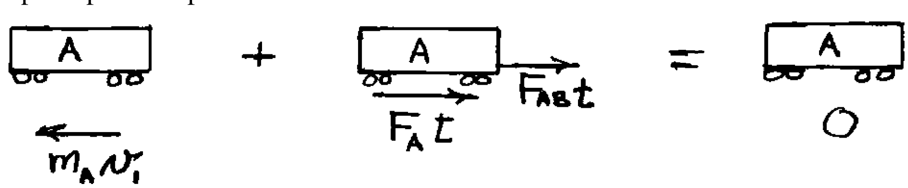
$$F = F_A + F_B + F_C = 28{,}000 \text{ lb}$$

$$-mv_1 + \text{Ft} = mv_2$$

$$t = \frac{m(v_1 - v_2)}{F} = \frac{(8074)(44)}{28{,}000} = 12.688 \text{ s} \qquad t = 12.69 \text{ s} \blacktriangleleft$$

(*b*) Coupling force F_{AB}:

Apply the principle of impulse-momentum to car *A* alone.



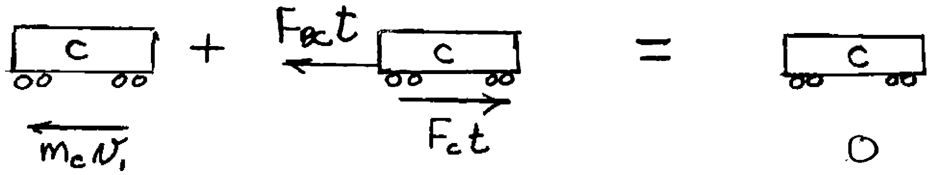
$$-m_A v_1 + F_A t + F_{AB} t = 0$$

$$-(2484)(44) + (28{,}000)(12.688) + F_{AB}(12.688) = 0$$

$$F_{AB} = -19{,}390 \text{ lb} \qquad F_{AB} = 19{,}390 \text{ lb (compression)} \blacktriangleleft$$

Coupling force F_{BC}:

Apply the principle of impulse-momentum to car C alone.

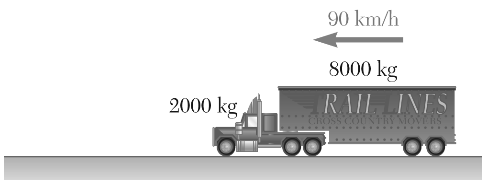


$$-m_C v_1 + F_C t - F_{BC} t = 0$$

$$-(2484)(44) + (0) - F_{BC}(12.688) = 0$$

$$F_{BC} = -8620 \text{ lb} \qquad F_{BC} = 8620 \text{ lb (compression)} \blacktriangleleft$$

PROBLEM 13.131

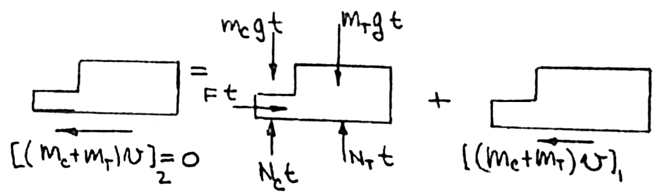


A trailer truck with a 2000-kg cab and an 8000-kg trailer is traveling on a level road at 90 km/h. The brakes on the trailer fail and the antiskid system of the cab provides the largest possible force which will not cause the wheels of the cab to slide. Knowing that the coefficient of static friction is 0.65, determine (*a*) the shortest time for the rig to come to a stop, (*b*) the force in the coupling during that time.

SOLUTION

$$v = 90 \text{ km/h} = 25 \text{ m/s}$$

(*a*) The shortest time for the rig to come to a stop will be when the friction force on the wheels is maximum. The downward force exerted by the trailer on the cab is assumed to be zero. Since the trailer brakes fail, all of the braking force is supplied by the wheels of the cab, which is maximum when the wheels of the cab are at impending sliding.



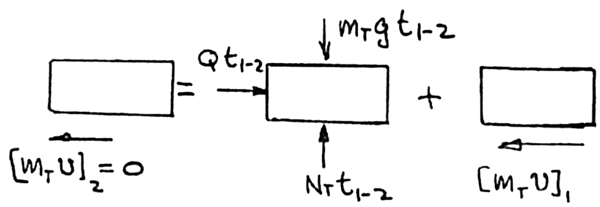
$$Ft_{1-2} = \mu_s N_C t_{1-2} \quad N_C = m_C g = (2000)g$$
$$Ft_{1-2} = (0.65)(2000)gt$$

$$[(m_C + m_T)v]_2 = -Ft + [(m_C + m_T)v]_1$$

$$\overset{+}{\longleftarrow} \quad 0 = -(0.65)(2000 \text{ kg})(9.81 \text{ m/s}^2)(t_{1-2}) = 10{,}000 \text{ kg}(25 \text{ m/s})$$

$$t_{1-2} = 19.60 \text{ s} \blacktriangleleft$$

(*b*) For the trailer:

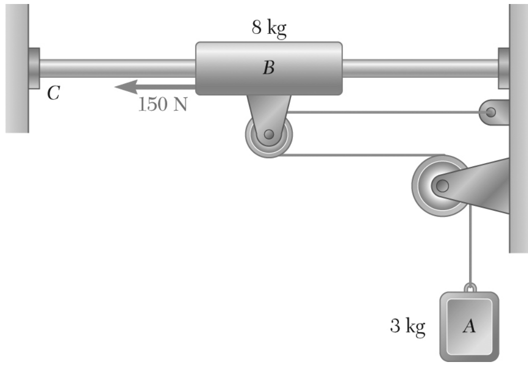


$$\overset{+}{\longleftarrow} \quad [m_T v]_2 = -Qt_{1-2} + [m_T v]_1$$

From (*a*), $t_{1-2} = 19.60$ s

$$0 = -Q(19.60 \text{ s}) + (8000 \text{ kg})(25 \text{ m/s})$$
$$Q = 10{,}204 \text{ N}$$

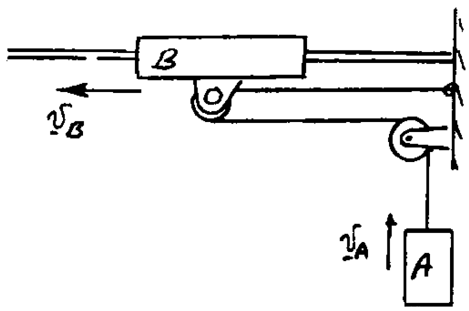
$$Q = 10.20 \text{ kN (compression)} \blacktriangleleft$$



PROBLEM 13.132

The system shown is at rest when a constant 150-N force is applied to collar B. Neglecting the effect of friction, determine (a) the time at which the velocity of collar B will be 2.5 m/s to the left, (b) the corresponding tension in the cable.

SOLUTION



Constraint of cord. When the collar B moves 1 unit to the left, the weight A moves up 2 units. Thus

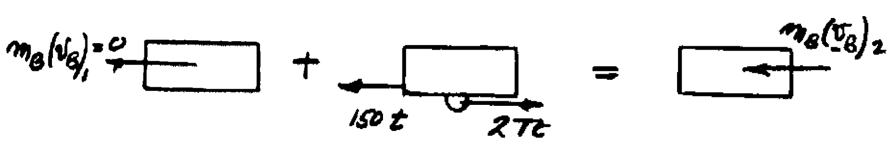
$$v_A = 2v_B \qquad v_B = \frac{1}{2}v_A$$

Masses and weights: $m_A = 3\text{ kg}$ $\quad W_A = 29.43\text{ N}$

$m_B = 8\text{ kg}$

Let T be the tension in the cable.

Principle of impulse and momentum applied to collar B.



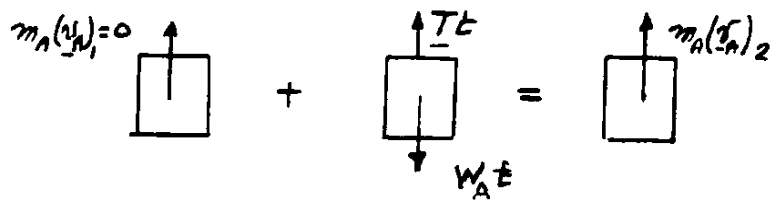
$$\overset{+}{\longleftarrow}:\ 0 + 150t - 2Tt = m_B(v_B)_2$$

For $(v_B)_2 = 2.5\text{ m/s}$ $\qquad 150t - 2Tt = (8\text{ kg})(2.5\text{ m/s})$

$$150t - 2Tt = 20 \qquad (1)$$

PROBLEM 13.132 (Continued)

Principle of impulse and momentum applied to weight A.



$$+\uparrow:\quad 0 + Tt - W_A t = m_A (v_A)_2$$

$$Tt + W_A t = m_A (2V_{B2})$$

$$Tt - 29.43t = (3 \text{ kg})(2)(2.5 \text{ m/s})$$

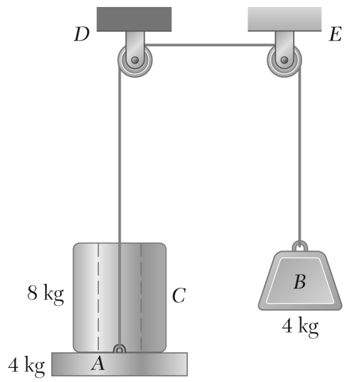
$$Tt - 29.43t = 15 \qquad (2)$$

To eliminate T multiply Eq. (2) by 2 and add to Eq. (1).

(a) Time: $91.14t = 50$ $\qquad t = 0.549 \text{ s}$ ◀

From Eq. (2), $T = \dfrac{15}{t} + 29.43$

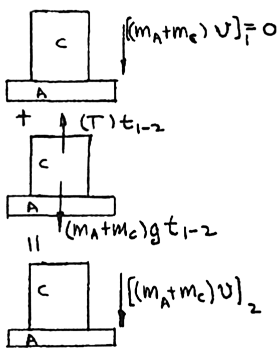
(b) Tension in the cable. $\qquad T = 56.8 \text{ N}$ ◀



PROBLEM 13.133

An 8-kg cylinder C rests on a 4-kg platform A supported by a cord which passes over the pulleys D and E and is attached to a 4-kg block B. Knowing that the system is released from rest, determine (a) the velocity of block B after 0.8 s, (b) the force exerted by the cylinder on the platform.

SOLUTION



(a) Blocks A and C:

$$\downarrow +$$

$$[(m_A+m_C)v]_1 - T(t_{1-2}) + (m_A+m_C)gt_{1-2} = [(m_A+m_C)v]_2$$

$$0 + (12g - T)(0.8) = 12v \qquad (1)$$

Block B:

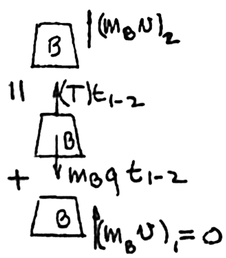
$$\uparrow +$$

$$[m_B v]_1 + (T)t_{1-2} - m_B g t_{1-2} = (m_B v)_2$$

$$0 + (T - 4g)(0.8) = 4v \qquad (2)$$

Adding (1) and (2), (eliminating T)

$$(12g - 4g)(0.8) = (12+4)$$



$$v = \frac{(8\text{ kg})(9.81\text{ m/s}^2)(0.8\text{ s})}{16\text{ kg}} \qquad v = 3.92\text{ m/s} \blacktriangleleft$$

(b) Collar A:

$$\downarrow +$$

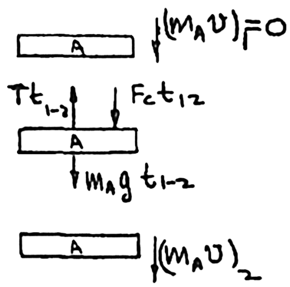
$$(m_A v)_1 = 0 \quad 0 + (F_C + m_A g) \qquad (3)$$

From Eq. (2) with $v = 3.92$ m/s

$$T = \frac{4v}{0.8} + 4g$$

$$T = \frac{(4\text{ kg})(3.92\text{ m/s})}{(0.8\text{ s})} + (4\text{ kg})(9.81\text{ m/s}^2)$$

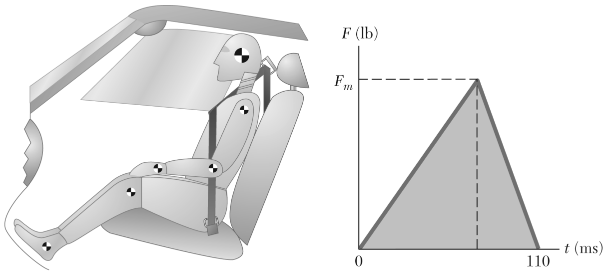
$$T = 58.84\text{ N}$$



Solving for F_C in (3)

$$F_C = \frac{(4\text{ kg})(3.92\text{ m/s})}{(0.8\text{ s})} - (4\text{ kg})(9.81\text{ m/s}^2) + 58.84\text{ N}$$

$$F_C = 39.2\text{ N} \blacktriangleleft$$

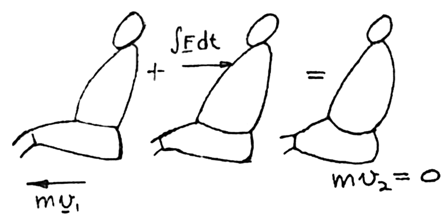


PROBLEM 13.134

An estimate of the expected load on over-the-shoulder seat belts is to be made before designing prototype belts that will be evaluated in automobile crash tests. Assuming that an automobile traveling at 45 mi/h is brought to a stop in 110 ms, determine (*a*) the average impulsive force exerted by a 200-lb man on the belt, (*b*) the maximum force F_m exerted on the belt if the force-time diagram has the shape shown.

SOLUTION

(*a*) Force on the belt is opposite to the direction shown.



$$v_1 = 45 \text{ mi/h} = 66 \text{ ft/s},$$
$$W = 200 \text{ lb}$$

$$m\mathbf{v}_1 - \int \mathbf{F} dt = m\mathbf{v}_2$$

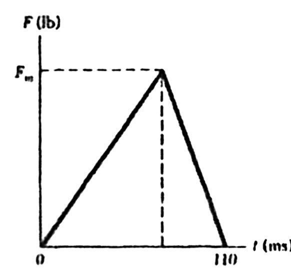
$$\int F dt = F_{\text{ave}} \Delta t$$

$$\Delta t = 0.110 \text{ s}$$

$$\frac{(200 \text{ lb})(66 \text{ ft/s})}{(32.2 \text{ ft/s}^2)} - F_{\text{ave}}(0.110 \text{ s}) = 0$$

$$F_{\text{ave}} = \frac{(200)(66)}{(32.2)(0.110)} = 3727 \text{ lb}$$

$F_{\text{ave}} = 3730 \text{ lb}$ ◀

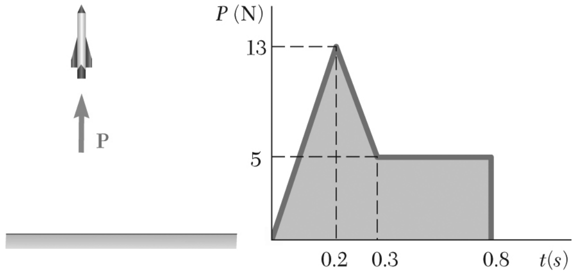


(*b*) Impulse = area under $F-t$ diagram $= \frac{1}{2} F_m (0.110 \text{ s})$

From (*a*), impulse $= F_{\text{ave}} \Delta t = (3727 \text{ lb})(0.110 \text{ s})$

$$\frac{1}{2} F_m (0.110) = (3727)(0.110)$$

$F_m = 7450 \text{ lb}$ ◀



PROBLEM 13.135

A 60-g model rocket is fired vertically. The engine applies a thrust **P** which varies in magnitude as shown. Neglecting air resistance and the change in mass of the rocket, determine (*a*) the maximum speed of the rocket as it goes up, (*b*) the time for the rocket to reach its maximum elevation.

SOLUTION

Mass: $m = 0.060 \text{ kg}$

Weight: $mg = (0.060)(9.81) = 0.5886 \text{ N}$

Forces acting on the model rocket:

Thrust: $P(t)\text{(given function of } t) \uparrow$

Weight: $W \text{ (constant)} \downarrow$

Support: $S \text{ (acts until } P > W) \uparrow$

Over $0 < t < 0.2$ s:

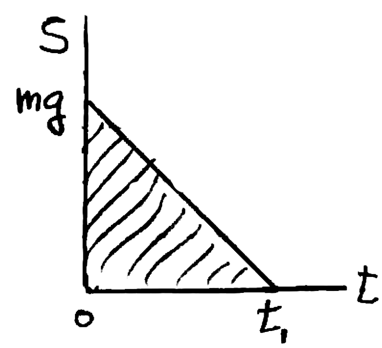
$$P = \frac{13}{0.2} t = 65t$$
$$W = 0.5886 \text{ N}$$

Before the rocket lifts off, $S = W - P = 0.5886 - 65t$

S become zero when $t = t_1$.

$$0 = 0.5886 - 65t_1 \qquad t_1 = 0.009055 \text{ s.}$$

Impulse due to S: $(t > t_1)$



$$\int_0^t S dt = \int_0^{t_1} S dt$$
$$= \frac{1}{2}\ mgt_1$$
$$= (0.5)(0.5886)(0.009055)$$
$$= 0.00266 \text{ N} \cdot \text{s}$$

The maximum speed occurs when $\dfrac{dv}{dt} = a = 0.$

At this time, $W - P = 0,$ which occurs at $t_2 = 0.8$ s.

(*a*) Maximum speed (upward motion):

Apply the principle of impulse-momentum to the rocket over $0 \le t \le t_2$.

$$\int_0^{0.8} P dt = \text{area under the given thrust-time plot.}$$

$$= \frac{1}{2}(0.2)(13) + \frac{1}{2}(0.1)(13+5) + (0.8-0.3)(5)$$
$$= 4.7 \text{ N} \cdot \text{s}$$

$$\int_0^{0.8} W dt = (0.5886)(0.8) = 0.47088 \text{ N} \cdot \text{S}$$

$$m_1 v_1 + \int_0^{0.8} P dt + \int_0^{0.8} S dt - \int_0^{0.8} W dt = m v_2$$
$$0 + 4.7 + 0.00266 - 0.47088 = 0.060\ v_2$$

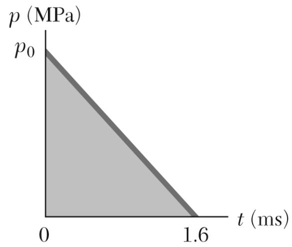
$v_2 = 70.5$ m/s ◀

(*b*) Time t_3 to reach maximum height: $(v_3 = 0)$

$$m v_1 + \int_0^{t_3} P dt + \int_0^{t_3} S dt - W t_3 = m v_3$$

$$0 + 4.7 + 0.00266 - 0.5886 t_3 = 0$$

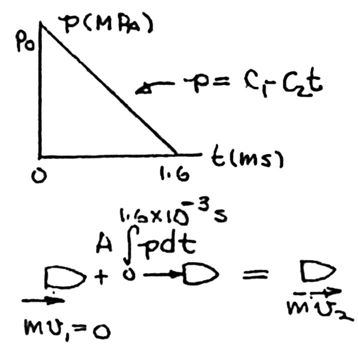
$t_3 = 7.99$ s ◀



PROBLEM 13.136

A simplified model consisting of a single straight line is to be obtained for the variation of pressure inside the 10-mm-diameter barrel of a rifle as a 20-g bullet is fired. Knowing that it takes 1.6 ms for the bullet to travel the length of the barrel and that the velocity of the bullet upon exit is 700 m/s, determine the value of p_0.

SOLUTION



At $t = 0,$

$$p = p_0 = c_1 - c_2 t$$
$$c_1 = p_0$$

At $t = 1.6\times10^{-3}\text{ s},$

$$p = 0$$
$$0 = c_1 - c_2(1.6\times10^{-3}\text{ s})$$
$$c_2 = \frac{p_0}{1.6\times10^{-3}\text{ s}}$$

$$m = 20\times10^{-3}\text{ kg}$$

$$0 + A\int_0^{1.6\times10^{-3}\text{ s}} p\,dt = mv_2$$

$$A = \frac{\pi(10\times10^{-3})^2}{4}$$
$$A = 78.54\times10^{-6}\text{ m}^2$$

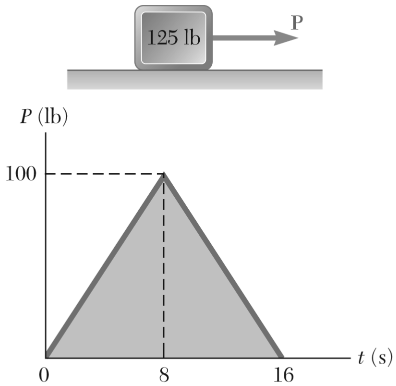
$$0 + A\int_0^{1.6\times10^{-3}\text{ s}} (c_1 - c_2 t)\,dt = \frac{20\times10^{-3}}{g}$$

$$(78.54\times10^{-6}\text{ m}^2)\left[(c_1)(1.6\times10^{-3}\text{ s}) - \frac{(c_2)(1.6\times10^{-3}\text{ s})^2}{2}\right] = (20\times10^{-3}\text{ kg})(700\text{ m/s})$$

$$1.6\times10^{-3}c_1 - 1.280\times10^{-6}c_2 = 178.25\times10^3$$

$$(1.6\times10^{-3}\text{ m}^2\cdot\text{s})\,p_0 - \frac{(1.280\times10^{-6}\text{ m}^2\cdot\text{s}^2)}{(1.6\times10^{-3}\text{ s})}\,p_0 = 178.25\times10^3\text{ kg}\cdot\text{m/s}$$

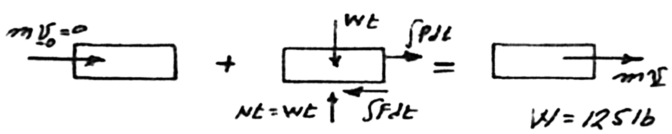
$$p_0 = 222.8\times10^6\text{ N/m}^2 \qquad p_0 = 223\text{ MPa} \blacktriangleleft$$



PROBLEM 13.137

A 125-lb block initially at rest is acted upon by a force **P** which varies as shown. Knowing that the coefficients of friction between the block and the horizontal surface are $\mu_s = 0.50$ and $\mu_k = 0.40$, determine (*a*) the time at which the block will start moving, (*b*) the maximum speed reached by the block, (*c*) the time at which the block will stop moving.

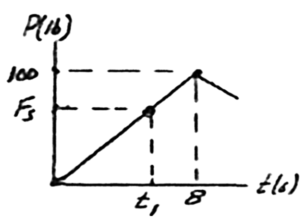
SOLUTION



$$\xrightarrow{+} 0 + \int Pdt - \int Fdt = mv$$

At any time:
$$v = \frac{1}{m}\left[\int Pdt - \int Fdt\right] \qquad (1)$$

(*a*) Block starts moving at *t*.



$$P = F_s = \mu_s W = (0.50)(125 \text{ lb}) = 62.5 \text{ lb}$$

$$\frac{t_1}{F_s} = \frac{8 \text{ s}}{100 \text{ lb}}; \quad \frac{t_1}{62.5 \text{ lb}} = \frac{8 \text{ s}}{100 \text{ lb}}$$

$t_1 = 5.00$ s ◀

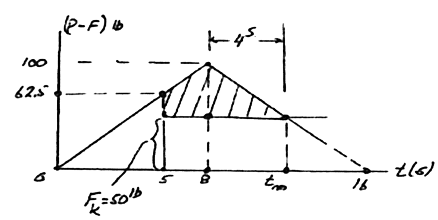
(*b*) Maximum velocity: At $t = t_m$

where $P = F_k = \mu_k W = 0.4(125) = 50$ lb

Block moves at $t = 5$ s.

Shaded area is maximum net impulse $\int Pdt - \int F_R dt$

when $t = t_{m1}$ $\quad v = v_m$



Eq. (1): $$v_m = \frac{1}{m}\begin{bmatrix}\text{shaded}\\ \text{area}\end{bmatrix} = \frac{1}{m}\left[\frac{1}{2}(12.5+50)(3) + \frac{1}{2}(50)(4)\right] = \frac{1}{m}(193.75)$$

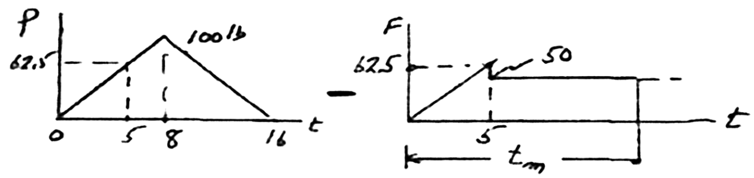
$$v_m = \frac{1}{\frac{125 \text{ lb}}{32.2}}[193.75] = 49.91 \text{ ft/s}$$

$\mathbf{v}_m = 49.9$ ft/s ⟶ ◀

PROBLEM 13.137 (Continued)

(*c*) Block stops moving when $\left[\int P dt - \int F dt\right] = 0;$ or $\int Q dt = \int F dt$

Assume $t_m > 16$ s.

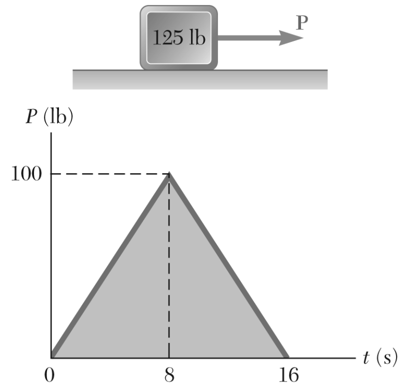


$$\int P dt = \frac{1}{2}(100)(16) = 800 \text{ lb} \cdot \text{s}$$

$$\int F dt = \frac{1}{2}(62.5)(5) + (50)(t_m - 5)$$

$$\int P dt - \int F dt = 800 - [156.25 + 50(t_m - 5)] = 0 \qquad t_m = 17.875 \text{ s}$$

$t_m > 16$ s OK $\qquad t_m = 17.88$ s ◀



PROBLEM 13.138

Solve Problem 13.137, assuming that the weight of the block is 175 lb.

PROBLEM 13.137 A 125-lb block initially at rest is acted upon by a force **P** which varies as shown. Knowing that the coefficients of friction between the block and the horizontal surface are $\mu_s = 0.50$ and $\mu_k = 0.40,$ determine (*a*) the time at which the block will start moving, (*b*) the maximum speed reached by the block, (*c*) the time at which the block will stop moving.

SOLUTION

See solution of Problem 13.137.

$$W = 175 \text{ lb} \qquad v = \frac{1}{m}\left[\int P dt - \int F dt\right] \qquad (1)$$

(*a*) Block starts moving: $P = F_s = \mu_s W = (0.50)(175) = 87.5 \text{ lb}$

See first figure of Problem 13.137.

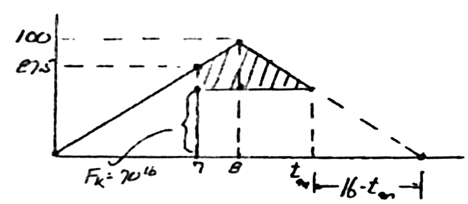
$$\frac{t_1}{F_s} = \frac{8 \text{ s}}{100 \text{ lb}}; \quad \frac{t_1}{87.5 \text{ lb}} = \frac{8 \text{ s}}{100 \text{ s}} \qquad t_1 = 7.00 \text{ s} \blacktriangleleft$$

(*b*) Maximum velocity: $P = F_k = \mu_k W = 0.4(175) = 70 \text{ lb}$

$$\frac{16 - t_m}{70 \text{ lb}} = \frac{8 \text{ s}}{100 \text{ lb}}$$

$$16 - t_m = 70\left(\frac{8}{100}\right) = 5.6$$

$$t_m = 10.40 \text{ s}$$



Eq. (1):

$$v_m = \frac{1}{m}\begin{bmatrix}\text{shaded}\\ \text{area}\end{bmatrix}$$

$$= \frac{1}{m}\left[\frac{1}{2}(17.5 + 30)(1.0) + \frac{1}{2}(30)(10.4 - 8)\right]$$

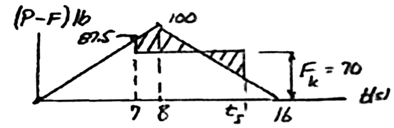
$$= \frac{1}{m}(59.75)$$

$$v_m = \frac{1}{\frac{175 \text{ lb}}{32.2}}[59.75]$$

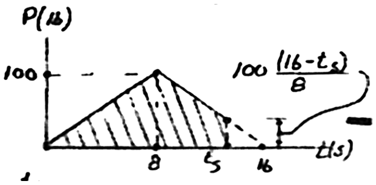
$$= 10.994 \text{ ft/s} \qquad \mathbf{v}_m = 10.99 \text{ ft/s} \longrightarrow \blacktriangleleft$$

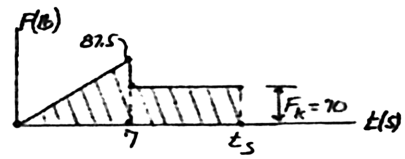
PROBLEM 13.138 (Continued)

(*c*) Block stops moving when net impulse $\left[\int (P-F)dt\right]=0$



Assume $t_s, < 16$ s.





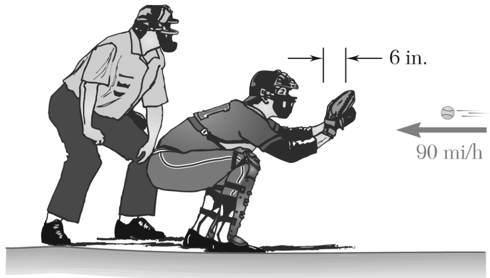
$$\int_0^{t_s} Pdt = \frac{1}{2}(100)(8) + \frac{1}{2}\left[100 + 100\frac{(16-t_s)}{8}\right](t_s - 8)$$

$$= \frac{1}{2}(100)(16) - \frac{1}{2}\left(\frac{100}{8}\right)(16-t_s)^2$$

$$\int_0^{t_s} Fdt = \frac{1}{2}(87.5)(7) + (70)(t_s - 7)$$

$$\int Pdt - \int Fdt = 800 - \frac{100}{16}(16-t_s)^2 - 306.25 - 70(t_s - 7) = 0$$

Solving for t_s, $t_s = 13.492$ s $t_s = 13.49$ s ◀



PROBLEM 13.139

A baseball player catching a ball can soften the impact by pulling his hand back. Assuming that a 5-oz ball reaches his glove at 90 mi/h and that the player pulls his hand back during the impact at an average speed of 30 ft/s over a distance of 6 in., bringing the ball to a stop, determine the average impulsive force exerted on the player's hand.

SOLUTION

$$v = 90\text{ mi/h} = 132\text{ ft/s}$$

$$m = \frac{5}{16}/\text{g}$$

$$t = \frac{d}{v_{av}} = \frac{\left(\frac{6}{12}\right)}{30} = \left(\frac{1}{60}\right)\text{ s}$$

$$\xrightarrow{+}\ 0 = F_{av}t + mv \quad F_{av} = \frac{Wv}{gt}$$

$$F_{av} = \frac{mv}{t}$$

$$= \frac{\left(\frac{5}{16}\text{ lb}\right)(132\text{ ft/s})}{(32.2\text{ ft/s}^2)\left(\frac{1}{60}\text{ s}\right)}$$

$F_{av} = 76.9\text{ lb}$ ◀

PROBLEM 13.140

A 1.62 ounce golf ball is hit with a golf club and leaves it with a velocity of 100 mi/h. We assume that for $0 \le t \le t_0$, where t_0 is the duration of the impact, the magnitude F of the force exerted on the ball can be expressed as $F = F_m \sin(\pi t/t_0)$. Knowing that $t_0 = 0.5$ ms, determine the maximum value F_m of the force exerted on the ball.

SOLUTION

$$W = 1.62 \text{ ounces} = 0.10125 \text{ lb} \quad m = 3.1444 \times 10^{-3} \text{ slug}$$

$$t = 0.5 \text{ ms} = 0.5 \times 10^{-3} \text{s}$$

$$v = 100 \text{ mi/h} = 146.67 \text{ ft/s}$$

The impulse applied to the ball is

$$\int_0^{t_0} F dt = \int_0^{t_0} F_m \sin \frac{\pi t}{t_0} dt = -\frac{F_m t_0}{\pi} \cos \frac{\pi t}{t_0} \bigg|_0^{t_0}$$

$$= -\frac{F_m t_0}{\pi} (\cos \pi - \cos 0) = \frac{2 F_m t_0}{\pi}$$

Principle of impulse and momentum.

$$m\mathbf{v}_1 + \int_0^{t_0} \mathbf{F}\, dt = m\mathbf{v}_2$$

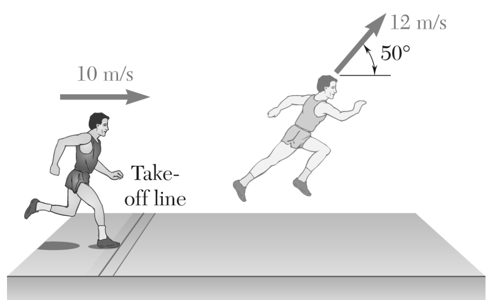
with $\mathbf{v}_1 = 0$,

$$0 + \frac{2 F_m t_0}{\pi} = m v_2$$

Solving for F_m,

$$F_m = \frac{\pi m v_2}{2 t_0} = \frac{\pi (3.1444 \times 10^{-3})(146.67)}{(2)(0.5 \times 10^{-3})} = 1.4488 \times 10^3 \text{lb}$$

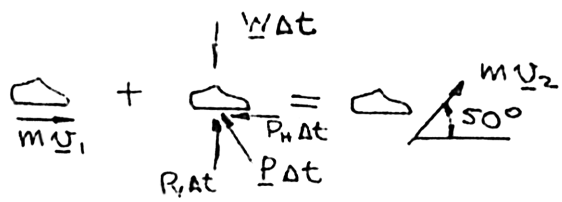
$F_m = 1.45$ kip ◀



PROBLEM 13.141

The triple jump is a track-and-field event in which an athlete gets a running start and tries to leap as far as he can with a hop, step, and jump. Shown in the figure is the initial hop of the athlete. Assuming that he approaches the takeoff line from the left with a horizontal velocity of 10 m/s, remains in contact with the ground for 0.18 s, and takes off at a 50° angle with a velocity of 12 m/s, determine the vertical component of the average impulsive force exerted by the ground on his foot. Give your answer in terms of the weight W of the athlete.

SOLUTION



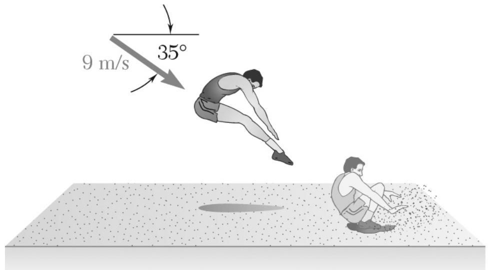
$$m\mathbf{v}_1 + (\mathbf{P} - \mathbf{W})\Delta t = m\mathbf{v}_2 \qquad \Delta t = 0.18 \text{ s}$$

Vertical components

$$0 + (P_v - W)(0.18) = \frac{W}{g}(12)(\sin 50°)$$

$$P_v = W + \frac{(12)(\sin 50°)}{(9.81)(0.18)}W$$

$$P_v = 6.21W \blacktriangleleft$$

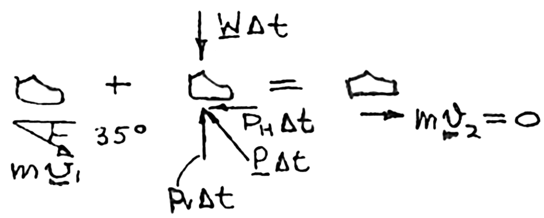


Landing pit

PROBLEM 13.142

The last segment of the triple jump track-and-field event is the jump, in which the athlete makes a final leap, landing in a sand-filled pit. Assuming that the velocity of a 80-kg athlete just before landing is 9 m/s at an angle of 35° with the horizontal and that the athlete comes to a complete stop in 0.22 s after landing, determine the horizontal component of the average impulsive force exerted on his feet during landing.

SOLUTION



$$m = 80 \text{ kg}$$

$$\Delta t = 0.22 \text{ s}$$

$$m\mathbf{v}_1 + (\mathbf{P} - \mathbf{W})\Delta t = m\mathbf{v}_2$$

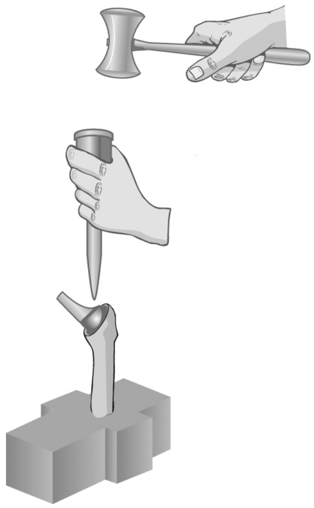
Horizontal components

$$m(9)(\cos 35°) - P_H(0.22) = 0$$

$$P_H = \frac{(80 \text{ kg})(9 \text{ m/s})(\cos 35°)}{(0.22 \text{ s})} = 2.6809 \text{ kN}$$

$P_H = 2.68$ kN ◀

PROBLEM 13.143



The design for a new cementless hip implant is to be studied using an instrumented implant and a fixed simulated femur. Assuming the punch applies an average force of 2 kN over a time of 2 ms to the 200 g implant determine (*a*) the velocity of the implant immediately after impact, (*b*) the average resistance of the implant to penetration if the implant moves 1 mm before coming to rest.

SOLUTION

$$m = 200 \text{ g} = 0.200 \text{ kg}$$
$$F_{\text{ave}} = 2 \text{ kN} = 2000 \text{ N}$$
$$\Delta t = 2 \text{ ms} = 0.002 \text{ s}$$

(*a*) Velocity immediately after impact:

Use principle of impulse and momentum:

$$v_1 = 0 \qquad v_2 = ? \qquad \text{Imp}_{1\to 2} = F_{\text{ave}}(\Delta t)$$
$$m\mathbf{v}_1 + \mathbf{Imp}_{1\to 2} = m\mathbf{v}_2$$
$$0 + F_{\text{ave}}(\Delta t) = mv_2$$
$$v_2 = \frac{F_{\text{ave}}(\Delta t)}{m} = \frac{(2000)(0.002)}{0.200}$$

$v_2 = 20.0 \text{ m/s}$ ◀

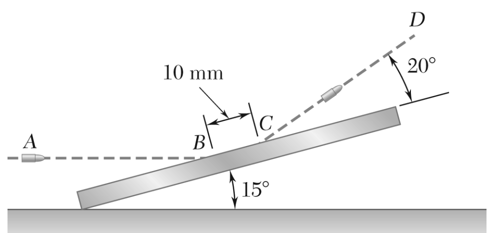
(*b*) Average resistance to penetration:

$$\Delta x = 1 \text{ mm} = 0.001 \text{ m}$$
$$v_2 = 20.0 \text{ ft/s}$$
$$v_3 = 0$$

Use principle of work and energy.

$$T_2 + U_{2\to 3} = T_3 \qquad \text{or} \qquad \frac{1}{2}mv_2^2 - R_{\text{ave}}(\Delta x) = 0$$
$$R_{\text{ave}} = \frac{mv_2^2}{2(\Delta x)} = \frac{(0.200)(20.0)^2}{(2)(0.001)} = 40\times 10^3 \text{ N}$$

$R_{\text{ave}} = 40.0 \text{ kN}$ ◀



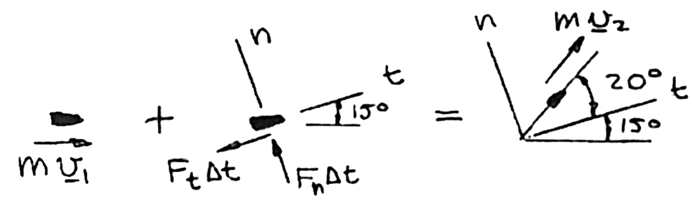
PROBLEM 13.144

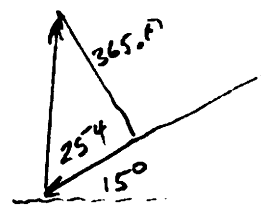
A 25-g steel-jacketed bullet is fired horizontally with a velocity of 600 m/s and ricochets off a steel plate along the path *CD* with a velocity of 400 m/s. Knowing that the bullet leaves a 10-mm scratch on the plate and assuming that its average speed is 500 m/s while it is in contact with the plate, determine the magnitude and direction of the average impulsive force exerted by the bullet on the plate.

SOLUTION

Impulse and momentum.

Bullet alone:





$$m\mathbf{v}_1 + \mathbf{F}\,\Delta t = m\mathbf{v}_2$$

t direction: $mv_1 \cos 15° - F_t \Delta t = mv_2 \cos 20°$

$$\nearrow + \; \mathbf{F}_t \Delta t = (0.025 \text{ kg})[600 \text{ m/s} \cos 15° - 400 \text{ m/s} \cos 20°] = 5.092 \text{ kg} \cdot \text{m/s}$$

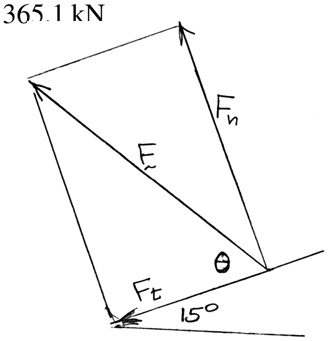
$$\Delta t = \frac{S_{BC}}{v_{AV}} = \frac{0.010 \text{ m}}{500 \text{ m/s}} = 20 \times 10^{-6} \text{ s}$$

$$\mathbf{F}_t = (5.092 \text{ kg} \cdot \text{m/s})/(20 \times 10^{-6} \text{ s}) = 254.6 \times 10^3 \text{ kg} \cdot \text{m/s}^2 = 254.6 \text{ kN}$$

n direction: $\nwarrow^{+} -mv_1 \sin 15° + F_n \Delta t = mv_2 \sin 20°$

$$\mathbf{F}_n \Delta t = (0.025 \text{ kg})[600 \text{ m/s} \sin 15° + 400 \text{ m/s} \sin 20°] = 7.3025 \text{ kg} \cdot \text{m/s}$$

$$\mathbf{F}_n = (43025 \text{ kg} \cdot \text{m/s})/(20 \times 10^{-6}) = 365.1 \times 10^3 \text{ kg} \cdot \text{m/s}^2 = 365.1 \text{ kN}$$



Force on bullet: $F = \sqrt{F_n^2 + F_t^2} = \sqrt{365.1^2 + 254.6^2} = 445 \text{ kN}$

$$\tan\theta = \frac{F_n}{F_t} = \frac{365.1}{254.6} \qquad \theta = 55.1°$$

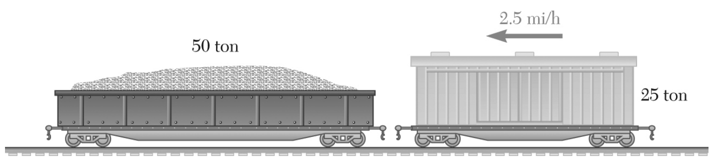
$$\theta - 15° = 40.1°$$

$$\mathbf{F} = 445 \text{ kN} \searrow 40.1°$$

Force on plate: $\mathbf{F}' = -\mathbf{F}$

$$\mathbf{F}' = 445 \text{ kN} \nwarrow 40.1° \blacktriangleleft$$

PROBLEM 13.145



A 25-ton railroad car moving at 2.5 mi/h is to be coupled to a 50 ton car which is at rest with locked wheels ($\mu_k = 0.30$). Determine (*a*) the velocity of both cars after the coupling is completed, (*b*) the time it takes for both cars to come to rest.

SOLUTION

Weight and mass: (Label cars A and B.)

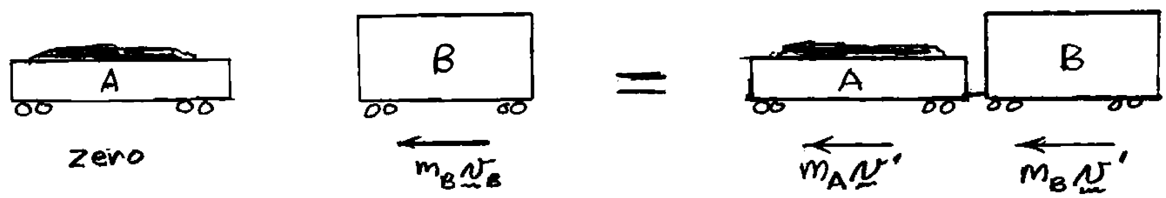
$$\text{Car } A: \quad W_A = 50 \text{ tons} = 100{,}000 \text{ lb}, \quad m_A = 3106 \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$\text{Car } B: \quad W_B = 25 \text{ tons} = 50{,}000 \text{ lb}, \quad m_B = 1553 \text{ lb}\cdot\text{s}^2/\text{ft}$$

Initital velocities: $\quad \mathbf{v}_A = 0$

$$v_B = 2.5 \text{ mi/h} = 3.6667 \text{ ft/s} \qquad \mathbf{v}_B = 3.6667 \text{ ft/s} \longleftarrow$$

(*a*) The momentum of the system consisting of the two cars is conserved immediately before and after coupling.



Let $\mathbf{v}'$ be the common velocity of that cars immediately after coupling. Apply conservation of momentum.

$$+\longleftarrow: \; m_B v_B = m_A v' + m_B v'$$

$$v' = \frac{m_B v_B}{m_A + m_B} = \frac{(3106)(3.6667)}{4569} = 2.444 \text{ ft/s} \qquad \mathbf{v}' = 1.667 \text{ mi/h} \longleftarrow \blacktriangleleft$$

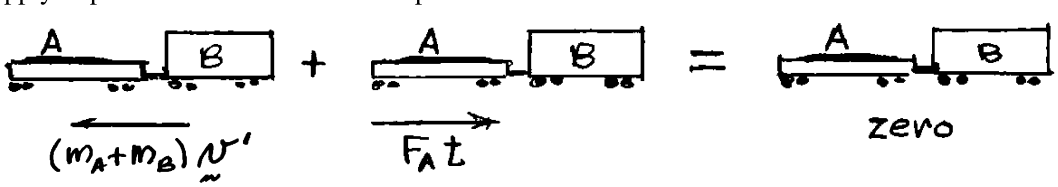
(*b*) After coupling: The friction force acts only on car A.

$$+\uparrow \Sigma F = 0_A: \quad N_A - W_A = 0 \qquad N_A = W_A$$

$$F_A = \mu_k N_A = \mu_k W_A \quad \text{(sliding)}$$

$$F_B = 0 \quad \text{(Car } B \text{ is rolling.)}$$

Apply impuslse-momentum to the coupled cars.



PROBLEM 13.145 (Continued)

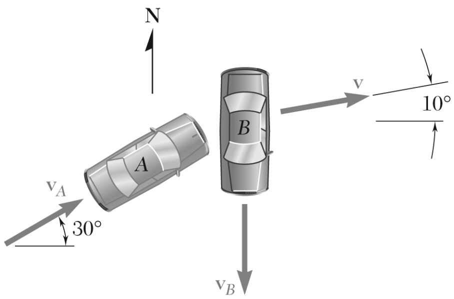
$$\xrightarrow{+}: -(m_A + m_B)v' + F_A t = 0$$

$$t = \frac{(m_A + m_B)v_1^1}{F_A} = \frac{m_B v_B}{\mu_k W_A}$$

$$t = \frac{(1553)(3.6667)}{(0.30)(100,000)} = 0.1898$$

$t = 0.190$ s ◀

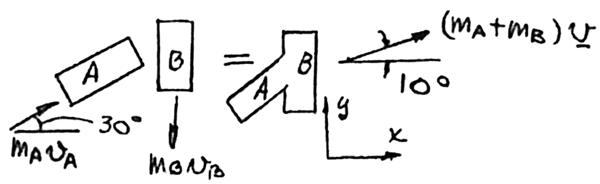
PROBLEM 13.146



At an intersection car B was traveling south and car A was traveling 30° north of east when they slammed into each other. Upon investigation it was found that after the crash the two cars got stuck and skidded off at an angle of 10° north of east. Each driver claimed that he was going at the speed limit of 50 km/h and that he tried to slow down but couldn't avoid the crash because the other driver was going a lot faster. Knowing that the masses of cars A and B were 1500 kg and 1200 kg, respectively, determine (a) which car was going faster, (b) the speed of the faster of the two cars if the slower car was traveling at the speed limit.

SOLUTION

(a) Total momentum of the two cars is conserved.



$$\Sigma mv,\ x: \qquad m_A v_A \cos 30° = (m_A + m_B) v \cos 10° \qquad (1)$$

$$\Sigma mv,\ y: \qquad m_A v_A \sin 30° - m_B v_B = (m_A + m_B) v \sin 10° \qquad (2)$$

Dividing (1) into (2),

$$\frac{\sin 30°}{\cos 30°} - \frac{m_B v_B}{m_A v_A \cos 30°} = \frac{\sin 10°}{\cos 10°}$$

$$\frac{v_B}{v_A} = \frac{(\tan 30° - \tan 10°)(m_A \cos 30°)}{m_B}$$

$$\frac{v_B}{v_A} = (0.4010)\frac{(1500)}{(1200)}\cos 30°$$

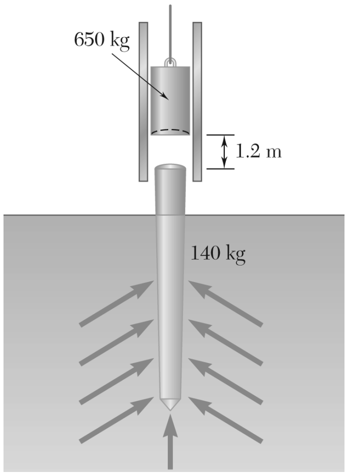
$$\frac{v_B}{v_A} = 0.434 \qquad v_A = 2.30\, v_B$$

Thus, A was going faster. ◀

(b) Since v_B was the slower car.

$$v_B = 50 \text{ km/h}$$

$$v_A = (2.30)(50) \qquad v_A = 115.2 \text{ km/h} \ ◀$$



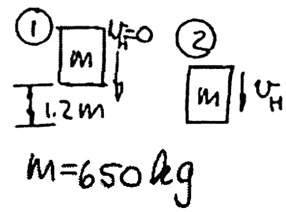
PROBLEM 13.147

The 650-kg hammer of a drop-hammer pile driver falls from a height of 1.2 m onto the top of a 140-kg pile, driving it 110 mm into the ground. Assuming perfectly plastic impact $(e = 0)$, determine the average resistance of the ground to penetration.

SOLUTION

Velocity of the hammer at impact:

Conservation of energy.



$$T_1 = 0$$

$$V_H = mg\ (1.2\text{ m})$$

$$V_H = (650\text{ kg})(9.81\text{ m/s}^2)(1.2\text{ m})$$

$$V_1 = 7652\text{ J}$$

$$T_2 = \frac{1}{2}m$$

$$V_H^2 = \frac{650}{2}v^2 = 325v_H^2$$

$$V_2 = 0$$

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 7652 = 325v^2$$

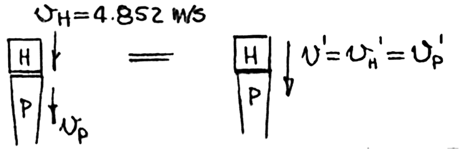
$$v^2 = 23.54\text{ m}^2/\text{s}^2$$

$$v = 4.852\text{ m/s}$$

Velocity of pile after impact:

Since the impact is plastic $(e = 0)$, the velocity of the pile and hammer are the same after impact.

Conservation of momentum:



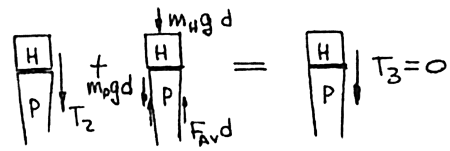
The ground reaction and the weights are non-impulsive.

PROBLEM 13.147 (Continued)

Thus, $m_H v_H = (m_H + m_p)v'$

$$v' = \frac{m_H v_H}{(m_H + m_p)} = \frac{(650)}{(650+140)}(4.852 \text{ m/s}) = 3.992 \text{ m/s}$$

Work and energy: $d = 0.110 \text{ m}$



$$T_2 + U_{2-3} = T_3$$

$$T_2 = \frac{1}{2}(m_H + m_H)(v')^2$$

$$T_3 = 0$$

$$T_2 = \frac{1}{2}(650+140)(3.992)^2$$

$$T_2 = 6.295 \times 10^3 \text{ J}$$

$$U_{2-3} = (m_H + m_p)gd - F_{AV}d$$

$$= (650+140)(9.81)(0.110) - F_{AV}(0.110)$$

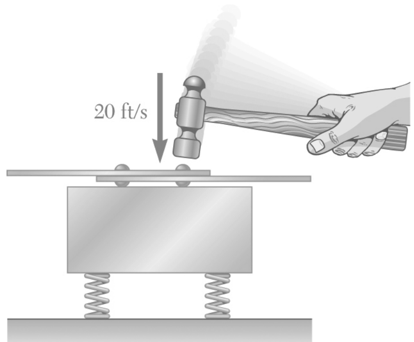
$$U_{2-3} = 852.49 - (0.110)F_{AV}$$

$$T_2 + U_{2-3} = T_3$$

$$6.295 \times 10^3 + 852.49 - (0.110)F_{AV} = 0$$

$$F_{AV} = (7147.5)/(0.110) = 64.98 \times 10^3 \text{ N}$$

$F_{AV} = 65.0 \text{ kN}$ ◀



PROBLEM 13.148

A small rivet connecting two pieces of sheet metal is being clinched by hammering. Determine the impulse exerted on the rivet and the energy absorbed by the rivet under each blow, knowing that the head of the hammer has a weight of 1.5 lbs and that it strikes the rivet with a velocity of 20 ft/s. Assume that the hammer does not rebound and that the anvil is supported by springs and (*a*) has an infinite mass (rigid support), (*b*) has a weight of 9 lb.

SOLUTION

Weight and mass:

$$\text{Hammer:}\quad W_H = 1.5 \text{ lb} \qquad m_H = 0.04658 \text{ lb}\cdot\text{s}^2/\text{ft}$$

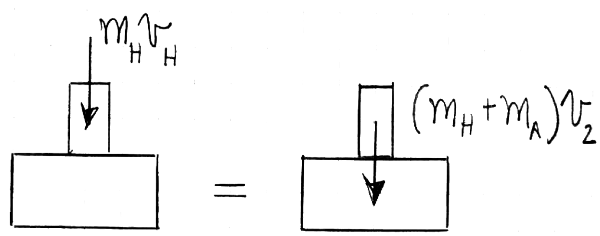
$$\text{Anvil:}\quad \text{Part } a\text{: } W_A = \infty \qquad m_A = \infty$$

$$\text{Part } b\text{: } W_A = 9 \text{ lb} \qquad m_A = 0.2795 \text{ lb}\cdot\text{s}^2/\text{ft}$$

Kinetic energy before impact:

$$T_1 = \frac{1}{2}m_H v_H^2 = \frac{1}{2}(0.04658)(20)^2 = 9.316 \text{ ft}\cdot\text{lb}$$

Let v_2 be the velocity common to the hammer and anvil immediately after impact. Apply the principle of conservation of momentum to the hammer and anvil over the duration of the impact.



$$+\downarrow\text{: } \Sigma m v_1 = \Sigma m v_2$$

$$m_H v_H = (m_H + m_A)v_2$$

$$v_2 = \frac{m_H v_H}{m_H + m_A} \qquad (1)$$

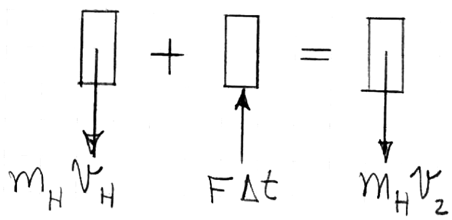
Kinetic energy after impact:

$$T_A = \frac{1}{2}(m_H + m_A)v_2^2 = \frac{1}{2}\frac{m_H^2 v_H^2}{m_H + m_A}$$

$$T_2 = \frac{m_H}{m_H + m_A}T_1 \qquad (2)$$

PROBLEM 13.148 (Continued)

Impulse exerted on the hammer:



$$+\downarrow: m_H v_H - F(\Delta t) = m_H v_2$$

$$F\Delta t = m_H(v_H - v_2) \qquad (3)$$

(*a*) $W_A = \infty$:

By Eq. (1), $v_2 = 0$

By Eq. (2), $T_2 = 0$

Energy absorbed: $T_1 - T_2 = 9.32 \text{ ft} \cdot \text{lb}$ ◀

By Eq. (3), $F(\Delta t) = (0.04658)(20 - 0) = 0.932 \text{ lb} \cdot \text{s}$

The impulse exerted on the rivet the same magnitude but opposite to direction.

$F\Delta t = 0.932 \text{ lb} \cdot \text{s}$ ◀

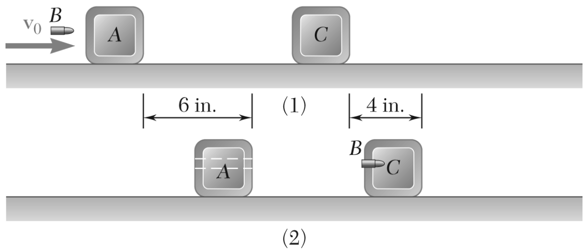
(*b*) $W_A = 9$ lb:

By Eq. (1), $v_2 = \dfrac{(0.04658)(20)}{0.32608} = 2.857 \text{ ft/s}$

By Eq. (2), $T_2 = \dfrac{(0.04658)(9.316)}{0.32608} = 1.331 \text{ ft} \cdot \text{lb}$

Energy absorbed: $T_1 - T_2 = 7.99 \text{ ft} \cdot \text{lb}$ ◀

By Eq. (3), $F(\Delta t) = (0.04658)(20 - 2.857)$ $F(\Delta t) = 0.799 \text{ lb} \cdot \text{s}$ ◀



PROBLEM 13.149

Bullet B weighs 0.5 oz and blocks A and C both weigh 3 lb. The coefficient of friction between the blocks and the plane is $\mu_k = 0.25$. Initially the bullet is moving at v_0 and blocks A and C are at rest (Figure 1). After the bullet passes through A it becomes embedded in block C and all three objects come to stop in the positions shown (Figure 2). Determine the initial speed of the bullet v_0.

SOLUTION

Masses:

Bullet: $$m_B = \frac{0.5}{(16)(32.2)} = 970.5 \times 10^{-6}\,\text{lb}\cdot\text{s}^2/\text{ft}$$

Blocks A and C: $$m_A = m_C = \frac{3}{32.2} = 93.168 \times 10^{-3}\,\text{lb}\cdot\text{s}^2/\text{ft}$$

Block C + bullet: $$m_C + m_B = 94.138 \times 10^{-3}\,\text{lb}\cdot\text{s}^2/\text{ft}$$

Normal forces for sliding blocks from $N - mg = 0$

Block A: $$N_A = m_A\, g = 3.00\text{ lb.}$$

Block C + bullet: $$N_C = (m_C + m_B)g = 3.03125\text{ lb.}$$

Let v_0 be the initial speed of the bullet;

v_1 be the speed of the bullet after it passes through block A;

v_A be the speed of block A immediately after the bullet passes through it;

v_C be the speed block C immediately after the bullet becomes embedded in it.

Four separate processes and their governing equations are described below.

1. The bullet hits block A and passes through it. Use the principle of conservation of momentum.

$$(v_A)_0 = 0$$

$$m_B v_0 + m_A (v_A)_0 = m_B v_1 + m_A v_A$$

$$v_0 = v_1 + \frac{m_A v_A}{m_B} \qquad (1)$$

2. The bullet hits block C and becomes embedded in it. Use the principle of conservation of momentum.

$$(v_C)_0 = 0$$

$$m_B v_1 + m_C (v_C)_0 = (m_B + m_C) v_C$$

$$v_1 = \frac{(m_B + m_C) v_C}{m_B} \qquad (2)$$

PROBLEM 13.149 (Continued)

3. Block A slides on the plane. Use principle of work and energy.

$$T_1 + U_{1\to 2} = T_2$$

$$\frac{1}{2}m_A v_A^2 - \mu_k N_A d_A = 0 \quad \text{or} \quad v_A = \sqrt{\frac{2\mu_k N_A d_A}{m_A}} \tag{3}$$

4. Block C with embedded bullet slides on the plane. Use principle of work and energy.

$$d_C = 4 \text{ in.} = 0.33333 \text{ ft}$$

$$T_1 + U_{1\to 2} = T_2$$

$$\frac{1}{2}(m_C + m_B)v_C^2 - \mu_k N_C d_C = 0 \quad \text{or} \quad v_C = \sqrt{\frac{2\mu_k N_C d_C}{m_C + m_B}} \tag{4}$$

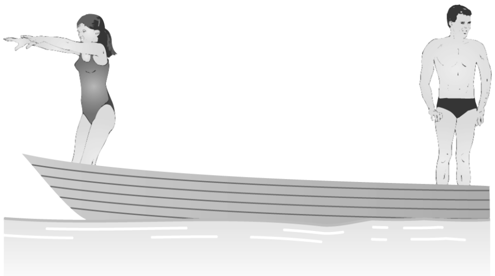
Applying the numerical data:

From Eq. (4), $$v_C = \sqrt{\frac{(2)(0.25)(3.03125)(0.33333)}{94.138\times 10^{-3}}}$$
$$= 2.3166 \text{ ft/s}$$

From Eq. (3), $$v_A = \sqrt{\frac{(2)(0.25)(3.00)(0.5)}{93.168\times 10^{-3}}}$$
$$= 2.8372 \text{ ft/s}$$

From Eq. (2), $$v_1 = \frac{(94.138\times 10^{-3})(2.3166)}{970.5\times 10^{-6}}$$
$$= 224.71 \text{ ft/s}$$

From Eq. (1), $$v_0 = 224.71 + \frac{(93.138\times 10^{-3})(2.8372)}{970.5\times 10^{-6}}$$ $v_0 = 497$ ft/s ◀



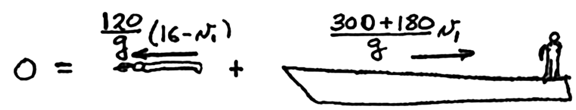
PROBLEM 13.150

A 180-lb man and a 120-lb woman stand at opposite ends of a 300-lb boat, ready to dive, each with a 16-ft/s velocity relative to the boat. Determine the velocity of the boat after they have both dived, if (*a*) the woman dives first, (*b*) the man dives first.

SOLUTION

(*a*) Woman dives first:

Conservation of momentum:



$$-\frac{120}{g}(16-v_1)+\frac{300+180}{g}v_1=0$$

$$v_1=\frac{(120)(16)}{600}=3.20\text{ ft/s}\longrightarrow$$

Man dives next. Conservation of momentum:

$$\frac{300+180}{g}v_1=-\frac{300}{g}v_2+\frac{180}{g}(16-v_2)$$

$$v_2=\frac{480v_1-(180)(16)}{480}=2.80\text{ ft/s}\qquad \mathbf{v}_2=2.80\text{ ft/s}\longleftarrow \blacktriangleleft$$

(*b*) Man dives first:

Conservation of momentum:

$$\frac{180}{g}(16-v_1')-\frac{300+120}{g}v_1'=0$$

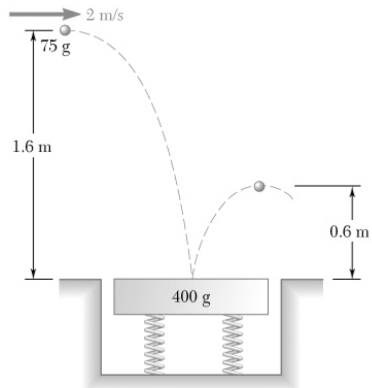
$$v_1'=\frac{(180)(16)}{600}=4.80\text{ ft/s}\longleftarrow$$

Woman dives next. Conservation of momentum:

$$-\frac{300+120}{g}v_1'=\frac{300}{g}v_2'+\frac{120}{g}(16-v_2')$$

$$v_2'=\frac{-420v_1'+(120)(16)}{420}=-0.229\text{ ft/s}$$

$$\mathbf{v}_2'=0.229\text{ ft/s}\longleftarrow \blacktriangleleft$$

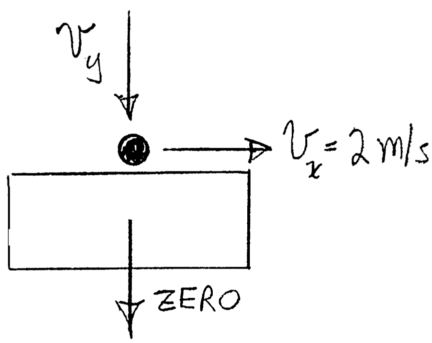


PROBLEM 13.151

A 75-g ball is projected from a height of 1.6 m with a horizontal velocity of 2 m/s and bounces from a 400-g smooth plate supported by springs. Knowing that the height of the rebound is 0.6 m, determine (*a*) the velocity of the plate immediately after the impact, (*b*) the energy lost due to the impact.

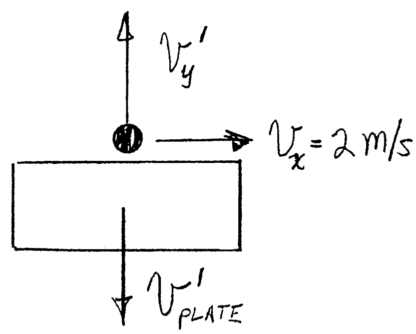
SOLUTION

Just before impact



$$v_y = \sqrt{2g(1.6)} = 5.603 \text{ m/s}$$

Just after impact



$$v_y = \sqrt{2g\,(0.6)} = 3.431 \text{ m/s}$$

(*a*) Conservation of momentum: $(+y\downarrow)$

$$m_{\text{ball}} v_y + 0 = -m_{\text{ball}} v'_y + m_{\text{plate}} v'_{\text{plate}}$$

$$(0.075)(5.603) + 0 = -0.075(3.431) + 0.4 v'_{\text{plate}}$$

$$v'_{\text{plate}} = 1.694 \text{ m/s} \downarrow \blacktriangleleft$$

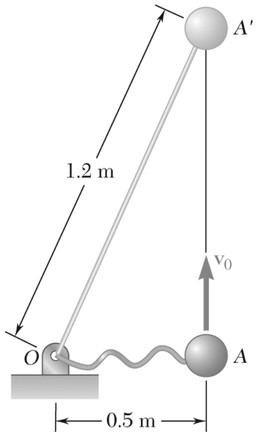
(*b*) Energy loss

Initial energy $$(T+V)_1 = \frac{1}{2}(0.075)(2)^2 + 0.075g(1.6)$$

Final energy $$(T+V)_2 = \frac{1}{2}(0.075)(2)^2 + 0.075g(0.6) + \frac{1}{2}(0.4)(1.694)^2$$

$$\text{Energy lost} = (1.3272 - 1.1653)\,\text{J} = 0.1619\,\text{J} \blacktriangleleft$$

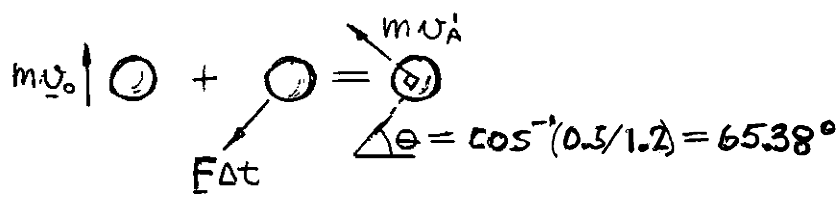
PROBLEM 13.152



A 2-kg sphere A is connected to a fixed Point O by an inextensible cord of length 1.2 m. The sphere is resting on a frictionless horizontal surface at a distance of 1.5 ft from O when it is given a velocity $\mathbf{v}_0$ in a direction perpendicular to line OA. It moves freely until it reaches position A', when the cord becomes taut. Determine the maximum allowable velocity $\mathbf{v}_0$ if the impulse of the force exerted on the cord is not to exceed 3 N·s.

SOLUTION

For the sphere at A' immediately before and after the cord becomes taut



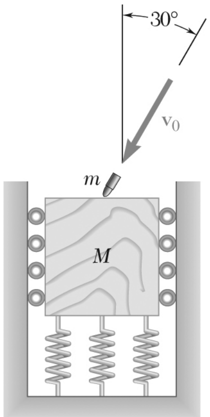
$$mv_0 + F\Delta t = mv_{A'}$$

$$\overset{+}{\nearrow}{}_\theta \quad mv_0 \sin\theta - F\Delta t = 0 \qquad F\Delta t = 3\ \text{N}\cdot\text{s}$$

$$m = 2\ \text{kg}$$

$$2(\sin 65.38°)v_0 = 3$$

$$v_0 = 1.650\ \text{m/s} \blacktriangleleft$$



PROBLEM 13.153

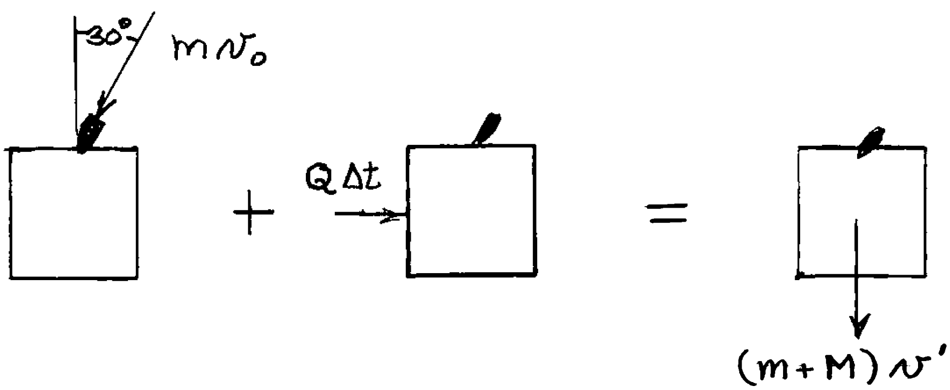
A 1-oz bullet is traveling with a velocity of 1400 ft/s when it impacts and becomes embedded in a 5-lb wooden block. The block can move vertically without friction. Determine (*a*) the velocity of the bullet and block immediately after the impact, (*b*) the horizontal and vertical components of the impulse exerted by the block on the bullet.

SOLUTION

Weight and mass. Bullet: $w = 1 \text{ oz} = \dfrac{1}{16} \text{ lb} \qquad m = 0.001941 \text{ lb} \cdot \text{s}^2/\text{ft}.$

Block: $W = 5 \text{ lb} \qquad M = 0.15528 \text{ lb} \cdot \text{s}^2/\text{ft}.$

(*a*) Use the principle of impulse and momentum applied to the bullet and the block together.



$$\Sigma m\mathbf{v}_1 + \Sigma \mathbf{Imp}_{1\to 2} = m\mathbf{v}_2$$

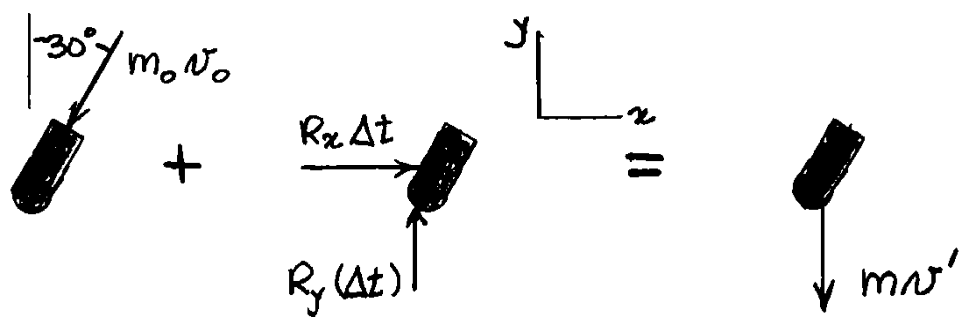
Components $\downarrow$: $\quad mv_0 \cos 30° + 0 = (m + M)v'$

$$v' = \frac{mv_0 \cos 30°}{m + M} = \frac{(0.001941)(1400)\cos 30°}{0.157221}$$

$$v' = 14.968 \text{ ft/s}$$

$\mathbf{v}' = 14.97 \text{ ft/s} \downarrow$ ◀

(*b*) Use the principle of impulse and momentum applied to the bullet alone.



x-components: $-mv_0 \sin 30° + R_x \Delta t = 0$

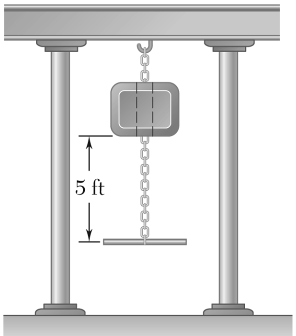
$$R_x \Delta t = mv_0 \sin 30° = (0.001941)(1400)\sin 30°$$

$$= 1.3587 \text{ lb}\cdot\text{s} \qquad R_x \Delta t = 1.359 \text{ lb}\cdot\text{s} \blacktriangleleft$$

y-components: $-mv_0 \cos 30° + R_y \Delta t = -mv'$

$$R_y \Delta t = m(v_0 \cos 30° - v')$$

$$= (0.001941)(1400 \cos 30° - 14.968) \qquad R_y \Delta t = 2.32 \text{ lb}\cdot\text{s} \blacktriangleleft$$



PROBLEM 13.154

In order to test the resistance of a chain to impact, the chain is suspended from a 240-lb rigid beam supported by two columns. A rod attached to the last link is then hit by a 60-lb block dropped from a 5-ft height. Determine the initial impulse exerted on the chain and the energy absorbed by the chain, assuming that the block does not rebound from the rod and that the columns supporting the beam are (*a*) perfectly rigid, (*b*) equivalent to two perfectly elastic springs.

SOLUTION

Velocity of the block just before impact:

$$T_1 = 0 \qquad V_1 = Wh = (60 \text{ lb})(5 \text{ ft}) = 300 \text{ lb} \cdot \text{ft}$$

$$T_2 = \frac{1}{2}mv^2 \quad V_2 = 0$$

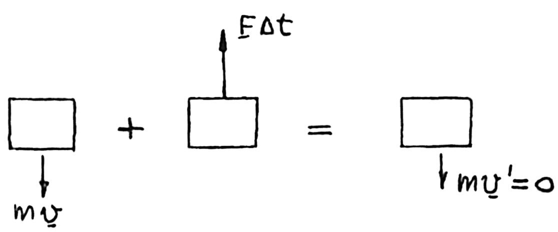
$$T_1 + V_1 = T_2 + V_2$$

$$0 + 300 = \frac{1}{2}\left(\frac{60}{g}\right)v^2$$

$$v = \sqrt{\frac{(600)(32.2)}{60}}$$

$$= 17.94 \text{ ft/s}$$

(*a*) Rigid columns:



$$+\!\uparrow \; -mv + F\Delta t = 0 \qquad \left(\frac{60}{g}\right)(17.94) = F\Delta t$$

$$F\Delta t = 33.43 \text{ lb} \cdot \text{s} \uparrow \text{on the block.} \qquad F\Delta t = 33.4 \text{ lb} \cdot \text{s} \blacktriangleleft$$

All of the kinetic energy of the block is absorbed by the chain.

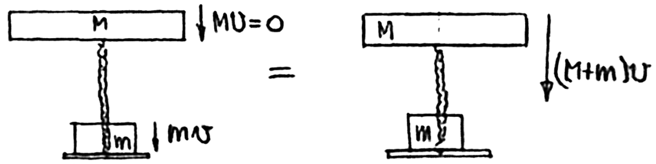
$$T = \frac{1}{2}\left(\frac{60}{g}\right)(17.94)^2$$

$$= 300 \text{ ft} \cdot \text{lb} \qquad E = 300 \text{ ft} \cdot \text{lb} \blacktriangleleft$$

PROBLEM 13.154 (Continued)

(*b*) Elastic columns:

Momentum of system of block and beam is conserved.



$$mv = (M + m)v'$$

$$v' = \frac{m}{(m+M)}v = \frac{60}{300}(17.94 \text{ ft/s}) \qquad v' = 3.59 \text{ ft/s}$$

Referring to figure in part (*a*),

$$-mv + F\Delta t = -mv'$$

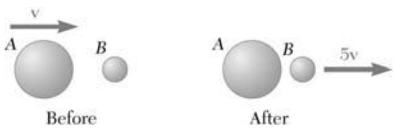
$$F\Delta t = m(v - v')$$

$$= \left(\frac{60}{g}\right)(17.94 - 3.59) \qquad F\Delta t = 26.7 \text{ lb}\cdot\text{s} \blacktriangleleft$$

$$E = \frac{1}{2}mv^2 - \frac{1}{2}mv'^2 - \frac{1}{2}Mv'^2$$

$$= \frac{60}{2g}[(17.94)^2 - (3.59)^2] - \frac{240}{2g}(3.59)^2 \qquad E = 240 \text{ ft}\cdot\text{lb} \blacktriangleleft$$

PROBLEM 13.CQ6



A 5 kg ball A strikes a 1 kg ball B that is initially at rest. Is it possible that after the impact A is not moving and B has a speed of $5v$?

(*a*) Yes

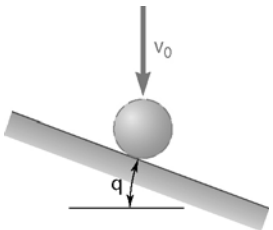
(*b*) No

Explain your answer.

SOLUTION

Answer: (*b*) No.

Conservation of momentum is satisfied, but the coefficient of restitution equation is not. The coefficient of restitution must be less than 1.

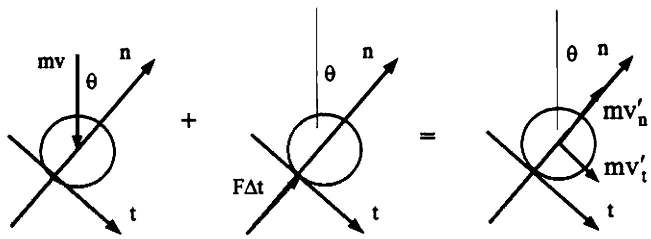


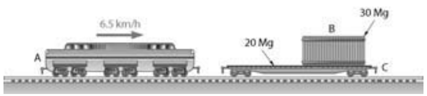
PROBLEM 13.F6

A sphere with a speed v_0 rebounds after striking a frictionless inclined plane as shown. Draw impulse-momentum diagrams that could be used to find the velocity of the sphere after the impact.

SOLUTION

Answer:





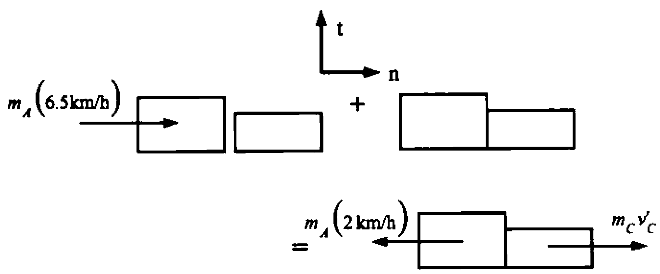
PROBLEM 13.F7

An 80-Mg railroad engine *A* coasting at 6.5 km/h strikes a 20-Mg flatcar *C* carrying a 30-Mg load *B* which can slide along the floor of the car ($\mu_k = 0.25$). The flatcar was at rest with its brakes released. Instead of *A* and *C* coupling as expected, it is observed that *A* rebounds with a speed of 2 km/h after the impact. Draw impulse-momentum diagrams that could be used to determine (*a*) the coefficient of restitution and the speed of the flatcar immediately after impact, and (*b*) the time it takes the load to slide to a stop relative to the car.

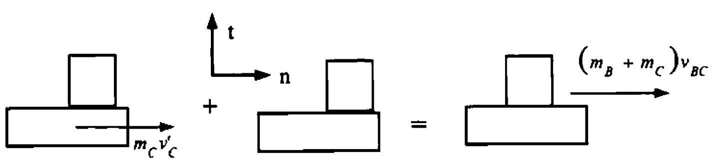
SOLUTION

Answer:

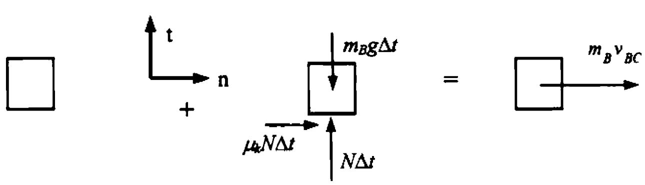
(*a*) Look at *A* and *C* (the friction force between *B* and *C* is not impulsive) to find the velocity after impact.

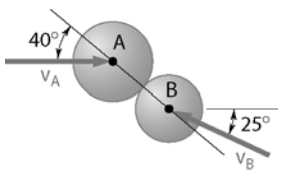


(*b*) Consider just *B* and *C* to find their final velocity.



Consider just *B* to find the time.



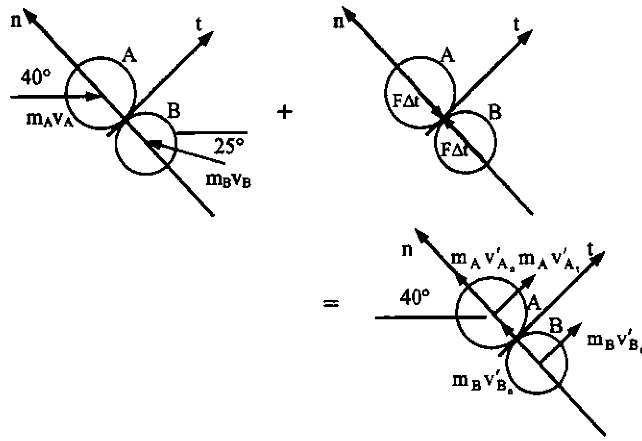


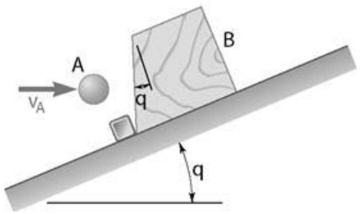
PROBLEM 13.F8

Two frictionless balls strike each other as shown. The coefficient of restitution between the balls is e. Draw the impulse-momentum diagrams that could be used to find the velocities of A and B after the impact.

SOLUTION

Answer:



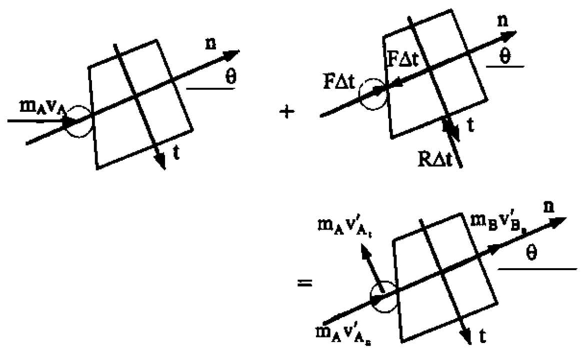


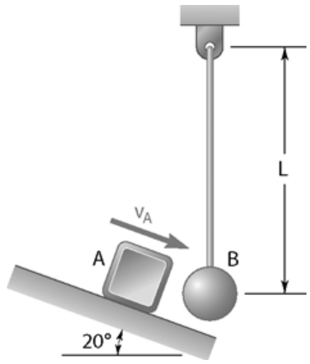
PROBLEM 13.F9

A 10-kg ball *A* moving horizontally at 12 m/s strikes a 10-kg block *B*. The coefficient of restitution of the impact is 0.4 and the coefficient of kinetic friction between the block and the inclined surface is 0.5. Draw impulse-momentum diagrams that could be used to determine the speeds of *A* and *B* after the impact.

SOLUTION

Answer:



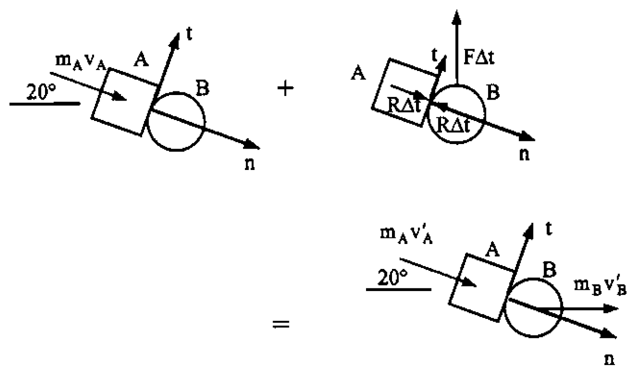


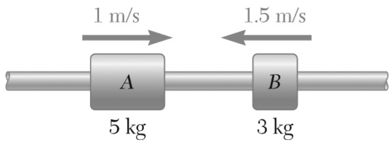
PROBLEM 13.F10

Block *A* of mass m_A strikes ball *B* of mass m_B with a speed of v_A as shown. Draw impulse-momentum diagrams that could be used to determine the speeds of *A* and *B* after the impact and the impulse during the impact.

SOLUTION

Answer:





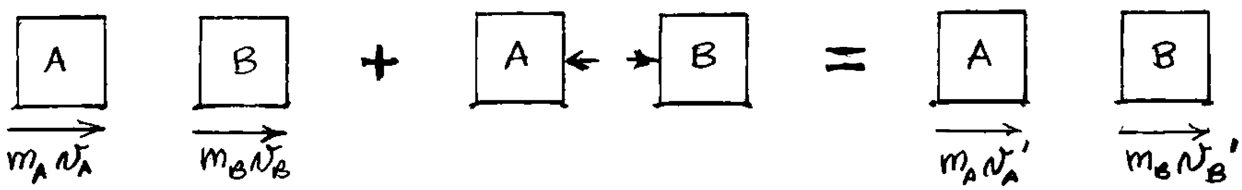
PROBLEM 13.155

The coefficient of restitution between the two collars is known to be 0.70. Determine (*a*) their velocities after impact, (*b*) the energy loss during impact.

SOLUTION

Impulse-momentum principle (collars *A* and *B*):

$$\Sigma m\mathbf{v}_1 + \Sigma \mathbf{Imp}_{1\to 2} = \Sigma m\mathbf{v}_2$$



Horizontal components $\xrightarrow{+}$: $m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$

Using data, $$(5)(1) + (3)(-1.5) = 5v_A' + 3v_B'$$

or $$5v_A' + 3v_B' = 0.5 \qquad (1)$$

Apply coefficient of restitution.

$$v_B' - v_A' = e(v_A - v_B)$$
$$v_B' - v_A' = 0.70[1 - (-0.5)]$$
$$v_B' - v_A' = 1.75 \qquad (2)$$

(*a*) Solving Eqs. (1) and (2) simultaneously for the velocities,

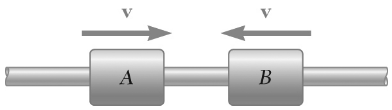
$$v_A' = -0.59375 \text{ m/s} \qquad \mathbf{v}_A = 0.594 \text{ m/s} \longleftarrow \blacktriangleleft$$

$$v_B' = 1.15625 \text{ m/s} \qquad \mathbf{v}_B = 1.156 \text{ m/s} \longrightarrow \blacktriangleleft$$

Kinetic energies: $T_1 = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 = \frac{1}{2}(5)(1)^2 + \frac{1}{2}(3)(-1.5)^2 = 5.875 \text{ J}$

$$T_2 = \frac{1}{2}m_A(v_A')^2 + \frac{1}{2}m_B(v_B')^2 = \frac{1}{2}(5)(-0.59375)^2 + \frac{1}{2}(3)(1.15625)^2 = 2.8867 \text{ J}$$

(*b*) Energy loss: $$T_1 - T_2 = 2.99 \text{ J} \blacktriangleleft$$



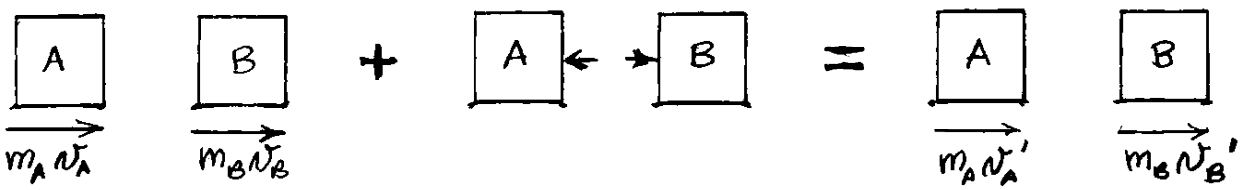
PROBLEM 13.156

Collars A and B, of the same mass m, are moving toward each other with identical speeds as shown. Knowing that the coefficient of restitution between the collars is e, determine the energy lost in the impact as a function of m, e and v.

SOLUTION

Impulse-momentum principle (collars A and B):

$$\Sigma m\mathbf{v}_1 + \Sigma \mathbf{Imp}_{1\to 2} = \Sigma m\mathbf{v}_2$$



Horizontal components $\xrightarrow{+}$: $m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$

Using data, $\quad mv + m(-v) = mv_A' + mv_B'$

or $\quad v_A' + v_B' = 0 \qquad (1)$

Apply coefficient of restitution.

$$v_B' - v_A' = e(v_A - v_B)$$
$$v_B' - v_A' = e[v - (-v)]$$
$$v_B' - v_A' = 2ev \qquad (2)$$

Subtracting Eq. (1) from Eq. (2), $\quad -2v_A = 2ev$

$$v_A = -ev \qquad \mathbf{v}_A = ev \longleftarrow$$

Adding Eqs. (1) and (2), $\quad 2v_B = 2ev$

$$v_B = ev \qquad \mathbf{v}_B = ev \longrightarrow$$

Kinetic energies: $T_1 = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 = \frac{1}{2}mv^2 + \frac{1}{2}m(-v)^2 = mv^2$

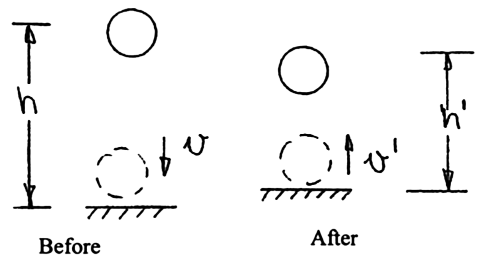
$$T_2 = \frac{1}{2}m_A(v_A')^2 + \frac{1}{2}m_B(v_B')^2 = \frac{1}{2}m(ev)^2 + \frac{1}{2}m(ev)^2 = e^2mv^2$$

Energy loss: $\quad T_1 - T_2 = (1-e^2)\,mv^2$ ◀

PROBLEM 13.157

One of the requirements for tennis balls to be used in official competition is that, when dropped onto a rigid surface from a height of 100 in., the height of the first bounce of the ball must be in the range 53 in. $\leq h \leq 58$ in. Determine the range of the coefficient of restitution of the tennis balls satisfying this requirement.

SOLUTION



Uniform accelerated motion:

$$v = \sqrt{2gh}$$

$$v' = \sqrt{2gh'}$$

Coefficient of restitution: $$e = \frac{v'}{v}$$

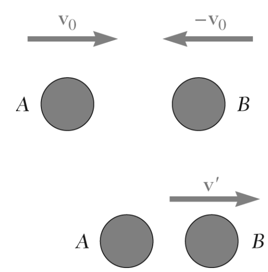
$$e = \sqrt{\frac{h'}{h}}$$

Height of drop $$h = 100 \text{ in.}$$

Height of bounce $$53 \text{ in.} \leq h' \leq 58 \text{ in.}$$

Thus, $$\sqrt{\frac{53}{100}} \leq e \leq \sqrt{\frac{58}{100}}$$

$0.728 \leq e \leq 0.762$ ◀

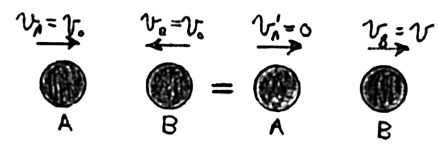


PROBLEM 13.158

Two disks sliding on a frictionless horizontal plane with opposite velocities of the same magnitude v_0 hit each other squarely. Disk A is known to have a weight of 6-lb and is observed to have zero velocity after impact. Determine (a) the weight of disk B, knowing that the coefficient of restitution between the two disks is 0.5, (b) the range of possible values of the weight of disk B if the coefficient of restitution between the two disks is unknown.

SOLUTION

Total momentum conserved:



$$\xrightarrow{+} m_A v_A + m_B v_B = m_A v_A' + m_B v'$$

$$(m_A)v_0 + m_B(-v_0) = 0 + m_B v'$$

$$v' = \left(\frac{m_A}{m_B} - 1\right)v_0 \qquad (1)$$

Relative velocities:

$$v_B' - v_A' = e(v_A - v_B)$$
$$v' = 2ev_0 \qquad (2)$$

Subtracting Eq. (2) from Eq. (1) and dividing by v_0,

$$\frac{m_A}{m_B} - 1 - 2e = 0 \qquad \frac{m_A}{m_B} = 1 + 2e \qquad m_B = \frac{m_A}{1+2e}$$

Since weight is proportional to mass,

$$W_B = \frac{W_A}{1+2e} \qquad (3)$$

(a) With $W_A = 6$ lb and $e = 0.5$,

$$W_B = \frac{6}{1+(2)(0.5)} \qquad W_B = 3.00 \text{ lb} ◀$$

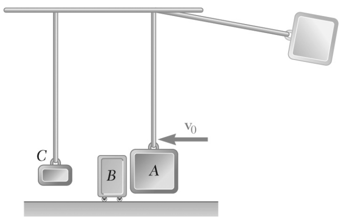
(b) With $W_A = 6$ lb and $e = 1$,

$$W_B = \frac{6}{1+(2)(1)} = 2 \text{ lb}$$

With $W_A = 6$ lb and $e = 0$,

$$W_B = \frac{6}{1+(2)(0)} = 6 \text{ lb}$$

Range: $2.00 \text{ lb} \le W_B \le 6.00 \text{ lb}$ ◀



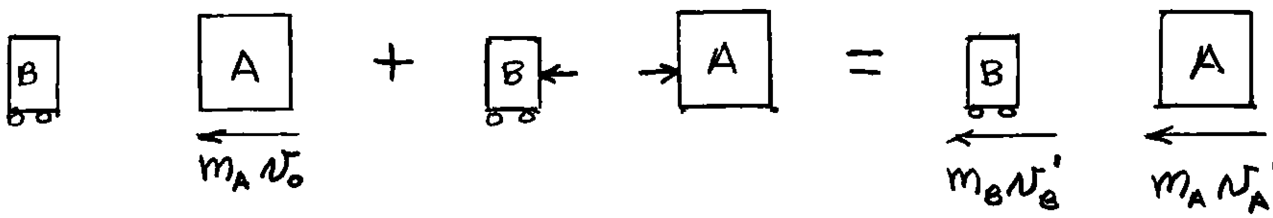
PROBLEM 13.159

To apply shock loading to an artillery shell, a 20-kg pendulum A is released from a known height and strikes impactor B at a known velocity $\mathbf{v_0}$. Impactor B then strikes the 1-kg artillery shell C. Knowing the coefficient of restitution between all objects is e, determine the mass of B to maximize the impulse applied to the artillery shell C.

SOLUTION

First impact: A impacts B. $\quad m_A = 20\text{ kg},\ m_B = ?$

Impulse-momentum: $\quad \Sigma m\mathbf{v} + \Sigma\mathbf{Imp}_{1\to 2} = \Sigma m\mathbf{v}_2$



Components directed left:

$$m_A v_0 = m_A v_A' + m_B v_B'$$

$$20v_0 = 20v_A' + m_B v_B' \qquad (1)$$

Coefficient of restitution:

$$v_B' - v_A' = e(v_A - v_B)$$

$$v_B' - v_A' = ev_0$$

$$v_A' = v_B' - ev_0 \qquad (2)$$

Substituting Eq. (2) into Eq. (1) yields

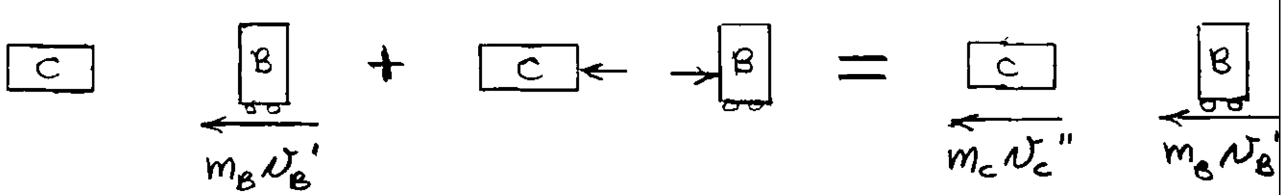
$$20v_0 = 20(v_B' - ev_0) + m_B v_B'$$

$$20v_0(1+e) = (+m_B)v_B'$$

$$v_B' = \frac{20v_0(1+e)}{20+m_B} \qquad (3)$$

Second impact: B impacts C. $\quad m_B = ?,\ m_C = 1\text{ kg}$

Impulse-momentum: $\quad \Sigma m\mathbf{v}_2 + \Sigma\mathbf{Imp}_{2\to 3} = \Sigma m\mathbf{v}_3$



PROBLEM 13.159 (Continued)

Components directed left:

$$m_B v'_B = m_B v''_B + m_C v''_C$$

$$m_B v'_B = m_B v''_B + v''_C \qquad (4)$$

Coefficient of restitution: $$v''_C - v''_B = e(v'_B - v'_C)$$

$$v''_C - v''_B = e v'_B$$

$$v''_B - v''_C = e v'_C \qquad (5)$$

Substituting Eq. (4) into Eq. (5) yields

$$m_B v'_B = m_B(v''_C - e v'_B) + m_C v''_C$$

$$m_B v'_B(1+e) = (1+m_B)v''_C$$

$$v''_C = \frac{m_B v'_B(1+e)}{1+m_B} \qquad (6)$$

Substituting Eq. (3) for v'_B in Eq. (6) yields

$$v''_C = \frac{20 m_B v_0 (1+e)^2}{(20+m_B)(1+m_B)}$$

The impulse applied to the shell C is

$$m_C v''_C = \frac{(1)(20) m_B v_0 (1+e)^2}{(20+m_B)(1+m_B)}$$

To maximize this impulse choose m_B such that

$$Z = \frac{m_B}{(20+m_B)(1+m_B)}$$

is maximum. Set dZ/dm_B equal to zero.

$$\frac{dZ}{dm_B} = \frac{(20+m_B)(1+m_B) - m_B[(20+m_B)+(1+m_B)]}{(20+m_B)^2(1+m_B)^2} = 0$$

$$20 + 21 m_B + m_B^2 - m_B(21 + 2m_B) = 0$$

$$20 - m_B^2 = 0$$

$m_B = 4.47$ kg ◀

PROBLEM 13.160

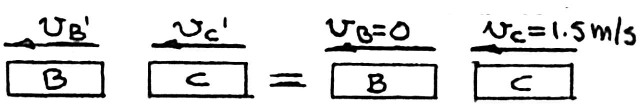
Two identical cars A and B are at rest on a loading dock with brakes released. Car C, of a slightly different style but of the same weight, has been pushed by dockworkers and hits car B with a velocity of 1.5 m/s. Knowing that the coefficient of restitution is 0.8 between B and C and 0.5 between A and B, determine the velocity of each car after all collisions have taken place.

SOLUTION

$$m_A = m_B = m_C = m$$

Collision between B and C:

The total momentum is conserved:



$$\xleftarrow{+} \quad mv'_B + mv'_C = mv_B + mv_C$$

$$v'_B + v'_C = 0 + 1.5 \tag{1}$$

Relative velocities:

$$(v_B - v_C)(e_{BC}) = (v'_C - v'_B)$$
$$(-1.5)(0.8) = (v'_C - v'_B)$$
$$-1.2 = v'_C - v'_B \tag{2}$$

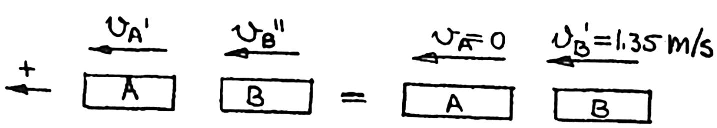
Solving (1) and (2) simultaneously,

$$v'_B = 1.35 \text{ m/s}$$
$$v'_C = 0.15 \text{ m/s}$$

$\mathbf{v}'_C = 0.150 \text{ m/s} \longleftarrow$ ◀

Since $v'_B > v'_C$, car B collides with car A.

Collision between A and B:



$$mv'_A + mv''_B = mv_A + mv'_B$$
$$v'_A + v''_B = 0 + 1.35 \tag{3}$$

PROBLEM 13.160 (Continued)

Relative velocities:

$$(v_A - v'_B)e_{AB} = (v''_B - v'_A)$$
$$(0 - 1.35)(0.5) = v''_B - v'_A$$
$$v'_A - v''_B = 0.675 \qquad (4)$$

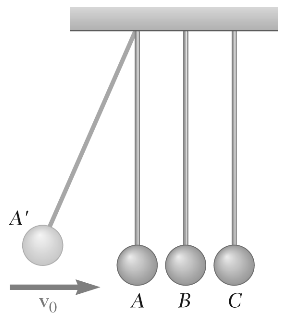
Solving (3) and (4) simultaneously,

$$2v'_A = 1.35 + 0.675$$

$$\mathbf{v}'_A = 1.013 \text{ m/s} \longleftarrow \blacktriangleleft$$

$$\mathbf{v}''_B = 0.338 \text{ m/s} \longleftarrow \blacktriangleleft$$

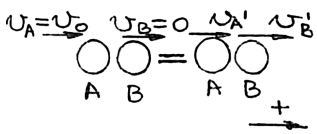
Since $v'_C < v''_B < v'_A$, there are no further collisions.



PROBLEM 13.161

Three steel spheres of equal weight are suspended from the ceiling by cords of equal length which are spaced at a distance slightly greater than the diameter of the spheres. After being pulled back and released, sphere *A* hits sphere *B*, which then hits sphere *C*. Denoting by *e* the coefficient of restitution between the spheres and by $\mathbf{v}_0$ the velocity of *A* just before it hits *B*, determine (*a*) the velocities of *A* and *B* immediately after the first collision, (*b*) the velocities of *B* and *C* immediately after the second collision. (*c*) Assuming now that *n* spheres are suspended from the ceiling and that the first sphere is pulled back and released as described above, determine the velocity of the last sphere after it is hit for the first time. (*d*) Use the result of Part *c* to obtain the velocity of the last sphere when $n = 5$ and $e = 0.9$.

SOLUTION



(*a*) First collision (between *A* and *B*):

The total momentum is conserved:

$$mv_A + mv_B = mv_A' + mv_B'$$

$$v_0 = v_A' + v_B' \qquad (1)$$

Relative velocities:

$$(v_A - v_B)e = (v_B' - v_A')$$

$$v_0 e = v_B' - v_A' \qquad (2)$$

Solving Equations (1) and (2) simultaneously,

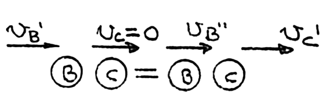
$$v_A' = \frac{v_0(1-e)}{2} \blacktriangleleft$$

$$v_B' = \frac{v_0(1+e)}{2} \blacktriangleleft$$

(*b*) Second collision (between *B* and *C*):

The total momentum is conserved.

$$mv_B' + mv_C = mv_B'' + mv_C'$$



Using the result from (*a*) for v_B'

$$\frac{v_0(1+e)}{2} + 0 = v_B'' + v_C' \qquad (3)$$

Relative velocities:

$$(v_B' - 0)e = v_C' - v_B''$$

PROBLEM 13.161 (Continued)

Substituting again for v'_B from (a)

$$v_0 \frac{(1+e)}{2}(e) = v'_C - v''_B \tag{4}$$

Solving equations (3) and (4) simultaneously,

$$v'_C = \frac{1}{2}\left[\frac{v_0(1+e)}{2} + v_0(1+e)\frac{(e)}{2}\right]$$

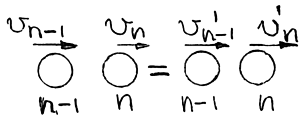
$$v'_C = \frac{v_0(1+e)^2}{4} \blacktriangleleft$$

$$v''_B = \frac{v_0(1-e^2)}{4} \blacktriangleleft$$

(c) For n spheres

n balls

$(n-1)$th collision,



we note from the answer to part (b) with $n = 3$

$$v'_n = v'_3 = v'_C = \frac{v_0(1+e)^2}{4}$$

or $$v'_3 = \frac{v_0(1+e)^{(3-1)}}{2^{(3-1)}}$$

Thus, for n balls

$$v'_n = \frac{v_0(1+e)^{(n-1)}}{2^{(n-1)}} \blacktriangleleft$$

(d) For $n = 5$, $e = 0.90$,

from the answer to part (c) with $n = 5$

$$v'_B = \frac{v_0(1+0.9)^{(5-1)}}{2^{(5-1)}}$$

$$= \frac{v_0(1.9)^4}{(2)^4}$$

$$v'_B = 0.815 v_0 \blacktriangleleft$$

PROBLEM 13.162

At an amusement park there are 200-kg bumper cars A, B, and C that have riders with masses of 40 kg, 60 kg, and 35 kg respectively. Car A is moving to the right with a velocity $\mathbf{v}_A = 2$ m/s and car C has a velocity $\mathbf{v}_B = 1.5$ m/s to the left, but car B is initially at rest. The coefficient of restitution between each car is 0.8. Determine the final velocity of each car, after all impacts, assuming (a) cars A and C hit car B at the same time, (b) car A hits car B before car C does.

SOLUTION

Assume that each car with its rider may be treated as a particle. The masses are:

$$m_A = 200 + 40 = 240 \text{ kg},$$
$$m_B = 200 + 60 = 260 \text{ kg},$$
$$m_C = 200 + 35 = 235 \text{ kg}.$$

Assume velocities are positive to the right. The initial velocities are:

$$v_A = 2 \text{ m/s} \quad v_B = 0 \quad v_C = -1.5 \text{ m/s}$$

Let v_A', v_B', and v_C' be the final velocities.

(a) Cars A and C hit B at the same time. Conservation of momentum for all three cars.

$$m_A v_A + m_B v_B + m_C v_C = m_A v_A' + m_B v_B' + m_C v_C'$$
$$(240)(2) + 0 + (235)(-1.5) = 240 v_A' + 260 v_B' + 235 v_C' \qquad (1)$$

Coefficient of restition for cars A and B.

$$v_B' - v_A' = e(v_A - v_B) = (0.8)(2 - 0) = 1.6 \qquad (2)$$

Coefficient of restitution for cars B and C.

$$v_C' - v_B' = e(v_B - v_C) = (0.8)[0 - (-1.5)] = 1.2 \qquad (3)$$

Solving Eqs. (1), (2), and (3) simultaneously,

$$v_A' = -1.288 \text{ m/s} \quad v_B' = 0.312 \text{ m/s} \quad v_C' = 1.512 \text{ m/s}$$

$\mathbf{v}_A' = 1.288$ m/s $\longleftarrow$ ◀

$\mathbf{v}_B' = 0.312$ m/s $\longrightarrow$ ◀

$\mathbf{v}_C' = 1.512$ m/s $\longrightarrow$ ◀

(*b*) Car *A* hits car *B* before *C* does.

First impact. Car *A* hits car *B*. Let v_A' and v_B' be the velocities after this impact. Conservation of momentum for cars *A* and *B*.

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

$$(240)(2) + 0 = 240 v_A' + 260 v_B' \tag{4}$$

Coefficient of restitution for cars *A* and *B*.

$$v_B' - v_A' = e(v_A - v_B) = (0.8)(2 - 0) = 1.6 \tag{5}$$

Solving Eqs. (4) and (5) simultaneously,

$$v_A' = 0.128 \text{ m/s}, \quad v_B' = 1.728 \text{ m/s}$$

$$\mathbf{v}_A' = 0.128 \text{ m/s} \longrightarrow$$

$$\mathbf{v}_B' = 1.728 \text{ m/s} \longrightarrow$$

Second impact. Cars *B* and *C* hit. Let v_B'' and v_C'' be the velocities after this impact. Conservation of momentum for cars *B* and *C*.

$$m_B v_B' + m_C v_C = m_B v_B'' + m_C v_C''$$

$$(260)(1.728) + (235)(-1.5) = 260 v_B'' + 235 v_C'' \tag{6}$$

Coefficient of restitution for cars *B* and *C*.

$$v_C'' - v_B'' = e(v_B' - v_C) = (0.8)[1.728 - (-1.5)] = 2.5824 \tag{7}$$

Solving Eqs. (6) and (7) simultaneously,

$$v_B'' = -1.03047 \text{ m/s} \quad v_C'' = 1.55193 \text{ m/s}$$

$$\mathbf{v}_B'' = 1.03047 \text{ m/s} \longleftarrow$$

$$\mathbf{v}_C'' = 1.55193 \text{ m/s} \longrightarrow$$

Third impact. Cars *A* and *B* hit again. Let v_A''' and v_B''' be the velocities after this impact. Conservation of momentum for cars *A* and *B*.

$$m_A v_A' + m_B v_B'' = m_A v_A''' + m_B v_B'''$$

$$(240)(0.128) + (260)(-1.03047) = 240 v_A''' + 260 v_B''' \tag{8}$$

Coefficient of restitution for cars *A* and *B*.

$$v_B''' - v_A''' = e(v_A' - v_B'') = (0.8)[0.128 - (-1.03047)] = 0.926776 \tag{9}$$

Solving Eqs. (8) and (9) simultaneously,

$$v_A''' = -0.95633 \text{ m/s}$$

$$v_B''' = -0.02955 \text{ m/s}$$

$$\mathbf{v}_A''' = 0.95633 \text{ m/s} \longleftarrow$$

$$\mathbf{v}_B''' = 0.02955 \text{ m/s} \longleftarrow$$

PROBLEM 13.162 (Continued)

There are no more impacts. The final velocities are:

$$\mathbf{v}_A''' = 0.956 \text{ m/s} \longleftarrow \blacktriangleleft$$

$$\mathbf{v}_B''' = 0.0296 \text{ m/s} \longleftarrow \blacktriangleleft$$

$$\mathbf{v}_C'' = 1.552 \text{ m/s} \longrightarrow \blacktriangleleft$$

We may check our results by considering conservation of momentum of all three cars over all three impacts.

$$\begin{aligned} m_A v_A + m_B v_B + m_C v_C &= (240)(2) + 0 + (235)(-1.5) \\ &= 127.5 \text{ kg} \cdot \text{m/s} \end{aligned}$$

$$\begin{aligned} m_A v_A''' + m_B v_B''' + m_C v_C'' &= (240)(-0.95633) + (260)(-0.02955) + (235)(1.55193) \\ &= 127.50 \text{ kg} \cdot \text{m/s.} \end{aligned}$$

PROBLEM 13.163

At an amusement park there are 200-kg bumper cars A, B, and C that have riders with masses of 40 kg, 60 kg, and 35 kg respectively. Car A is moving to the right with a velocity $\mathbf{v}_A = 2$ m/s when it hits stationary car B. The coefficient of restitution between each car is 0.8. Determine the velocity of car C so that after car B collides with car C the velocity of car B is zero.

SOLUTION

Assume that each car with its rider may be treated as a particle. The masses are:

$$m_A = 200 + 40 = 240 \text{ kg}$$
$$m_B = 200 + 60 = 260 \text{ kg}$$
$$m_C = 200 + 35 = 235 \text{ kg}$$

Assume velocities are positive to the right. The initial velocities are:

$$v_A = 2 \text{ m/s}, \quad v_B = 0, \quad v_C = ?$$

First impact. Car A hits car B. Let v'_A and v'_B be the velocities after this impact. Conservation of momentum for cars A and B.

$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B$$
$$(240)(2) + 0 = 240 v'_A + 260 v'_B \qquad (1)$$

Coefficient of restitution for cars A and B.

$$v'_B - v'_A = e(v_A - v_B) = (0.8)(2 - 0) = 1.6 \qquad (2)$$

Solving Eqs. (1) and (2) simultaneously,

$$v'_A = 0.128 \text{ m/s}$$
$$v'_B = 1.728 \text{ m/s}$$

$$\mathbf{v}'_A = 0.128 \text{ m/s} \longrightarrow$$

$$\mathbf{v}'_B = 1.728 \text{ m/s} \longrightarrow$$

Second impact. Cars B and C hit. Let v''_B and v''_C be the velocities after this impact. $v''_B = 0$. Coefficient of restitution for cars B and C.

$$v''_C - v''_B = e(v'_B - v_C) = (0.8)(1.728 - v_C)$$
$$v''_C = 1.3824 - 0.8 v_C$$

PROBLEM 13.163 (Continued)

Conservation of momentum for cars B and C.

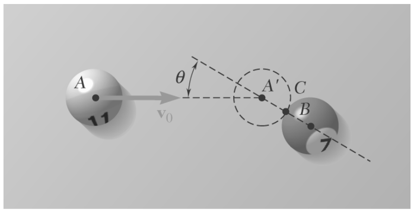
$$m_B v'_B + m_C v_C = m_B v''_B + m_C v''_C$$

$$(260)(1.728) + 235 v_C = (260)(0) + (235)(1.3824 - 0.8 v_C)$$

$$(235)(1.8) v_C = (235)(1.3824) - (260)(1.728)$$

$$v_C = -0.294 \text{ m/s} \qquad \mathbf{v}_C = 0.294 \text{ m/s} \longleftarrow \blacktriangleleft$$

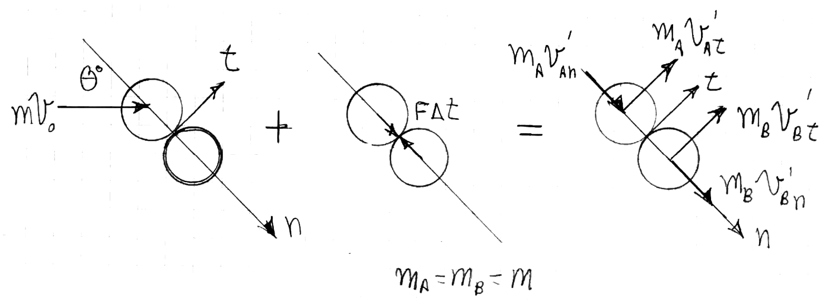
Note: There will be another impact between cars A and B.



PROBLEM 13.164

Two identical billiard balls can move freely on a horizontal table. Ball A has a velocity $\mathbf{v}_0$ as shown and hits ball B, which is at rest, at a Point C defined by $\theta = 45°$. Knowing that the coefficient of restitution between the two balls is $e = 0.8$ and assuming no friction, determine the velocity of each ball after impact.

SOLUTION



Ball A: *t-dir*
$$\cancel{m} v_0 \sin\theta = \cancel{m} v'_{At} \Rightarrow v'_{At} = v_0 \sin\theta$$

Ball B: *t-dir*
$$0 = m_B v'_{Bt} \Rightarrow v'_{Bt} = 0$$

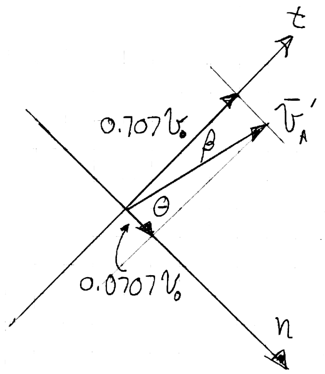
Balls $A + B$: *n-dir*
$$\cancel{m} v_0 \cos\theta + 0 = \cancel{m} v'_{An} + \cancel{m} v'_{Bn} \quad (1)$$

Coefficient of restitution
$$v'_{Bn} - v'_{An} = e(v_{An} - v_{Bn})$$
$$v'_{Bn} - v'_{An} = e(v_0 \cos\theta - 0) \quad (2)$$

Solve (1) and (2)
$$v'_{An} = v_0\left(\frac{1-e}{2}\cos\theta\right); \quad v'_{Bn} = v_0\left(\frac{1+e}{2}\right)\cos\theta$$

With numbers
$$e = 0.8; \quad \theta = 45°$$
$$v'_{At} = v_0 \sin 45° = 0.707\, v_0$$
$$v'_{An} = v_0\left(\frac{1-0.8}{2}\cos 45°\right) = 0.0707\ v_0$$
$$v'_{Bt} = 0$$
$$v'_{Bn} = v_0\left(\frac{1+0.8}{2}\right)\cos 45° = 0.6364\, v_0$$

(*A*)

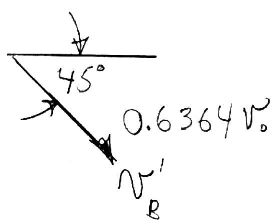


$$|v''_A| = [(0.707\,v_0)^2 + (0.0707 v_0)^2]^{\frac{1}{2}} = 0.711 v_0$$

$$\beta = \tan^{-1}\left(\frac{0.0707}{0.707}\right) = 5.7106°$$

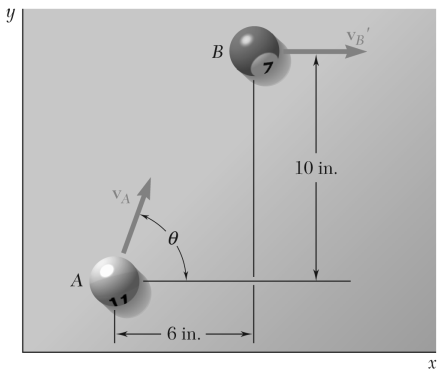
So $\theta = 45 - 5.7106 = 39.3°$

(*B*)



$\vec{v}'_A = 0.711 v_0$ ∠39.3° ◀

$\vec{v}'_B = 0.636 v_0$ ∖45° ◀

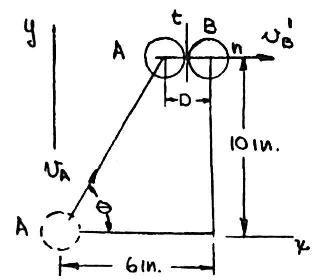


PROBLEM 13.165

The coefficient of restitution is 0.9 between the two 2.37-in. diameter billiard balls *A* and *B*. Ball *A* is moving in the direction shown with a velocity of 3 ft/s when it strikes ball *B*, which is at rest. Knowing that after impact *B* is moving in the *x* direction, determine (*a*) the angle θ, (*b*) the velocity of *B* after impact.

SOLUTION

(*a*) Since v_B' is in the *x*-direction and (assuming no friction), the common tangent between *A* and *B* at impact must be parallel to the *y*-axis,



$$\tan\theta = \frac{10}{6-D}$$

$$\theta = \tan^{-1}\frac{10}{6-2.37}$$

$$= 70.04°$$

$\theta = 70.0°$ ◀

(*b*) Conservation of momentum in $x(n)$ direction:

$$mv_A\cos\theta + m(v_B)_n = m(v_A')_n + mv_B'$$

$$(3)(\cos 70.04) + 0 = (v_A')_n + v_B'$$

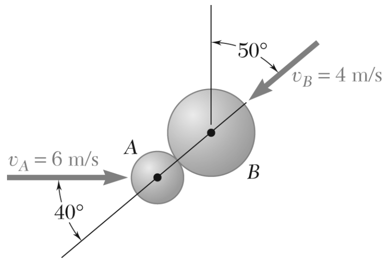
$$1.0241 = (v_A')_n + (v_B') \quad (1)$$

Relative velocities in the *n* direction:

$e = 0.9$ $\qquad (v_A\cos\theta - (v_B)_n)e = v_B' - (v_A')_n$

$$(1.0241 - 0)(0.9) = v_B' - (v_A')_n \quad (2)$$

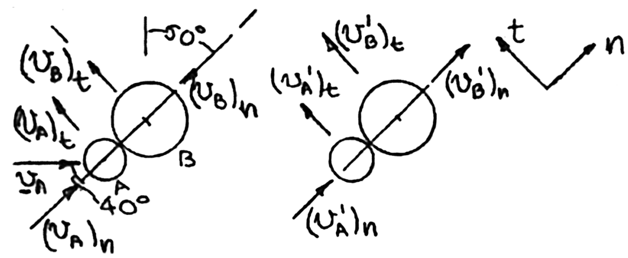
$(1)+(2)$ $\qquad 2v_B' = 1.0241(1.9)$ $\qquad \mathbf{v}_B' = 0.972 \text{ ft/s} \longrightarrow$ ◀



PROBLEM 13.166

A 600-g ball *A* is moving with a velocity of magnitude 6 m/s when it is hit as shown by a 1-kg ball *B* which has a velocity of magnitude 4 m/s. Knowing that the coefficient of restitution is 0.8 and assuming no friction, determine the velocity of each ball after impact.

SOLUTION



Before After

$$v_A = 6 \text{ m/s}$$
$$(v_A)_n = (6)(\cos 40°) = 4.596 \text{ m/s}$$
$$(v_A)_t = -6(\sin 40°) = -3.857 \text{ m/s}$$
$$v_B = (v_B)_n = -4 \text{ m/s}$$
$$(v_B)_t = 0$$

t-direction:

Total momentum conserved:

$$m_A(v_A)_t + m_B(v_B)_t = m_A(v'_B)_t + m_B(v'_B)_t$$
$$(0.6 \text{ kg})(-3.857 \text{ m/s}) + 0 = (0.6 \text{ kg})(v'_A)_t + (1 \text{ kg})(v'_B)t$$
$$-2.314 \text{ m/s} = 0.6\,(v'_A)_t + (v'_B)_t \qquad (1)$$

Ball *A* alone:

Momentum conserved:

$$m_A(v_A)_t = m_A(v'_A)_t \qquad -3.857 = (v'_A)_t$$
$$(v'_A)_t = -3.857 \text{ m/s} \qquad (2)$$

Replacing $(v'_A)_t$ in (2) in Eq. (1)

$$-2.314 = (0.6)(-3.857) + (v'_B)_t$$
$$-2.314 = -2.314 + (v'_B)_t$$
$$(v'_B)_t = 0$$

PROBLEM 13.166 (Continued)

n-direction:

Relative velocities:

$$[(v_A)_n - (v_B)_n]e = (v'_B)_n - (v'_A)_n$$
$$[(4.596) - (-4)](0.8) = (v'_B)_n - (v'_A)_n$$
$$6.877 = (v'_B)_n - (v'_A)_n \qquad (3)$$

Total momentum conserved:

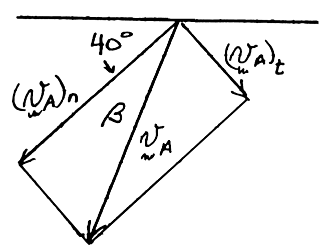
$$m_A(v_A)_n + m_B(v_B)_n = m_A(v'_A)_n + m_B(v'_B)_n$$
$$(0.6\text{ kg})(4.596\text{ m/s}) + (1\text{ kg})(-4\text{ m/s}) = (1\text{ kg})(v'_B)_n + (0.6\text{ kg})(v'_A)_n$$
$$-1.2424 = (v'_B)_n + 0.6(v'_A)_n \qquad (4)$$

Solving Eqs. (4) and (3) simultaneously,

$$(v'_A)_n = 5.075\text{ m/s}$$
$$(v'_B)_n = 1.802\text{ m/s}$$

Velocity of A:

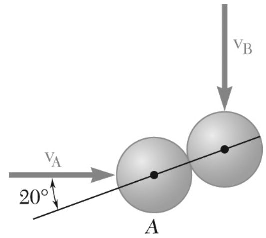
$$\tan\beta = \frac{|(v_A)_t|}{|(v_A)_n|} = \frac{3.857}{5.075}$$
$$\beta = 37.2° \qquad \beta + 40° = 77.2°$$
$$v'_A = \sqrt{(3.857)^2 + (5.075)^2} = 6.37\text{ m/s}$$



$\mathbf{v}'_A = 6.37\text{ m/s} \measuredangle 77.2°$ ◀

Velocity of B: $\mathbf{v}'_B = 1.802\text{ m/s} \measuredangle 40°$ ◀

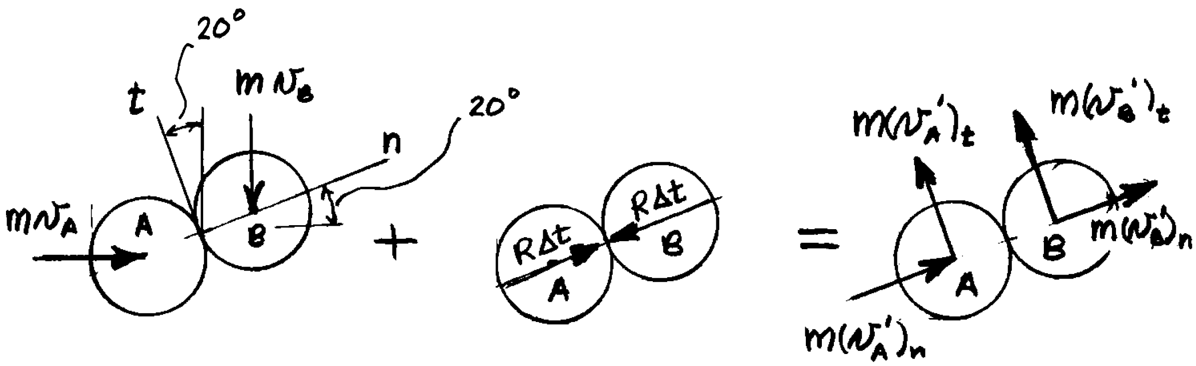
PROBLEM 13.167



Two identical hockey pucks are moving on a hockey rink at the same speed of 3 m/s and in perpendicular directions when they strike each other as shown. Assuming a coefficient of restitution $e = 0.9$, determine the magnitude and direction of the velocity of each puck after impact.

SOLUTION

Use principle of impulse-momentum: $\Sigma m\mathbf{v}_1 + \Sigma \mathbf{Imp}_{1\to 2} = \Sigma m\mathbf{v}_2$



t-direction for puck *A*:

$$-mv_A \sin 20° + 0 = m(v_A')_t$$
$$(v_A')_t = v_A \sin 20° = 3\sin 20° = 1.0261 \text{ m/s}$$

t-direction for puck *B*:

$$-mv_B \cos 20° + 0 = m(v_B')_t$$
$$(v_B')_t = v_B \cos 20° = -3\cos 20° = -2.8191 \text{ m/s}$$

n-direction for both pucks:

$$mv_A \cos 20° - mv_B \sin 20° = m(v_A')_n + m(v_B')_n$$
$$(v_A')_n + (v_B')_n = v_A \cos 20° - v_B \sin 20°$$
$$= 3\cos 20° - 3\sin 20° \qquad (1)$$

Coefficient of restitution: $e = 0.9$

$$(v_B')_n - (v_A')_n = e[(v_A)_n - (v_B)_n]$$
$$= 0.9[3\cos 20° - (-3)\sin 20°] \qquad (2)$$

Solving Eqs. (1) and (2) simultaneously,

$$(v_A')_n = -0.8338 \text{ m/s} \qquad (v_B')_n = 2.6268 \text{ m/s}$$

Summary:

$$(\mathbf{v}'_A)_n = 0.8338 \text{ m/s} \measuredangle 20^\circ$$

$$(\mathbf{v}'_A)_t = 1.0261 \text{ m/s} \measuredangle 70^\circ$$

$$(\mathbf{v}'_B)_n = 2.6268 \text{ m/s} \measuredangle 20^\circ$$

$$(\mathbf{v}'_B)_t = 2.8191 \text{ m/s} \measuredangle 70^\circ$$

$$v_A = \sqrt{(0.8338)^2 + (1.0261)^2} = 1.322 \text{ m/s}$$

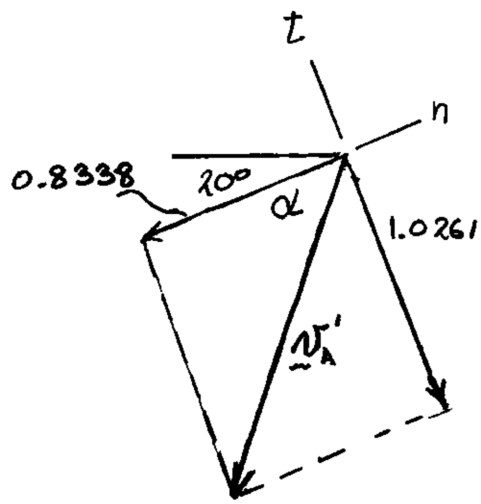
$$\tan\alpha = \frac{1.0261}{0.8338} \qquad \alpha = 50.9^\circ \qquad \alpha + 20^\circ = 70.9^\circ$$

$$\mathbf{v}'_A = 1.322 \text{ m/s} \measuredangle 70.9^\circ \blacktriangleleft$$

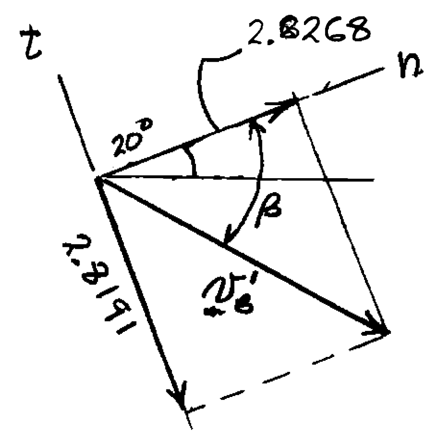
$$v'_B = \sqrt{(2.6268)^2 + (2.8191)^2} = 3.85 \text{ m/s}$$

$$\tan\beta = \frac{2.8191}{2.6268} \qquad \beta = 47.0^\circ \qquad \beta - 20^\circ = 27.0^\circ$$

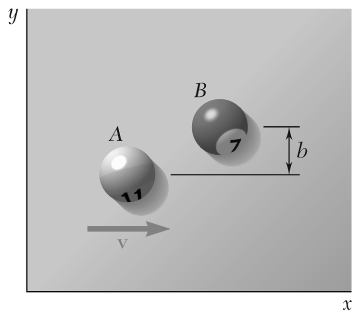
$$\mathbf{v}'_B = 3.85 \text{ m/s} \measuredangle 27.0^\circ \blacktriangleleft$$



Velocity of Puck A



Velocity of Puck B

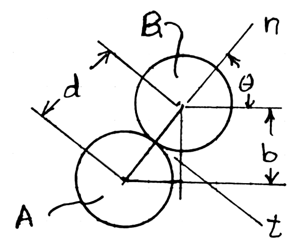


PROBLEM 13.168

Two identical pool balls of 57.15-mm diameter, may move freely on a pool table. Ball B is at rest and ball A has an initial velocity $\mathbf{v} = v_0\mathbf{i}$. ($a$) Knowing that $b = 50$ mm and $e = 0.7$, determine the velocity of each ball after impact. (b) Show that if $e = 1$, the final velocities of the balls from a right angle for all values of b.

SOLUTION

Geometry at instant of impact:



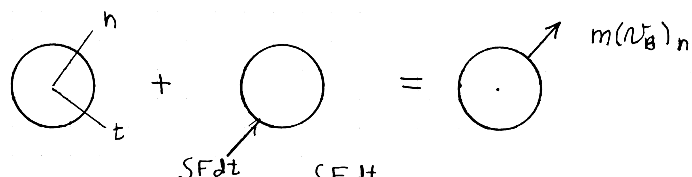
$$\sin\theta = \frac{b}{d} = \frac{50}{57.15}$$

$$\theta = 61.032^\circ$$

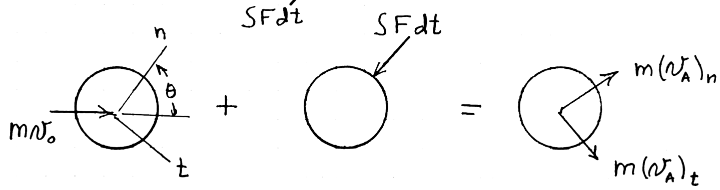
Directions n and t are shown in the figure.

Principle of impulse and momentum:

Ball B:



Ball A:



Ball A, t-direction: $\quad mv_0 \sin\theta + 0 = m(v_A)_t \quad (v_A)_t = v_0 \sin\theta \quad$ (1)

Ball B, t-direction: $\quad 0 + 0 = m(v_B)_t \quad (v_B)_t = 0 \quad$ (2)

Balls A and B, n-direction: $\quad mv_0 \cos\theta + 0 + m(v_A)_n + m(v_B)_n$

$$(v_A)_n + (v_B)_n = v_0 \cos\theta \quad (3)$$

Coefficient of restitution: $\quad (v_B)_n - (v_A)_n = e[v_0 \cos\theta] \quad$ (4)

(a) $\quad e = 0.7.$ From Eqs. (1) and (2),

$$(v_A)_t = 0.87489 v_0 \quad (1)'$$

$$(v_B)_t = 0 \quad (2)'$$

From Eqs. (3) and (4),

$$(v_A)_n + (v_B)_n = 0.48432v_0 \tag{3$'$}$$

$$(v_B)_n - (v_A)_n = (0.7)(0.48432v_0) \tag{4$'$}$$

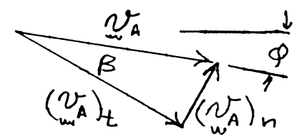
Solving Eqs. (5) and (6) simultaneously,

$$(v_A)_n = 0.072648v_0 \qquad (v_B)_n = 0.41167v_0$$

$$v_A = \sqrt{(v_A)_n^2 + (v_A)_t^2}$$

$$= \sqrt{(0.072648v_0)^2 + (0.87489v_0^2}$$

$$= 0.87790v_0$$



$$\tan\beta = \frac{(v_A)_n}{(v_A)_t} = \frac{0.072648v_0}{0.87489v_0} = 0.083037$$

$$\beta = 4.7468°$$

$$\varphi = 90° - \theta - \beta$$

$$= 90° - 61.032° - 4.7468°$$

$$= 24.221°$$

$\mathbf{v}_A = 0.878v_0 \searrow 24.2°$ ◀

$\mathbf{v}_B = 0.412v_0 \measuredangle 61.0°$ ◀

(*b*) $e = 1$. Eqs. (3) and (4) become

$$(v_A)_n + (v_B)_n = v_0 \cos\theta \tag{3$''$}$$

$$(v_B)_n - (v_A)_n = v_0 \cos\theta \tag{4$''$}$$

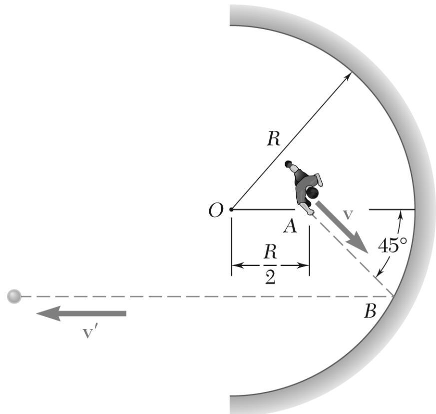
Solving Eqs. (3)$''$ and (4)$''$ simultaneously,

$$(v_A)_n = 0, \quad (v_B)_t = v_0 \cos\theta$$

But $\qquad (v_A)_t = v_0 \sin\theta, \quad \text{and} \quad (v_B)_t = 0$

$\mathbf{v}_A$ is in the *t*-direction and $\mathbf{v}_B$ is in the *n*-direction; therefore, the velocity vectors form a right angle.

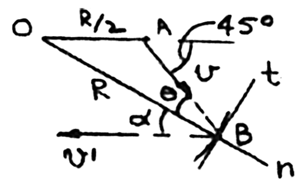
PROBLEM 13.169



A boy located at Point A halfway between the center O of a semicircular wall and the wall itself throws a ball at the wall in a direction forming an angle of 45° with OA. Knowing that after hitting the wall the ball rebounds in a direction parallel to OA, determine the coefficient of restitution between the ball and the wall.

SOLUTION

Law of sines:



$$\frac{\sin\theta}{\frac{R}{2}} = \frac{\sin 135°}{R}$$

$$\theta = 20.705°$$

$$\alpha = 45° - 20.705°$$

$$= 24.295°$$

Conservation of momentum for ball in t-direction:

$$-v\sin\theta = -v'\sin\alpha$$

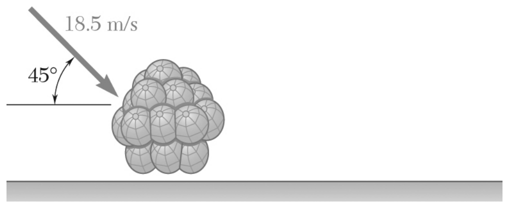
Coefficient of restitution in n: $$v(\cos\theta)e = v'\cos\alpha$$

Dividing, $$\frac{\tan\theta}{e} = \tan\alpha$$

$$e = \frac{\tan 20.705°}{\tan 24.295°}$$

$e = 0.837$ ◀

PROBLEM 13.170

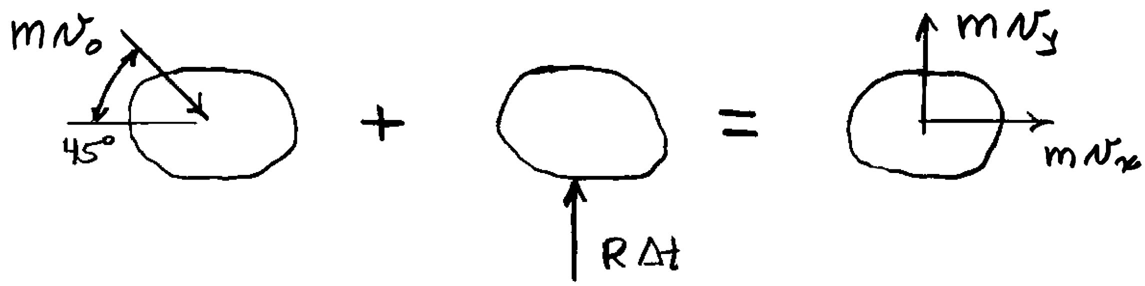


The Mars Pathfinder spacecraft used large airbags to cushion its impact with the planet's surface when landing. Assuming the spacecraft had an impact velocity of 18 m/s at an angle of 45° with respect to the horizontal, the coefficient of restitution is 0.85 and neglecting friction, determine (*a*) the height of the first bounce, (*b*) the length of the first bounce. (Acceleration of gravity on the Mars = 3.73 m/s^2.)

SOLUTION

Use impulse-momentum principle.

$$\Sigma m\mathbf{v}_1 + \Sigma \mathbf{Imp}_{1\to 2} = \Sigma m\mathbf{v}_2$$



The horizontal direction (x to the right) is the tangential direction and the vertical direction (y upward) is the normal direction.

Horizontal components: $mv_0 \sin 45° = 0 = mv_x$

$$v_x = v_0 \sin 45°. \qquad \mathbf{v}_x = 12.728 \text{ m/s} \longrightarrow$$

Vertical components, using coefficient of restitution $e = 0.85$

$$v_y - 0 = e[0 - (-v_0 \cos 45°)]$$

$$v_y = (0.85)(18 \cos 45°) \qquad \mathbf{v}_y = 10.819 \text{ m/s} \uparrow$$

The motion during the first bounce is projectile motion.

Vertical motion: $$y = (v_y)_0 t - \frac{1}{2} g t^2$$

$$v_y = (v_y)_0 - gt$$

Horizontal motion: $$x = v_x t$$

PROBLEM 13.170 (Continued)

(*a*) Height of first bounce:

$$v_y = 0: \qquad 0 = (v_y)_0 = gt$$

$$t = \frac{(v_y)_0}{g} = \frac{10.819 \text{ m/s}}{3.73 \text{ m/s}^2} = 2.901 \text{ s}$$

$$y = (10.819)(2.901) - \frac{1}{2}(3.73)(2.901)^2$$

$y = 15.69$ m ◀

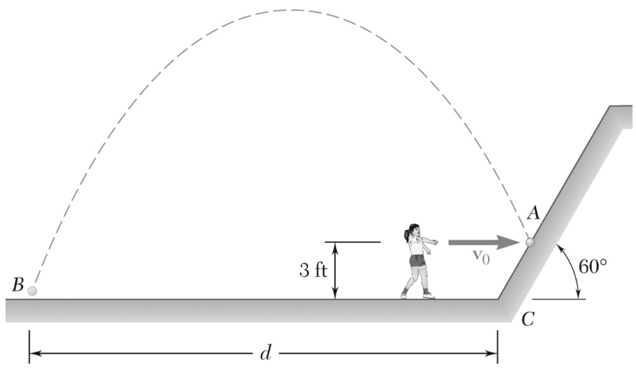
(*b*) Length of first bounce:

$$y = 0: \qquad 10.819t - \frac{1}{2}(3.73)t^2 = 0$$

$$t = 5.801 \text{ s}$$

$$x = (12.728)(5.801)$$

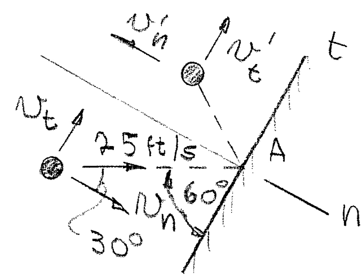
$x = 73.8$ m ◀



PROBLEM 13.171

A girl throws a ball at an inclined wall from a height of 3 ft, hitting the wall at A with a horizontal velocity $\mathbf{v}_0$ of magnitude 25 ft/s. Knowing that the coefficient of restitution between the ball and the wall is 0.9 and neglecting friction, determine the distance d from the foot of the wall to the Point B where the ball will hit the ground after bouncing off the wall.

SOLUTION



Momentum in t direction is conserved

$$mv\sin 30° = mv_t'$$

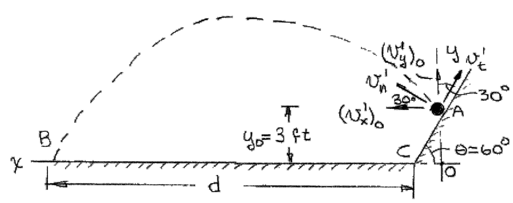
$$(25)(\sin 30°) = v_t'$$

$$v_t' = 12.5 \text{ ft/s}$$

Coefficient of restitution in n-direction

$$(v\cos 30°)e = v_n'$$

$$(25)(\cos 30°)(0.9) = v_n' \qquad v_n' = 19.49 \text{ ft/s}$$



Write v' in terms of x and y components

$$(v_x')_0 = v_n'(\cos 30°) - v_t'(\sin 30°) = 19.49(\cos 30°) - 12.5(\sin 30°)$$
$$= 10.63 \text{ ft/s}$$

$$(v_y')_0 = v_n'(\sin 30°) + v_t'(\cos 30°) = 19.49(\sin 30°) + 12.5(\cos 30°)$$
$$= 20.57 \text{ ft/s}$$

PROBLEM 13.171 (Continued)

Projectile motion

$$y = y_0 + (v'_y)_0 t - \frac{1}{2}gt^2 = 3 \text{ ft} + (20.57 \text{ ft/s})t - (32.2 \text{ ft/s}^2)\frac{t^2}{2}$$

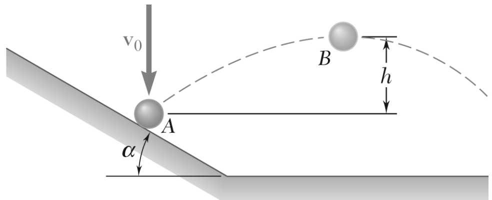
At B,

$$y = 0 = 3 + 20.57t_B - 16.1t_B^2; \qquad t_B = 1.4098 \text{ s}$$

$$x_B = x_0 + (v'_x)_0 t_B = 0 + 10.63(1.4098); \qquad x_B = 14.986 \text{ ft}$$

$$d = x_B - 3\cot 60° = (14.986 \text{ ft}) - (3 \text{ ft})\cot 60° = 13.254 \text{ ft}$$

$d = 13.25$ ft ◀



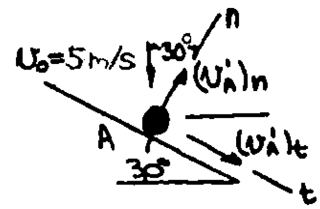
PROBLEM 13.172

A sphere rebounds as shown after striking an inclined plane with a vertical velocity $\mathbf{v}_0$ of magnitude $v_0 = 5$ m/s. Knowing that $\alpha = 30°$ and $e = 0.8$ between the sphere and the plane, determine the height h reached by the sphere.

SOLUTION

Rebound at A

Conservation of momentum in the t-direction:



$$mv_0 \sin 30° = m(v_A')_t$$

$$(v_A')_t = (5 \text{ m/s})(\sin 30°) = 2.5 \text{ m/s}$$

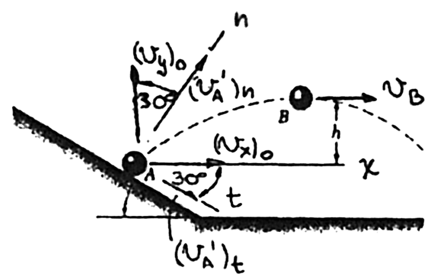
Relative velocities in the n-direction:

$$(-v_0 \cos 30° - 0)e = 0 - (v_A')_n$$

$$(v_A')_n = (0.8)(5 \text{ m/s})(\cos 30°) = 3.4641 \text{ m/s}$$

Projectile motion between A and B:

After rebound



$$(v_x)_0 = (v_A')_t \cos 30° + (v_A')_n \sin 30°$$

$$(v_x)_0 = (2.5)(\cos 30°) + (3.4641) \sin 30 = 3.8971 \text{ m/s}$$

$$(v_y)_0 = -(v_A')_t \sin 30° + (v_A')_n \cos 30°$$

$$(v_y)_0 = -(2.5)(\sin 30°) + (3.4641) \cos 30° = 1.750 \text{ m/s}$$

x-direction:

$$x = (v_x)_0 t \qquad v_x = (v_x)_0$$

$$x = 3.8971t \quad v_x = 3.8971 \text{ m/s} = v_B$$

y-direction:

$$y = (v_y)_0 t - \frac{1}{2} g t^2$$

$$v_y = (v_y)_0 - gt$$

At A:

$$v_y = 0 = (v_y)_0 - g t_{AB}$$

$$t_{AB} = (v_y)_0 / g = \frac{1.75 \text{ m/s}}{9.81 \text{ m/s}^2}$$

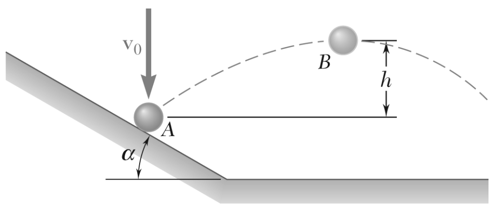
$$t_{A-B} = 0.17839 \text{ s}$$

At B:

$$y = h = (v_y)_0 t_{A-B} - \frac{g t_{A-B}^2}{2}$$

$$h = (1.75)(0.17839) - \frac{9.81}{2}(0.17839)^2$$

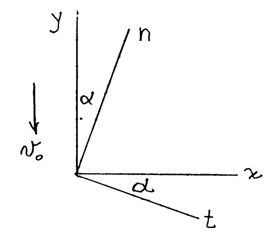
$h = 0.156$ m ◀



PROBLEM 13.173

A sphere rebounds as shown after striking an inclined plane with a vertical velocity $\mathbf{v}_0$ of magnitude $v_0 = 6$ m/s. Determine the value of α that will maximize the horizontal distance the ball travels before reaching its maximum height h assuming the coefficient of restitution between the ball and the ground is (*a*) $e = 1$, (*b*) $e = 0.8$.

SOLUTION



Directions x, y, n, and t are shown in the sketch.

Analysis of the impact: Use the principle of impulse and momentum for components in the t-direction.

$$mv_0 \sin\alpha + 0 = m(v_t')_1$$

$$(v_t)_1 = v_0 \sin\alpha \qquad (1)$$

Coefficient of restitution:

$$(\mathbf{v}_n)_1 = -e(\mathbf{v}_n)_0$$

$$(v_n)_1 = ev_0 \cos\alpha \qquad (2)$$

x and y components of velocity immediately after impact:

$$(v_x)_1 = (v_n)_1 \sin\alpha + (v_t)_1 \cos\alpha = v_0(1+e)\sin\alpha\cos\alpha$$

$$= \frac{1}{2}v_0(1+e)\sin 2\alpha \qquad (3)$$

$$(v_y)_1 = (v_n)_1 \cos\alpha - (v_t)_1 \sin\alpha = v_0(e\cos^2\alpha - \sin^2\alpha)$$

$$= \frac{1}{2}v_0[e(1+\cos 2\alpha) - (1-\cos 2\alpha)]$$

$$= \frac{1}{2}v_0[(1+e)\cos 2\alpha - (1-e)] \qquad (4)$$

Projectile motion: Use coordinates x and y with the origin at the point of impact.

$$x_0 = 0$$

$$y_0 = 0$$

Vertical motion:

$$v_y = (v_y)_1 - gt$$

$$v_y = \frac{1}{2}v_0[(1+e)\cos 2\alpha - (1-e)] - gt$$

$v_y = 0$ at the position of maximum height where

$$t_2 = \frac{(v_y)_1}{g} = \frac{v_0}{2g}[(1+e)\cos\alpha - (1-e)] \qquad (5)$$

Horizontal motion: $$v_x = (v_x)_1 = \frac{1}{2}v_0(1+e)\sin 2\alpha$$

$$x = (v_x)_1 t$$

At the point of maximum height,

$$x_2 = (v_x)_1 t_2 = \frac{v_0^2}{4g}(1+e)\sin 2\alpha[(1+e)\cos 2\alpha - (1-e)]$$

Let $\theta = 2\alpha$ and $Z = 4gx_2/v_0^2(1+e)$. To determine the value of θ that maximizes x_2 (or Z), differentiate Z with respect to θ and set the derivative equal to zero.

$$Z = \sin\theta[(1+e)\cos\theta - (1-e)]$$

$$\frac{dZ}{d\theta} = \cos\theta[(1+e)\cos\theta - (1-e)] - (1+e)\sin^2\theta$$

$$= (1+e)\cos^2\theta - (1-e)\cos\theta - (1+e)(1-\cos^2\theta) = 0$$

$$2(1+e)\cos^2\theta - (1-e)\cos\theta - (1+e) = 0$$

This is a quadratic equation for $\cos\theta$.

(a) $e = 1$ $$4\cos^2\theta - 2 = 0$$

$$\cos^2\theta = \frac{1}{2}$$

$$\cos\theta = \pm\frac{\sqrt{2}}{2}$$

$$\theta = \pm 45° \quad \text{and} \quad \pm 135°$$

$$\alpha = 22.5° \quad \text{and} \quad 67.5°$$

Reject the negative values of θ which make x_2 negative.

Reject $\alpha = 67.5°$ since it makes a smaller maximum height.

$\alpha = 22.5°$ ◀

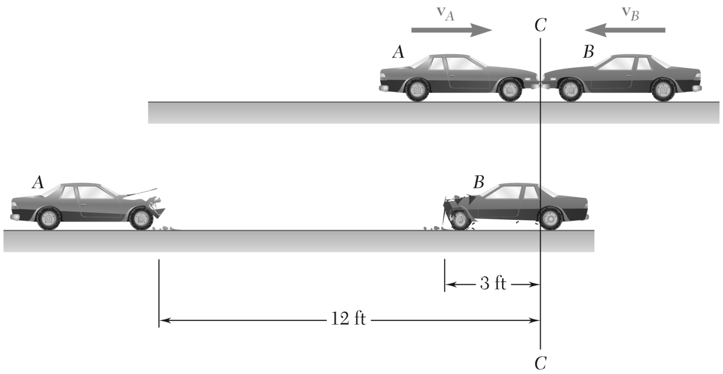
(b) $e = 0.8$ $$3.6\cos^2\theta - 0.2\cos\theta - 1.8 = 0$$

$$\cos\theta = 0.73543 \quad \text{and} \quad -0.67987$$

$$\theta = \pm 42.656° \quad \text{and} \quad \pm 132.833°$$

$$\alpha = \pm 21.328° \quad \text{and} \quad \pm 66.417°$$

$\alpha = 21.3°$ ◀

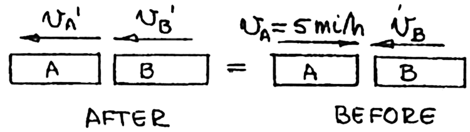


PROBLEM 13.174

Two cars of the same mass run head-on into each other at C. After the collision, the cars skid with their brakes locked and come to a stop in the positions shown in the lower part of the figure. Knowing that the speed of car A just before impact was 5 mi/h and that the coefficient of kinetic friction between the pavement and the tires of both cars is 0.30, determine (*a*) the speed of car B just before impact, (*b*) the effective coefficient of restitution between the two cars.

SOLUTION

(*a*) At C:



Conservation of total momentum:

$$m_A = m_B = m$$

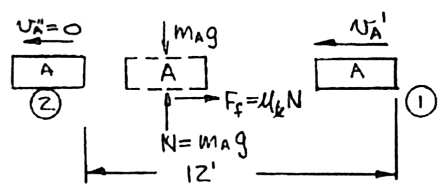
$$5 \text{ mi/h} = 7.333 \text{ ft/s}$$

$$\overset{+}{\longleftarrow} \quad m_A v_A + m_B v_B = m_A v'_A + m_B v'_B$$

$$-7.333 + v_B = v'_A + v'_B \qquad (1)$$

Work and energy.

Care A (after impact):



$$T_1 = \frac{1}{2} m_A (v'_A)^2$$

$$T_2 = 0$$

$$U_{1-2} = F_f (12)$$

$$U_{1-2} = \mu_k m_A g (12 \text{ ft})$$

$$T_1 + U_{1-2} = T_2$$

$$\frac{1}{2} m_A (v'_A)^2 - m_k m_A g (12) = 0$$

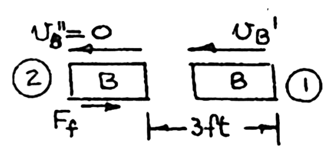
$$(v'_A)^2 = (2)(12 \text{ ft})(0.3)(32.2 \text{ ft/s}^2)$$

$$= 231.84 \text{ ft/s}^2$$

$$v'_A = 15.226 \text{ ft/s}$$

PROBLEM 13.174 (Continued)

Car B (after impact):



$$T_1 = \frac{1}{2} m_B (v_B')^2$$

$$T_2 = 0$$

$$U_{1-2} = \mu_k \, m_B \, g(3)$$

$$T_1 + U_{1-2} = T_2$$

$$\frac{1}{2} m_B (v_B')^2 - \mu_k m_B g(3)$$

$$v_B'^2 = (2)(3 \text{ ft})(0.3)(32.2 \text{ ft/s}^2)$$

$$(v_B')^2 = 57.96 \text{ ft/s}^2$$

$$v_B' = 7.613 \text{ ft/s}$$

From (1)

$$v_B = 7.333 + v_A' + v_B'$$
$$= 7.333 + 15.226 + 7.613$$

$$v_B = 30.2 \text{ ft/s} = 20.6 \text{ mi/h} \blacktriangleleft$$

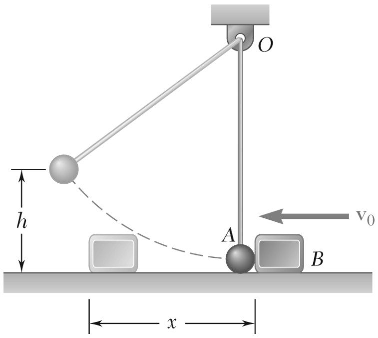
(b) Relative velocities:

$$\overset{+}{\longleftarrow} \quad (-v_A - v_B)e = v_B' - v_A'$$

$$(-7.333 - 30.2)e = 7.613 - 15.226$$

$$e = \frac{-(7.613)}{-(37.53)} = 0.2028$$

$$e = 0.203 \blacktriangleleft$$

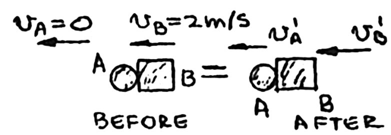


PROBLEM 13.175

A 1-kg block B is moving with a velocity $\mathbf{v}_0$ of magnitude $v_0 = 2$ m/s as it hits the 0.5-kg sphere A, which is at rest and hanging from a cord attached at O. Knowing that $\mu_k = 0.6$ between the block and the horizontal surface and $e = 0.8$ between the block and the sphere, determine after impact (*a*) the maximum height h reached by the sphere, (*b*) the distance x traveled by the block.

SOLUTION

Velocities just after impact



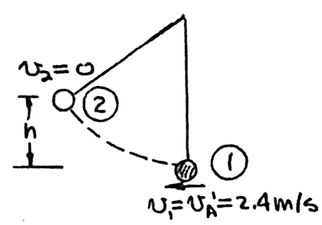
Total momentum in the horizontal direction is conserved:

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$
$$0 + (1\text{ kg})(2\text{ m/s}) = (0.5\text{ kg})(v_A') + (1\text{ kg})(v_B')$$
$$4 = v_A' + 2v_B' \qquad (1)$$

Relative velocities:

$$(v_A - v_B)e = (v_B' - v_A')$$
$$(0-2)(0.8) = v_B' - v_A'$$
$$-1.6 = v_B' - v_A' \qquad (2)$$

Solving Eqs. (1) and (2) simultaneously:



$$v_B' = 0.8\text{ m/s}$$
$$v_A' = 2.4\text{ m/s}$$

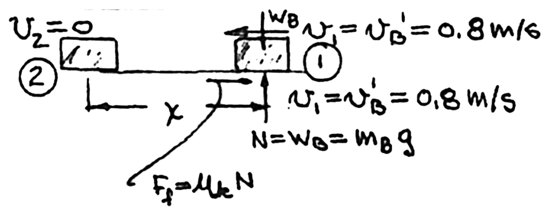
(*a*) Conservation of energy:

$$T_1 = \frac{1}{2} m_A v_1^2 \quad V_1 = 0$$
$$T_1 = \frac{1}{2} m_A (2.4\text{ m/s})^2 = 2.88 m_A$$
$$T_2 = 0$$
$$V_2 = m_A g h$$
$$T_1 + V_1 = T_2 + V_2 \qquad 2.88 \cancel{m}_A + 0 = 0 + \cancel{m}_A (9.81)h$$

$h = 0.294$ m ◀

PROBLEM 13.175 (Continued)

(*b*) Work and energy:



$$T_1 = \frac{1}{2} m_B v_1^2 = \frac{1}{2} m_B (0.8 \text{ m/s})^2 = 0.32 m_B \qquad T_2 = 0$$

$$U_{1-2} = -F_f x = -\mu_k N x = -\mu_x m_B g x = -(0.6)(m_B)(9.81)x$$

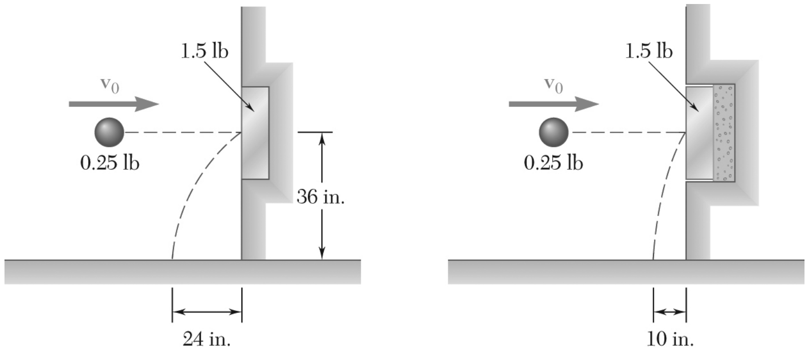
$$U_{1-2} = -5.886 m_B x$$

$$T_1 + U_{1-2} = T_2: \qquad 0.32 m_B - 5.886 m_B x = 0$$

$$x = 0.0544 \text{ m} \qquad\qquad x = 54.4 \text{ mm} \blacktriangleleft$$

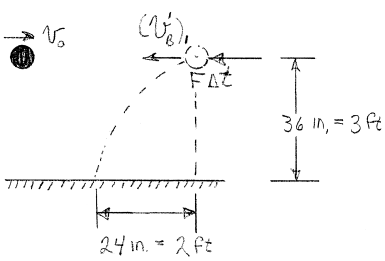
PROBLEM 13.176

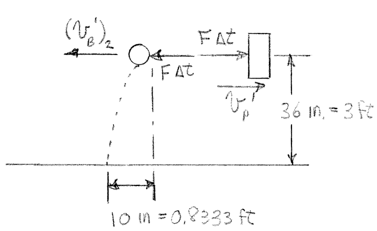
A 0.25-lb ball thrown with a horizontal velocity $\mathbf{v}_0$ strikes a 1.5-lb plate attached to a vertical wall at a height of 36 in. above the ground. It is observed that after rebounding, the ball hits the ground at a distance of 24 in. from the wall when the plate is rigidly attached to the wall (Figure 1) and at a distance of 10 in. when a foam-rubber mat is placed between the plate and the wall (Figure 2). Determine (*a*) the coefficient of restitution e between the ball and the plate, (*b*) the initial velocity $\mathbf{v}_0$ of the ball.



(1) (2)

SOLUTION





(*a*) Figure (1), ball alone relative velocities

$$v_0 e = (v_B')_1$$

Projectile motion

$$t = \text{time for the ball to hit ground}$$

$$2\text{ ft} = v_0 e t \tag{1}$$

Figure (2), ball and plate relative velocities

$$(v_B - v_A)e = v_P' + (v_B')_2$$

$$v_B = v_0, \qquad v_P = 0$$

$$v_0 e = v_P' + (v_B')_2 \tag{2}$$

Conservation of momentum

$$\xrightarrow{+} m_B v_B + m_P v_P = m_B v_B' + m_P v_P'$$

$$\frac{0.25}{g}v_0 + 0 = \frac{0.25}{g}(-v_B')_2 + \frac{1.5}{g}v_P'$$

$$0.25v_0 = -0.25(v'_B)_2 + 1.5v'_p \Rightarrow v_0 = -(v'_B)_2 - 6v'_p \qquad (3)$$

Solving (2) and (3) for $(v'_B)_2$, $\qquad (v'_B)_2 = \dfrac{(6e-1)}{7}v_0$

Projectile motion

$$0.8333 = \frac{(6e-1)}{7}v_0 t \qquad (4)$$

Dividing Equation (4) by Equation (1)

$$\frac{0.8333}{2} = \frac{6e-1}{7e};\ 2.91655e = 6e - 1$$

$e = 0.324$ ◀

(*b*) From Figure (1)

Projectile motion, $\qquad h = \dfrac{1}{2}gt^2;\ 3 = \dfrac{1}{2}(32.2)t^2$

$$6 = 32.2t^2 \qquad (5)$$

From Equation (1),

$$2 = v_0 et \Rightarrow t = \frac{2}{0.324v_0} = \frac{6.1728}{v_0}$$

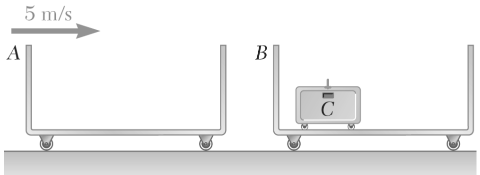
Using Equation (5)

$$6 = 32.2\left(\frac{6.1728}{v_0}\right)^2 \Rightarrow 6v_0^2 = 1226.947$$

$$v_0^2 = 204.49$$

$v_0 = 14.30$ ft/s ◀

PROBLEM 13.177

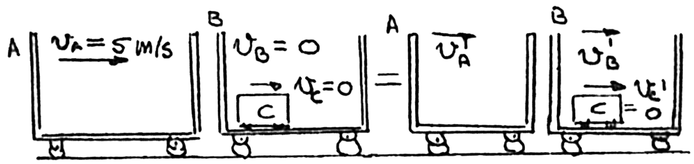


After having been pushed by an airline employee, an empty 40-kg luggage carrier A hits with a velocity of 5 m/s an identical carrier B containing a 15-kg suitcase equipped with rollers. The impact causes the suitcase to roll into the left wall of carrier B. Knowing that the coefficient of restitution between the two carriers is 0.80 and that the coefficient of restitution between the suitcase and the wall of carrier B is 0.30, determine (a) the velocity of carrier B after the suitcase hits its wall for the first time, (b) the total energy lost in that impact.

SOLUTION

(a) Impact between A and B:

Total momentum conserved:



$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B' \qquad m_A = m_B = 40 \text{ kg}$$

$$\xrightarrow{+} 5 \text{ m/s} + 0 = v_A' + v_B' \tag{1}$$

Relative velocities:

$$(v_A - v_B)e_{AB} = v_B' - v_A'$$

$$(5-0)(0.80) = v_B' - v_A' \tag{2}$$

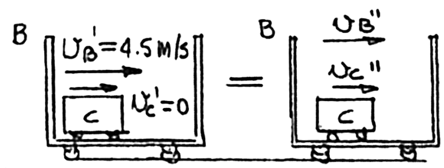
Adding Eqs. (1) and (2)

$$(5 \text{ m/s})(1+0.80) = 2v_B'$$

$$v_B' = 4.5 \text{ m/s} \longrightarrow$$

Impact between B and C (after A hits B)

Total momentum conserved:



$$\xrightarrow{+} m_B v_B' + m_C v_C' = m_B v_B'' + m_C v_C''$$

$$(40 \text{ kg})(4.5 \text{ m/s}) + 0 = (40 \text{ kg})\, v_B'' + (15 \text{ kg})\, v_C''$$

$$4.5 = v_B'' + 0.375 v_C'' \tag{3}$$

Relative velocities:

$$(v_B' - v_C')\, e_{BC} = v_C'' - v_B''$$

$$(4.5-0)(0.30) = v_C'' - v_B'' \tag{4}$$

PROBLEM 13.177 (Continued)

Adding Eqs. (4) and (3)

$$(4.5)(1+0.3) = (1.375)v_C''$$

$$v_C'' = 4.2545 \text{ m/s}$$

$$v_B'' = 4.5 - 0.375(4.2545)\, v_B'' = 2.90 \text{ m/s} \qquad v_B' = 2.90 \text{ m/s} \blacktriangleleft$$

(*b*)

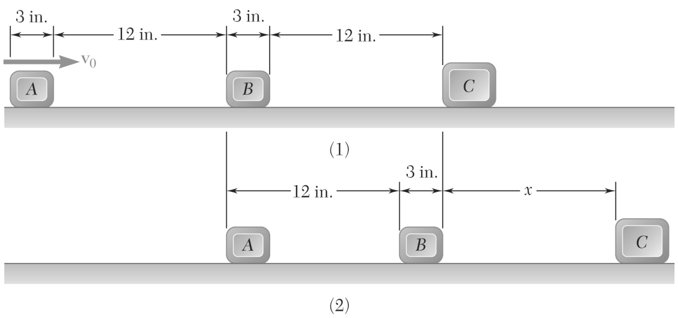
$$\Delta T_L = (T_B' + T_C') - (T_B'' + T_C'')$$

$$T_B' = \frac{1}{2} m_B (v_B')^2 = \left(\frac{40}{2}\text{ kg}\right)(4.5 \text{ m/s})^2 = 405 \text{ J}$$

$$T_C' = 0 \qquad T_B'' = \frac{1}{2} m_B (v_B'')^2 = \left(\frac{40}{2}\text{ kg}\right)(2.90)^2 = 168.72 \text{ J}$$

$$T_C'' = \frac{1}{2} m_C (v_C'')^2 = \left(\frac{15}{2}\text{ kg}\right)(4.2545 \text{ m/s})^2 = 135.76 \text{ J}$$

$$\Delta T_L = (405 + 0) - (168.72 + 135.76) = 100.5 \text{ J} \qquad \Delta T_L = 100.5 \text{ J} \blacktriangleleft$$



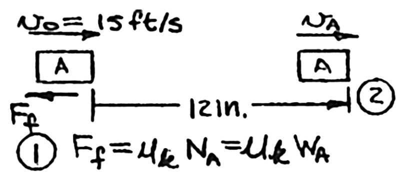
PROBLEM 13.178

Blocks A and B each weigh 0.8 lb and block C weighs 2.4 lb. The coefficient of friction between the blocks and the plane is $\mu_k = 0.30.$ Initially block A is moving at a speed $v_0 = 15\text{ ft/s}$ and blocks B and C are at rest (Fig. 1). After A strikes B and B strikes C, all three blocks come to a stop in the positions shown (Fig. 2). Determine (a) the coefficients of restitution between A and B and between B and C, (b) the displacement x of block C.

SOLUTION

(a) Work and energy

Velocity of A just before impact with B:



$$T_1 = \frac{1}{2}\frac{W_A}{g}v_0^2 \quad T_2 = \frac{1}{2}\frac{W_A}{g}\left(v_A^2\right)_2$$

$$U_{1-2} = -\mu_k W_A\,(1\text{ ft})$$

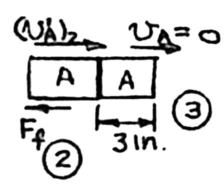
$$T_1 + U_{1-2} = T_2$$

$$\frac{1}{2}\frac{W_A}{g}v_0^2 - \mu_k W_A(1) = \frac{1}{2}\frac{W_A}{g}\left(v_A^2\right)_2$$

$$(v_A^2)_2 = v_0^2 - 2\mu_k g = (15\text{ ft/s})^2 - 2(0.3)(32.2\text{ ft/s}^2)(1\text{ ft})$$

$$(v_A^2)_2 = 205.68\text{ ft/s}^2, \quad (v_A)_2 = 14.342\text{ ft/s}$$

Velocity of A after impact with B: $(v_A')_2$



$$T_2' = \frac{1}{2}\frac{W_A}{g}(v_A')_2^2 \quad T_3 = 0$$

$$U_{2-3} = -\mu_k\, W_A(3/12)$$

$$T_2' + U_{2-3} = T_3, \quad \frac{1}{2}\frac{W_A}{g}(v_A')_2^2 - (\mu_k)(W_A/4) = 0$$

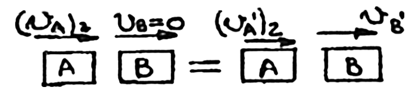
$$(v_A')_2^2 = 2(0.3)(32.2\text{ ft/s}^2)\left(\frac{1}{4}\text{ft}\right) = 4.83\text{ ft}^2/\text{s}^2$$

$$(v_A')_2 = 2.198\text{ ft/s}$$

PROBLEM 13.178 (Continued)

Conservation of momentum as A hits B:

$$(v_A)_2 = 14.342 \text{ ft/s}$$
$$(v_A')_2 = 2.198 \text{ ft/s}$$



$$\xrightarrow{+} \; m_A(v_A)_2 + m_B v_B = m_B(v_A')_2 + m_B v_B' \quad m_A = m_B$$
$$14.342 + 0 = 2.198 + v_B' \qquad v_B' = 12.144 \text{ ft/s}$$

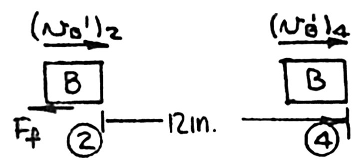
Relative velocities A and B:

$$\xrightarrow{+} \; [(v_A)_2 - v_B]e_{AB} = v_B' - (v_A')_2$$
$$(14.342 - 0)e_{AB} = 12.144 - 2.198$$

$e_{AB} = 0.694$ ◀

Work and energy.

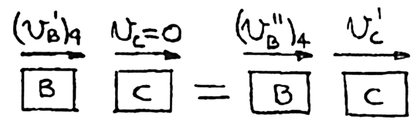
Velocity of B just before impact with C:



$$T_2 = \frac{1}{2}\frac{W_B}{g}(v_B')_2^2 = \frac{W_B}{2g}(12.144)^2$$
$$T_4 = \frac{1}{2}\frac{W_B}{g}(v_B')_4^2 = \frac{W_B}{2g}(v_B')_4^2$$
$$(v_B') = 12.144 \text{ ft/s} \qquad U_{2-4} = -\mu_k W_B(1 \text{ ft}) = (0.3)W_B$$
$$F_f = \mu_k W_B \qquad T_2 + U_{2-4} = T_4, \quad \frac{(12.144)^2}{2g} - 0.3 = \frac{(v_B')_4^2}{2g}$$
$$(v_B')_4 = 11.321 \text{ ft/s}$$

Conservation of momentum as B hits C:

$$m_B = \frac{0.8}{g}$$
$$m_C = \frac{2.4}{g}$$



$$(v_B')_4 = 11.321 \text{ ft/s}$$

$$\xrightarrow{+} \; m_B(v_B')_4 + m_C v_C = m_B(v_B'')_4 + m_C v_C'$$
$$\frac{0.8}{g}(11.321) + 0 = \frac{0.8}{g}(v_B'')_4 + \frac{(2.4)}{g}(v_C')$$
$$11.321 = (v_B'')_4 + 3v_C'$$

Velocity of B after B hits $C, (v''_B)_4 = 0.$

(Compare Figures (1) and (2).)

$$v'_C = 3.774 \text{ ft/s}$$

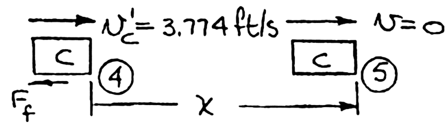
Relative velocities B and C:

$$((v'_B)_4 - v_C)e_{BC} = v'_C - (v''_B)_4$$
$$(11.321 - 0)e_{BC} = 3.774 - 0$$

$$e_{BC} = 0.333 \blacktriangleleft$$

(*b*) Work and energy, Block C:

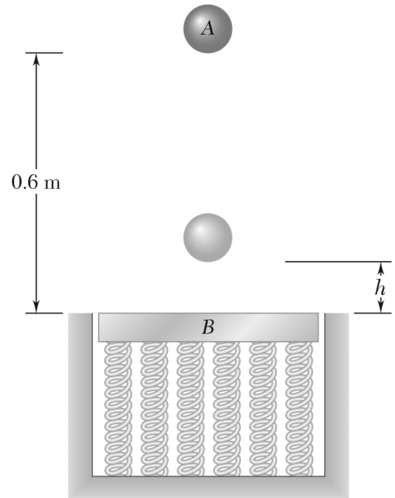
$$T_4 = \frac{1}{2}\frac{W_C}{g}(v_C)^2 \qquad T_5 = 0 \qquad U_{4-5} = -\mu_k W_C(x)$$



$$T_4 + U_{4-5} = T_5 \qquad \frac{1}{2}\frac{W_C}{g}(3.774)^2 - (0.3)W_C(x) = 0$$

$$x = \frac{(3.774)^2}{2(32.2)(0.3)} = 0.737 \text{ ft}$$

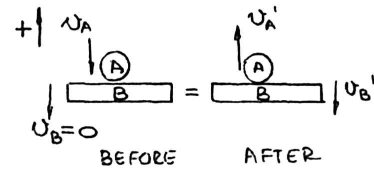
$$x = 8.84 \text{ in.} \blacktriangleleft$$



PROBLEM 13.179

A 0.5-kg sphere A is dropped from a height of 0.6 m onto a 1.0 kg plate B, which is supported by a nested set of springs and is initially at rest. Knowing that the coefficient of restitution between the sphere and the plate is $e = 0.8$, determine (*a*) the height h reached by the sphere after rebound, (*b*) the constant k of the single spring equivalent to the given set if the maximum deflection of the plate is observed to be equal to $3h$.

SOLUTION



Velocity of A and B after impact.

$$m_A = 0.5 \text{ kg}$$
$$m_B = 1.0 \text{ kg}$$

Sphere A falls. Use conservation of energy to find v_A, the speed just before impact. Use the plate surface as the datum.

$$T_1 = 0, \quad V_1 = m_A g h_0, \quad T_2 = \frac{1}{2} m_A v_A^2, \quad V_2 = 0$$

$$T_1 + V_1 = T_2 + V_2 \quad 0 + m_A g h_0 = \frac{1}{2} m_A v_A^2 + 0$$

With
$$h_0 = 0.6 \text{ m},$$
$$v_A = \sqrt{2 g h_0} = \sqrt{(2)(9.81)(0.6)}$$

$$\mathbf{v}_A = 3.4310 \text{ m/s} \downarrow$$

Analysis of the impact. Conservation of momentum.

$$m_A \mathbf{v}_A + m_B \mathbf{v}_B = m_A \mathbf{v}_A' + m_B \mathbf{v}_B' \quad \text{with} \quad \mathbf{v}_B = 0$$

Dividing by m_A and using y-components $+\uparrow$ with $(m_B/m_A = 2)$

$$-3.4310 + 0 = (v_A')_y + 2(v_B')_y \tag{1}$$

Coefficient of restitution.
$$(v_B')_y - (v_A')_y = e[(v_A)_y - (v_B)_y]$$

$$(v_B')_y - (v_A')_y = e(v_A)_y = -3.4310e \tag{2}$$

Solving Eqs. (1) and (2) simultaneously with $e = 0.8$ gives

$$(v'_A)_y = 0.68621 \text{ m/s}$$
$$(v'_B)_y = -2.0586 \text{ m/s}$$
$$\mathbf{v}'_A = 0.68621 \text{ m/s} \uparrow$$
$$\mathbf{v}'_B = 2.0586 \text{ m/s} \downarrow$$

(*a*) Sphere *A* rises. Use conservation of energy to find *h*.

$$T_1 = \frac{1}{2} m_A (v'_A)^2, \quad V_1 = 0, \quad T_2 = 0, \quad V_2 = m_A g h$$

$$T_1 + V_1 = T_2 + V_2: \qquad \frac{1}{2} m_A (v'_A)^2 + 0 = 0 + m_A g h$$

$$h = \frac{(v'_A)^2}{2g} = \frac{(0.68621)^2}{(2)(9.81)} \qquad h = 0.0240 \text{ m} \blacktriangleleft$$

(*b*) Plate *B* falls and compresses the spring. Use conservation of energy.

Let δ_0 be the initial compression of the spring and Δ be the additional compression of the spring after impact. In the initial equilibrium state,

$$+\uparrow \Sigma F_y = 0: \quad k\delta_0 - W_B = 0 \quad \text{or} \quad k\delta_0 = W_B \tag{3}$$

Just after impact: $T_1 = \frac{1}{2} m_B (v'_B)^2, \quad V_1 = \frac{1}{2} k \delta_0^2$

At maximum deflection of the plate, $T_2 = 0$

$$V_2 = (V_2)_g + (V_2)_e = -W_B \Delta + \frac{1}{2} k (\delta_0 + \Delta)^2$$

Conservation of energy: $T_1 + V_1 = T_2 + V_2$

$$\frac{1}{2} m_B (v'_B)^2 + \frac{1}{2} k \delta_0^2 = 0 - W_B \Delta + \frac{1}{2} k \delta_0^2 + k \delta_0 \Delta + \frac{1}{2} k \Delta^2$$

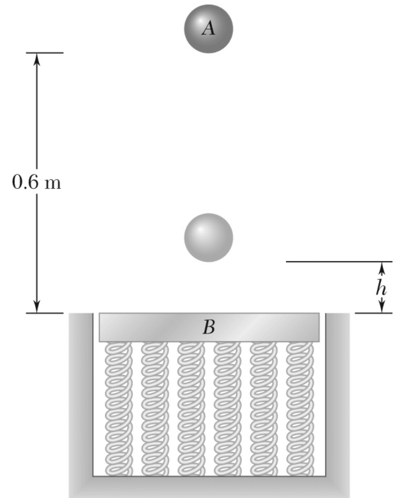
Invoking the result of Eq. (3) gives

$$\frac{1}{2} m_B (v'_B)^2 = \frac{1}{2} k \Delta^2 \tag{4}$$

Data: $m_B = 1.0 \text{ kg}, \quad v'_B = 2.0586 \text{ m/s}$

$$\Delta = 3h = (3)(0.024) = 0.072 \text{ m}$$

$$k = \frac{m_B (v'_B)^2}{\Delta^2} = \frac{(1.0)(2.0586)^2}{(0.072)^2} \qquad k = 817 \text{ N/m} \blacktriangleleft$$



PROBLEM 13.180

A 0.5-kg sphere A is dropped from a height of 0.6 m onto a 1.0-kg plate B, which is supported by a nested set of springs and is initially at rest. Knowing that the set of springs is equivalent to a single spring of constant $k = 900$ N/m, determine (*a*) the value of the coefficient of restitution between the sphere and the plate for which the height h reached by the sphere after rebound is maximum, (*b*) the corresponding value of h, (*c*) the corresponding value of the maximum deflection of the plate.

SOLUTION

$$m_A = 0.5 \text{ kg}$$
$$m_B = 1.0 \text{ kg}$$
$$k = 900 \text{ N/m}$$

<u>Sphere A falls</u>. Use conservation of energy to find v_A, the speed just before impact. Use the plate surface as the datum.

$$T_1 = 0 \qquad V_1 = m_A g h_0$$
$$T_2 = \frac{1}{2} m_A v_A^2, \qquad V_2 = 0$$

With $h_0 = 0.6$ m, $\quad v_A = \sqrt{2gh_0} = \sqrt{(2)(9.81)(0.6)}$

$$\mathbf{v}_A = 3.4310 \text{ m/s} \downarrow$$

<u>Analysis of impact</u>. Conservation of momentum.

$$m_A \mathbf{v}_A + m_B \mathbf{v}_B = m_A \mathbf{v}'_A + m_B v'_B \text{ with } \mathbf{v}_B = 0$$

Dividing by m_A and using y components $+\uparrow$ with $(m_B/m_A = 2)$

$$-3.4310 = (v'_A)_y + 2(v'_B)_y \tag{1}$$

<u>Coefficient of restitution</u>. $\quad (v'_B)_y - (v'_A)_y = e[(v_A)_y - (v_B)_y]$

$$(v'_B)_y - (v'_A)_y = e(v_A)_y = -3.4310e$$
$$(v'_B)_y = -3.4310 + (v'_A)_y \tag{2}$$

PROBLEM 13.180 (Continued)

Substituting into Eq. (1),

$$-3.4310 = (v'_A)_y + (2)[-3.4310e + (v'_A)_y]$$

$$(v'_A)_y = 1.1437\,(2e-1) \qquad (3)$$

From Eq. (2), $$(v'_B)_y = -1.1437(1+e) \qquad (4)$$

(*a*) Sphere *A* rises. Use conservation of energy to find *h*.

$$T_1 = \frac{1}{2}m_A(v'_A)^2, \qquad V_1 = 0$$

$$T_2 = 0, \qquad V_2 = m_A gh$$

$$T_1 + V_1 = T_2 + V_2: \quad \frac{1}{2}m_A(v'_A)^2 + 0 = 0 + m_A gh$$

$$h = \frac{(v'_A)^2}{2g} = \frac{(1.1437)^2(2e-1)^2}{(2)(9.81)}$$

Since h is to be maximum, e must be as large as possible.

Coefficient of restitution for maximum h: $e = 1.000$ ◀

(*b*) Corresponding value of *h*. $(v'_A) = 1.1437[(2)(1)-1] = 1.1437$ m/s

$$h = \frac{(v'_A)^2}{2g} = \frac{(1.1437)^2}{(2)(9.81)}$$

$h = 0.0667$ m ◀

(*c*) Plate *B* falls and compresses the spring. Use conservation of energy.

Let δ_0 be the initial compression of the spring and Δ be the additional compression of the spring after impact. In the initial equilibrium state,

$$+\!\uparrow \Sigma F_y = 0 \quad k\delta_0 - W_B = 0 \quad \text{or} \quad k\delta_0 = W_B \qquad (3)$$

Just after impact: $$T_1 = \frac{1}{2}m_B(v'_B)^2, \qquad V_1 = \frac{1}{2}k\delta_0^2$$

At maximum deflection of the plate, $T_2 = 0$

$$V_2 = (V_2)_g + (V_2)_e = -W_B\Delta + \frac{1}{2}k(\delta_0 + \Delta)^2$$

Conservation of energy: $T_1 + V_1 = T_2 + V_2$

$$\frac{1}{2}m_B(v'_B)^2 + \frac{1}{2}k\delta_0^2 = 0 - W_B\Delta + \frac{1}{2}k\delta_0^2 + k\delta_0\Delta + \frac{1}{2}k\Delta^2$$

PROBLEM 13.180 (Continued)

Invoking the result of Eq. (3) gives

$$\frac{1}{2}m_B(v'_B)^2 = \frac{1}{2}k\Delta^2$$

Data: $m_B = 1.0 \text{ kg},$

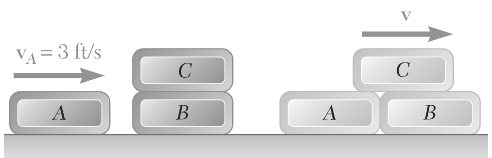
$$(v'_B)_y = -1.1437(1+1) = -2.2874 \text{ m/s}.$$

$$\mathbf{v}'_B = 2.2874 \text{ m/s}\downarrow, \quad k = 900 \text{ N/m}$$

$$\Delta^2 = \frac{m_B(v'_B)^2}{k} = \frac{(1.0)(2.2874)^2}{900} = 0.0058133 \text{ m}^2$$

$\Delta = 0.0762 \text{ m}$ ◀

PROBLEM 13.181

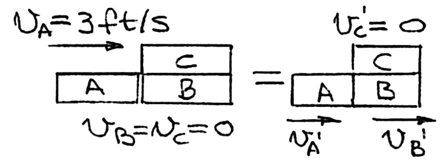


The three blocks shown are identical. Blocks B and C are at rest when block B is hit by block A, which is moving with a velocity $\mathbf{v}_A$ of 3 ft/s. After the impact, which is assumed to be perfectly plastic $(e = 0)$, the velocity of blocks A and B decreases due to friction, while block C picks up speed, until all three blocks are moving with the same velocity $\mathbf{v}$. Knowing that the coefficient of kinetic friction between all surfaces is $\mu_k = 0.20$, determine (a) the time required for the three blocks to reach the same velocity, (b) the total distance traveled by each block during that time.

SOLUTION

(a) Impact between A and B, conservation of momentum

$$mv_A + mv_B + mv_C = mv_A' + mv_B' + mv_C'$$



$$3 + 0 = v_A' + v_B' + 0$$

Relative velocities $(e = 0)$

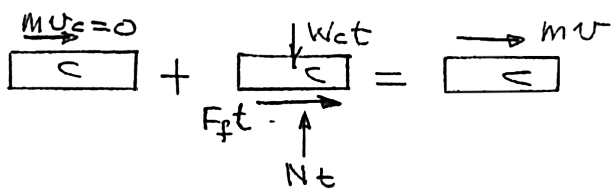
$$(v_A - v_B)e = v_B' - v_A' \qquad 3 = 2v_B'$$

$$0 = v_B' - v_A' \qquad v_B' = 1.5 \text{ ft/s}$$

$$v_A' = v_B'$$

$v =$ Final (common) velocity

Block C: Impulse and momentum

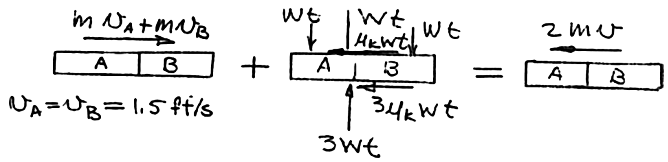


$$\xrightarrow{+} W_C v_C + F_f t = \frac{W_C}{g} v \qquad F_f = \mu_k W_C$$

$$0 + (0.2)t = \frac{v}{g} \qquad v = (0.2)gt \qquad (1)$$

PROBLEM 13.181 (Continued)

Blocks A and B: Impulse and momentum



$$2\frac{W}{g}(1.5) - 4(0.2)Wt = 2\frac{W}{g}v$$

$$1.5 - 0.4gt = v \tag{2}$$

Substitute v from Eq. (1) into Eq. (2)

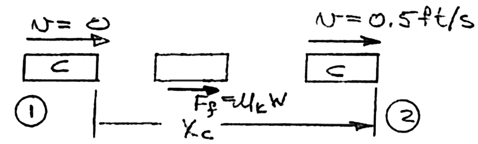
$$1.5 - 0.4gt = 0.2gt$$

$$t = \frac{(1.5 \text{ ft/s})}{0.6(32.2 \text{ ft/s}^2)} \qquad t = 0.0776 \text{ s} \blacktriangleleft$$

(*b*) <u>Work and energy</u>:

From Eq. (1) $\quad v = (0.2)(32.2)(0.0776) = 0.5$ ft/s

Block C:



$$T_1 = 0 \qquad T_2 = \frac{1}{2}\frac{W}{g}(v)^2 = \frac{W}{2g}(0.5)^2$$

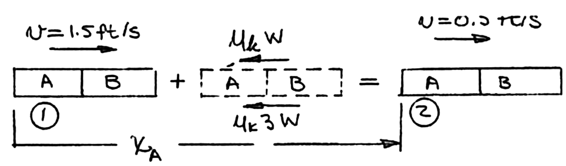
$$U_{1-2} = F_f x_C = \mu_k W x_C = 0.2W x_C$$

$$T_1 + U_{1-2} = T_2 \qquad 0 + (0.2)(W)x_C = \frac{1}{2}\frac{W}{g}v^2$$

$$x_C = \frac{(0.5 \text{ ft/s})^2}{0.2(2)(32.2 \text{ ft/s}^2)} = 0.01941 \text{ ft}$$

$$x_C = 0.01941 \text{ ft} \blacktriangleleft$$

Blocks A and B:



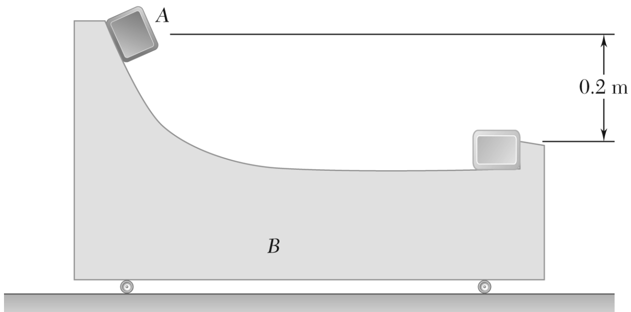
$$T_1 = \frac{1}{2}\left(2\frac{W}{g}\right)(1.5)^2 = 2.25W \qquad T_2 = \frac{1}{2}\left(2\frac{W}{g}\right)(0.5)^2 = 0.25W$$

$$U_{1-2} = -4\mu_k Wg x_A = -0.8Wg x_A$$

$$T_1 + U_{1-2} = T_2$$

$$2.25W - 4(0.2)W(32.2)x_A = 0.25W$$

$$x_A = 0.07764 \text{ ft} \qquad x_A = 0.0776 \text{ ft} \blacktriangleleft$$



PROBLEM 13.182

Block A is released from rest and slides down the frictionless surface of B until it hits a bumper on the right end of B. Block A has a mass of 10 kg and object B has a mass of 30 kg and B can roll freely on the ground. Determine the velocities of A and B immediately after impact when (a) $e = 0$, (b) $e = 0.7$.

SOLUTION

Let the x-direction be positive to the right and the y-direction vertically upward.

Let $(v_A)_x, (v_A)_y, (v_A)_x$ and $(v_B)_y$ be velocity components just before impact and $(v'_A)_x, (v'_A)'_y, (v'_B)_x,$ and $(v'_B)_y$ those just after impact. By inspection,

$$(v_A)_y = (v_B)_y = (v'_A)_y = (v'_B)_y = 0$$

Conservation of momentum for x-direction:

While block is sliding down: $\quad 0 + 0 = m_A(v_A)_x + m_B(v_B)_x \quad (v_B)_x = -\beta(v_A)_x \quad (1)$

Impact: $\quad 0 + 0 = m_A(v'_A)_x + m_B(v'_B)_x \quad (v'_B)_x = -\beta(v'_A)_x \quad (2)$

where $\quad \beta = m_A/m_B$

Conservation of energy during frictionless sliding:

Initial potential energies: $m_A gh$ for A, $\quad$ 0 for B.

Potential energy just before impact: $\quad V_1 = 0$

Initial kinetic energy: $\quad T_0 = 0$ (rest)

Kinetic energy just before impact: $\quad T_1 = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2$

$$T_0 + V_0 = T_1 + V_1$$

$$m_A gh = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 = \frac{1}{2}(m_A + m_B\beta^2)v_A^2$$

$$= \frac{1}{2}m_A(1+\beta)v_A^2$$

$$v_A^2 = (v_A)_x^2 = \frac{2gh}{1+\beta} \qquad v_A = \sqrt{\frac{2gh}{1+\beta}} \qquad (3)$$

PROBLEM 13.182 (Continued)

Velocities just before impact:

$$\mathbf{v}_A = \sqrt{\frac{2gh}{1+\beta}} \longrightarrow$$

$$\mathbf{v}_B = \beta\sqrt{\frac{2gh}{1+\beta}} \longleftarrow$$

Analysis of impact. Use Eq. (2) together with coefficient of restitution.

$$(v'_B)_x - (v'_A)_x = e[(v_A)_x - (v_B)_x]$$

$$-\beta(v'_A)_x - (v'_A)_x = e[(v_A)_x + \beta(v_A)_x]$$

$$(v'_A)_x = -e(v_A)_x \qquad (4)$$

Data:

$$m_A = 10 \text{ kg}$$

$$m_B = 30 \text{ kg}$$

$$h = 0.2 \text{ m}$$

$$g = 9.81 \text{ m/s}^2$$

$$\beta = \frac{10\,\text{kg}}{30\,\text{kg}} = 0.33333$$

From Eq. (3),

$$v_A = \sqrt{\frac{(2)(9.81)(0.2)}{1.33333}}$$

$$= 1.71552 \text{ m/s}$$

(*a*) $e = 0$: $(v'_A)_x = 0 \qquad (v'_B)_x = 0$

$$\mathbf{v}'_A = 0 \blacktriangleleft$$

$$\mathbf{v}'_B = 0 \blacktriangleleft$$

(*b*) $e = 0.7$:

$$(v'_A)_x = -(0.7)(1.71552)$$

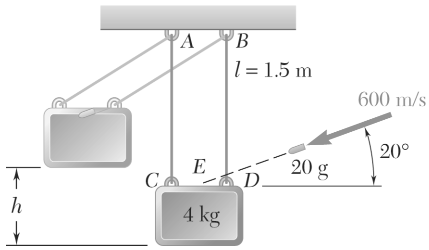
$$= -1.20086 \text{ m/s}$$

$$(v'_B)_x = -(0.33333)(1.20086)$$

$$= 0.40029 \text{ m/s}$$

$$\mathbf{v}'_A = 1.201 \text{ m/s} \longleftarrow \blacktriangleleft$$

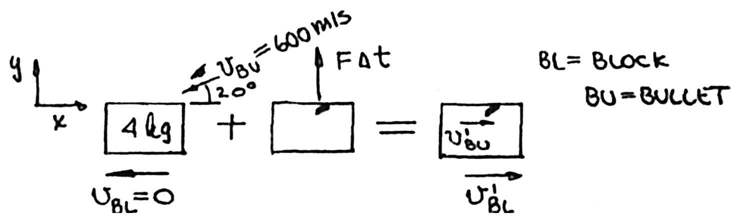
$$\mathbf{v}'_B = 0.400 \text{ m/s} \longrightarrow \blacktriangleleft$$



PROBLEM 13.183

A 20-g bullet fired into a 4-kg wooden block suspended from cords *AC* and *BD* penetrates the block at Point *E*, halfway between *C* and *D*, without hitting cord *BD*. Determine (*a*) the maximum height *h* to which the block and the embedded bullet will swing after impact, (*b*) the total impulse exerted on the block by the two cords during the impact.

SOLUTION



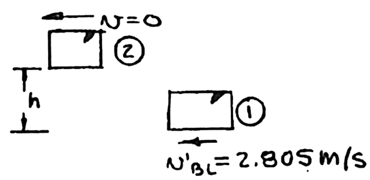
Total momentum in *x* is conserved:

$$\xrightarrow{+}\ m_{\text{bl}}v_{\text{bl}} + m_{\text{bu}}v_{\text{bu}}\cos 20° = m_{\text{bl}}v'_{\text{bl}} + m_{\text{bu}}v'_{\text{bu}} \qquad (v'_{\text{bl}} = v'_{\text{bu}})$$

$$0 + (0.02\text{ kg})(-600\text{ m/s})(\cos 20) = (4.02\text{ kg})(v'_{\text{bl}})$$

$$v'_{\text{bl}} = -2.805\text{ m/s}$$

Conservation of energy:



$$T_1 = \frac{1}{2}(m_{\text{bl}} + m_{\text{bu}})(v'_{\text{bl}})^2$$

$$T_1 = \left(\frac{4.02\text{ kg}}{2}\right)(2.805\text{ m/s})^2$$

$$T_1 = 15.815\text{ J}$$

$$V_1 = 0$$

$$T_2 = 0 \qquad V_2 = (m_{\text{bl}} + m_{\text{bu}})gh$$

$$V_2 = (4.02\text{ kg})(9.81\text{ m/s}^2)(h) = 39.44h$$

$$T_1 + V_1 = T_2 + V_2$$

$$15.815 + 0 = 0 + 39.44h$$

$$h = 0.401\text{ m}$$

$h = 401$ mm ◀

(*b*) Refer to figure in part (*a*).

Impulse-momentum in *y*-direction:

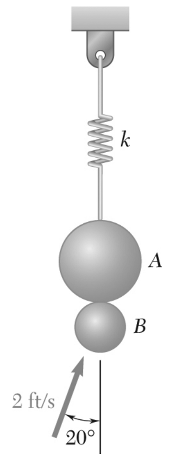
$$+\!\uparrow m_{\text{bu}}v_{\text{bu}}\sin 20 + F\Delta t = (m_{\text{bl}} + m_{\text{bu}})(v'_{\text{bl}})_y$$

$$(v_{\text{bl}})_y = 0$$

$$(0.02\text{ kg})(-600\text{ m/s})(\sin 20°) + F\Delta t = 0$$

$F\Delta t = 4.10\text{ N}\cdot\text{s}$ ◀

PROBLEM 13.184



A 2-lb ball A is suspended from a spring of constant 10 lb/in and is initially at rest when it is struck by 1-lb ball B as shown. Neglecting friction and knowing the coefficient of restitution between the balls is 0.6, determine (*a*) the velocities of A and B after the impact, (*b*) the maximum height reached by A.

SOLUTION

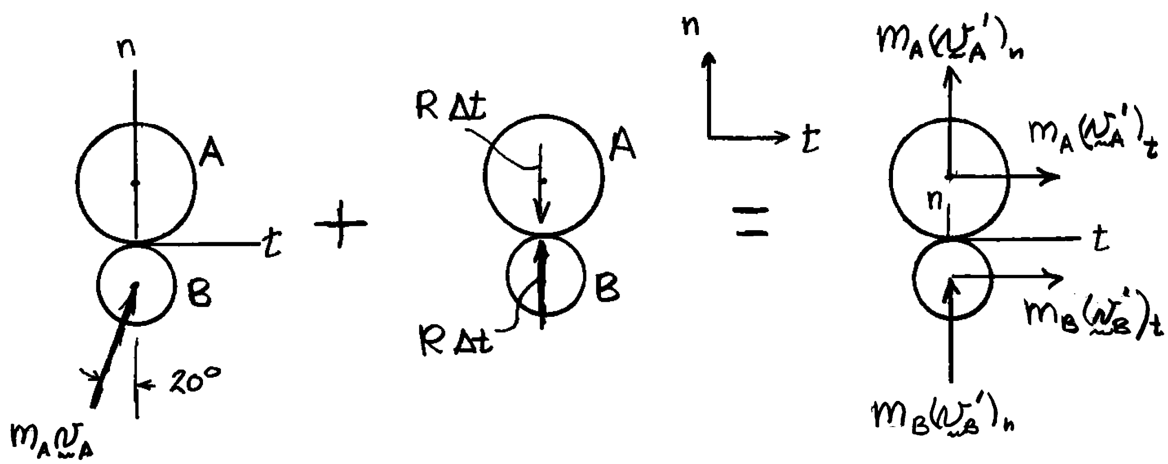
Masses: $m_A = \dfrac{2 \text{ lb}}{32.2 \text{ ft/s}^2} = 0.062112 \text{ lb}\cdot\text{s}^2/\text{ft} \qquad m_B = \dfrac{1 \text{ lb}}{32.2 \text{ ft/s}^2} = 0.031056 \text{ lb}\cdot\text{s}^2/\text{ft}$

Other data: $k = (10 \text{ lb/in.})(12 \text{ in./ft.}) = 120 \text{ lb/ft}, \quad e = 0.6$

$v_A = 0, \quad v_B = 2 \text{ ft/s}$

For analysis of the impact use the principle of impulse and momentum.

$$\Sigma m\mathbf{v}_1 + \Sigma \mathbf{Imp}_{1\to 2} = \Sigma m\mathbf{v}_2$$



t-direction for ball A:

$$0 + 0 = m_A(v'_A)_t \qquad (v'_A)_t = 0$$

t-direction for ball B: $\quad m_B v_B \sin 20° + 0 = m_B (v'_B)_t$

$$(v'_B)_t = v_B \sin 20° = (2)(\sin 20°) = 0.6840 \text{ ft/s}$$

n-direction for balls A and B:

$$m_B v_B \cos 20° + 0 = m_B (v'_B)_n + m_A (v'_A)_n$$

$$(v'_B)_n + \frac{m_A}{m_B}(v'_A)_n = v_B \cos 20°$$

$$(v'_B)_n + 2(v'_A)_n = (2)\cos 20° \qquad (1)$$

Coefficient of restitution:

$$(v'_B)_n - (v'_A)_n = e[(v_A)_n - (v_B)_n]$$
$$= e[0 - (v_B \cos 20°)]$$
$$= -(0.6)(2)\cos 20° \qquad (2)$$

Solving Eqs. (1) and (2) simultaneously,

$$(v'_A)_n = 1.00234 \text{ ft/s} \qquad (v'_B)_n = -0.12529 \text{ ft/s}$$

(a) Velocities after the impact:

$$\mathbf{v}'_A = 1.00234 \text{ ft/s}\uparrow$$

$\mathbf{v}'_A = 1.002 \text{ ft/s}\uparrow$ ◀

$$\mathbf{v}'_B = (0.6840 \text{ ft/s} \rightarrow) + (0.12529 \text{ ft/s}\downarrow)$$

$$v_B = \sqrt{(0.6840)^2 + (0.12529)^2} = 0.695 \text{ ft/s}$$

$$\tan\beta = \frac{0.12529}{0.6840} \qquad \beta = 10.4°$$

$\mathbf{v}'_B = 0.695 \text{ ft/s} \searrow 10.4°$ ◀

(b) Maximum height reached by A:

Use conservation of energy for ball A after the impact.

Position 1: Just after impact.

$$T_1 = \frac{1}{2}m_A(v'_A)^2 = \frac{1}{2}(0.062112)(1.00234)^2 = 0.0312021 \text{ ft}\cdot\text{lb}$$

Force in spring = weight of A

$$x_1 = -\frac{F}{k} = -\frac{W_A}{k} = -\frac{2 \text{ lb}}{120 \text{ lb/ft}} = -0.016667 \text{ ft}$$

$$(V_1)_e = \frac{1}{2}kx_1^2 = \frac{1}{2}k\left(\frac{W_B}{k}\right)^2 = \frac{W_A^2}{2k}$$

$$= \frac{(2 \text{ lb})^2}{(2)(120)} = 0.016667 \text{ ft}\cdot\text{lb}$$

$$(V_1)_g = 0 \qquad \text{(datum)}$$

PROBLEM 13.184 (Continued)

Position 2: Maximum height h.

$$V_2 = 0$$

$$T_2 = 0$$

$$(V_2)_e = \frac{1}{2}k(h + x_1)^2 = \frac{1}{2}(120)(h - 0.016667)^2$$

$$= 60h^2 - 2h + 0.016667$$

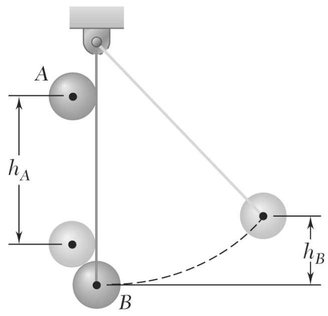
$$(V_2)_g = W_A h = (2 \text{ lb})h = 2h$$

Conservation of energy: $T_1 + V_1 = T_2 + V_2$

$$0.031202 + 0.016667 = 0 + 60h^2 - 2h + 0.016667 + 2h$$

$60h^2 = 0.031202 \qquad h = \pm 0.022804 \text{ ft}$

Using the positive root, $h = 0.274 \text{ in.}$ ◀



PROBLEM 13.185

Ball B is hanging from an inextensible cord. An identical ball A is released from rest when it is just touching the cord and drops through the vertical distance $h_A = 8$ in. before striking ball B. Assuming perfectly elastic impact $(e = 0.9)$ and no friction, determine the resulting maximum vertical displacement h_B of ball B.

SOLUTION

Ball A falls

$$T_1 = 0 \quad V_2 = 0$$

$$T_1 + V_1 = T_2 + V_2 \qquad \text{(Put datum at 2)}$$

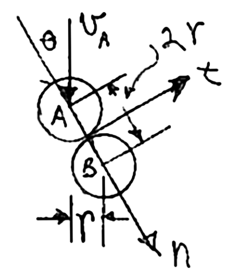
$$h = 8 \text{ in.} = 0.66667 \text{ ft}$$

$$mgh = \frac{1}{2} m v_A^2$$

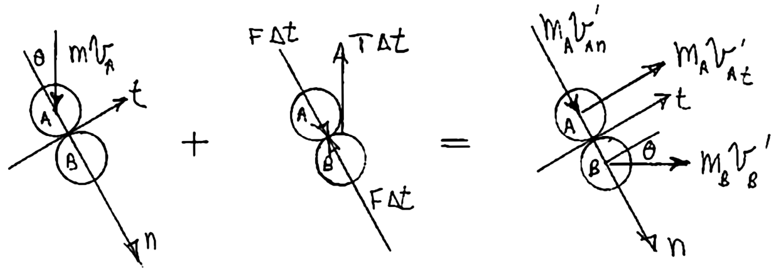
$$v_A = \sqrt{2gh} = \sqrt{(2)(32.2)(0.66667)} = 6.5524 \text{ ft/s}$$

Impact

$$\theta = \sin^{-1} \frac{r}{2r} = 30°$$



Impulse-Momentum



Unknowns: $\quad v'_B, v'_{At}, v'_{An}$

x-dir $\longrightarrow$

$$0 + 0 = m_B v'_B + m_A v'_{An} \sin 30° + m_A v'_{At} \cos 30° \qquad (1)$$

Noting that $m_A = m_B$ and dividing by m_A

$$v'_B + v'_{An} \sin 30° + v'_{At} \cos 30° = 0 \qquad (1)$$

Ball A alone:

<u>Momentum in t-direction</u>:

$$-m_A v_A \sin 30° + 0 = m_A v_{At}$$

$$v'_{At} = -v_A \sin 30° = -6.5524 \sin 30° = -3.2762 \text{ ft/s} \qquad (2)$$

<u>Coefficient of restitution</u>:

$$v'_{Bn} - v'_{An} = e(v_{An} - v_{en})$$

$$v'_B \sin 30° - v'_{An} = 0.9(v_A \cos 30° - 0) \qquad (3)$$

With known value for v_{At}, Eqs. (1) and (3) become

$$v'_B + v'_{An} \sin 30° = 3.2762 \cos 30°$$
$$v'_B \sin 30° - v'_{An} = (0.9)(6.5524) \cos 30°$$

Solving the two equations simultaneously,

$$v'_B = 4.31265 \text{ ft/s}$$
$$v'_{An} = -2.9508 \text{ ft/s}$$

After the impact, ball B swings upward. Using B as a free body

$$T' + V' = T_B + V_B$$

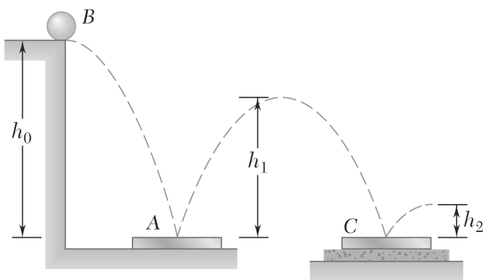
where $$T' = \frac{1}{2} m_B (v'_B)^2,$$
$$V' = 0,$$
$$T_B = 0$$

and $$V_B = m_B g h_B$$

$$\frac{1}{2} m_B (v'_B)^2 = m_B g h_B$$

$$h_B = \frac{1}{2} \frac{(v'_B)^2}{g}$$
$$= \frac{1}{2} \frac{(4.31265)^2}{32.2}$$
$$= 0.2888 \text{ ft}$$

$h_B = 3.47$ in. ◀

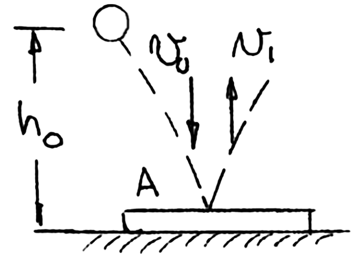


PROBLEM 13.186

A 70 g ball B dropped from a height $h_0 = 1.5$ m reaches a height $h_2 = 0.25$ m after bouncing twice from identical 210-g plates. Plate A rests directly on hard ground, while plate C rests on a foam-rubber mat. Determine (a) the coefficient of restitution between the ball and the plates, (b) the height h_1 of the ball's first bounce.

SOLUTION

(a) Plate on hard ground (first rebound):



Conservation of energy:

$$\cancel{\frac{1}{2}m_B v_y^2} + \frac{1}{2}m_B v_0^2 = m_B g h_0 + \cancel{\frac{1}{2}m_B v_x^2}$$

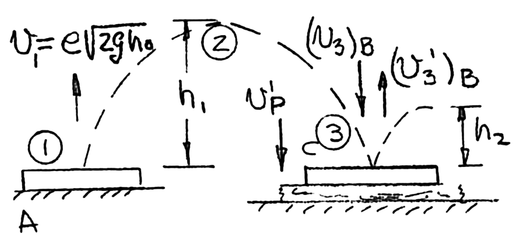
$$v_0 = \sqrt{2gh_0}$$

Relative velocities., n-direction:

$$v_0 e = v_1 \qquad v_1 = e\sqrt{2gh_0}$$

t-direction $\qquad v'_{Bx} = v_{Bx}$

Plate on foam rubber support at C.



Conservation of energy:

Points ① and ③: $\qquad V_1 = V_3 = 0$

$$\cancel{\frac{1}{2}m_B(v'_{Bx})^2} \times \frac{1}{2}m_B v_1^2 = \frac{1}{2}m_B(v_3)_B^2 + \cancel{\frac{1}{2}m_B(v'_{Bx})^2}$$

$$(v_3)_B = e\sqrt{2gh_0}$$

Conservation of momentum:

$\uparrow$+ At ③: $\qquad m_B(-v_3)_B + m_P v_P = m_B(v_3')_B - m_P v_P'$

$$\frac{m_P}{m_B} = \frac{210}{70} = 3 \qquad -e\sqrt{2gh_0} = (v_3')_B - 3v_P' \qquad (1)$$

Relative velocities:

$$[(-v_3)_B - (v_P)]e = -v'_P - (v'_3)_B$$

$$e^2\sqrt{2gh_0} + 0 = v'_P + (v'_3)_B \qquad (2)$$

Multiplying (2) by 3 and adding to (1)

$$4(v'_3)_B = \sqrt{2gh_0}(3e^2 - e)$$

Conservation of energy at ③, $\quad (v'_3)_B = \sqrt{2gh_2}$

Thus, $\quad 4\sqrt{2gh_2} = \sqrt{2gh_0}(3e^2 - e)$

$$3e^2 - e = 4\sqrt{\frac{h_2}{h_0}} = 4\sqrt{\frac{0.25}{1.5}} = 1.63299$$

$$3e^2 - e - 1.633 = 0 \qquad e = 0.923 \blacktriangleleft$$

(*b*) Points ① and ②:

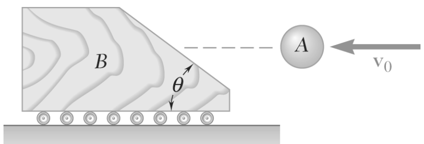
Conservation of energy.

$$\cancel{\frac{1}{2}m_B(v'_{Bx})^2} + \frac{1}{2}m_B v_1^2 = \cancel{\frac{1}{2}m_B(v'_{Bx})^2}; \quad \frac{1}{2}e^2(2gh_0) = gh_1$$

$$h_1 = e^2 h_0 = (0.923)^2(1.5)$$

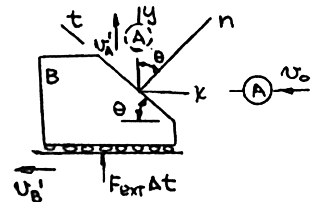
$$h_1 = 1.278 \text{ m} \blacktriangleleft$$

PROBLEM 13.187



A 700-g sphere A moving with a velocity $\mathbf{v}_0$ parallel to the ground strikes the inclined face of a 2.1-kg wedge B which can roll freely on the ground and is initially at rest. After impact the sphere is observed from the ground to be moving straight up. Knowing that the coefficient of restitution between the sphere and the wedge is $e = 0.6$, determine (a) the angle θ that the inclined face of the wedge makes with the horizontal, (b) the energy lost due to the impact.

SOLUTION



(a) <u>Momentum of sphere A alone is conserved in the t-direction</u>:

$$m_A v_0 \cos\theta = m_A v_A' \sin\theta$$

$$v_0 = v_A' \tan\theta \qquad (1)$$

<u>Total momentum is conserved in the x-direction</u>:

$$m_B v_B + m_A v_0 = m_B v_B' + (v_A')_x$$

$$v_B = 0, \quad (v_A')_x = 0$$

$$0 + 0.700\, v_0 = 2.1 v_B' + 0$$

$$v_B' = v_0/3 \qquad (2)$$

<u>Relative velocities in the n-direction</u>:

$$(-v_0 \sin\theta - 0)e = -v_B' \sin\theta - v_A' \cos\theta$$

$$(v_0)(0.6) = v_B' + v_A' \cot\theta \qquad (3)$$

Substituting v_B' from Eq. (2) into Eq. (3)

$$0.6 v_0 = 0.333\, v_0 + v_A' \cot\theta$$

$$0.267 v_0 = v_A' \cot\theta \qquad (4)$$

Divide (4) into (1)

$$\frac{1}{0.267} = \frac{\tan\theta}{\cot\theta} = \tan^2\theta$$

$$\tan\theta = 1.935 \qquad \theta = 62.7° \blacktriangleleft$$

(b) From (1)

$$v_0 = v_A' \tan\theta = v_A'(1.935)$$

$$v_A' = 0.5168 v_0, \quad v_B' = v_0/3 \qquad (2)$$

$$T_{\text{lost}} = \frac{1}{2} m_A v_A^2 - \frac{1}{2}\left(m_A (v_A')^2 + m_B v_B^2\right)$$

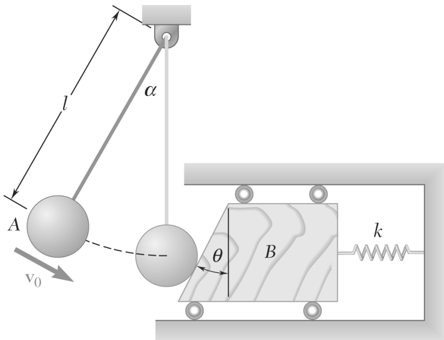
PROBLEM 13.187 (Continued)

$$T_{\text{lost}} = \frac{1}{2}(0.7)(v_0)^2 - \frac{1}{2}[(0.7)(0.5168v_0)^2 + (2.1)(v_0/3)^2]$$

$$T_{\text{lost}} = \frac{1}{2}[0.7 - 0.1870 - 0.2333]v_0^2$$

$$T_{\text{lost}} = 0.1400v_0^2 \text{ J}$$

$$T_{\text{lost}} = 0.1400v_0^2 \blacktriangleleft$$



PROBLEM 13.188

When the rope is at an angle of $\alpha = 30°$ the 1-lb sphere A has a speed $v_0 = 4$ ft/s. The coefficient of restitution between A and the 2-lb wedge B is 0.7 and the length of rope $l = 2.6$ ft. The spring constant has a value of 2 lb/in. and $\theta = 20°$. Determine (*a*) the velocities of A and B immediately after the impact, (*b*) the maximum deflection of the spring assuming A does not strike B again before this point.

SOLUTION

Masses: $m_A = (1/32.2)\ \text{lb}\cdot\text{s}^2/\text{ft} \qquad m_B = (2/32.2)\ \text{lb}\cdot\text{s}^2/\text{ft}$

<u>Analysis of sphere *A* as it swings down</u>:

Initial state: $\alpha = 30°, \quad h_0 = l(1-\cos\alpha) = (2.6)(1-\cos 30°) = 0.34833$ ft

$$V_0 = m_A g h_0 = (1)(0.34833) = 0.34833\ \text{lb}\cdot\text{ft}$$

$$T_0 = \frac{1}{2}m_A v_0^2 = \frac{1}{2}\left(\frac{1.0}{32.2}\right)(4)^2 = 0.24845\ \text{lb}\cdot\text{ft}$$

Just before impact: $\alpha = 0, \quad h_1 = 0, \quad V_1 = 0$

$$T_1 = \frac{1}{2}m_A v_A^2 = \frac{1}{2}\left(\frac{1.0}{32.2}\right)v_A^2 = \left(\frac{1.0}{64.4}\right)v_A^2$$

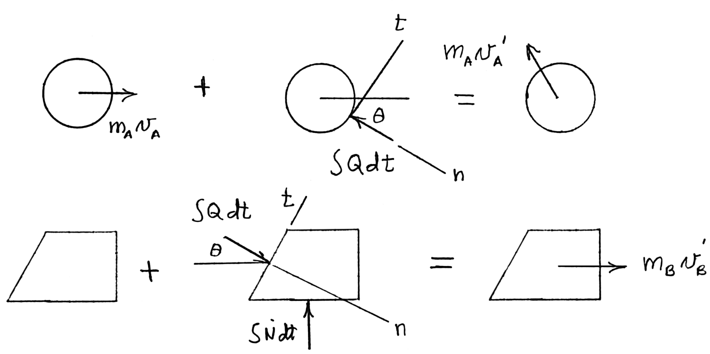
Conservation of energy: $T_0 + V_0 = T_1 + V_1$

$$0.24845 + 0.34833 = \frac{1}{64.4}v_A^2 + 0$$

$$v_A^2 = 38.433\ \text{ft}^2/\text{s}^2$$

$$\mathbf{v}_A = 6.1994\ \text{ft/s} \longrightarrow$$

<u>Analysis of the impact</u>. Use conservation of momentum together with the coefficient of restitution. $e = 0.7$.



Note that the rope does not apply an impulse since it becomes slack.

Sphere A: Momentum in t-direction:

$$m_A v_A \sin\theta + 0 = m_A (v'_A)_t$$
$$(v'_A)_t = v_A \sin\theta = 6.1994 \sin 20° = 2.1203 \text{ m/s}$$
$$(\mathbf{v}_A)_t = 2.1203 \text{ m/s} \angle 70°$$

Both A and B: Momentum in x-direction:

$$m_A v_A + 0 = m_A (v'_A)_n \cos\theta + m_A (v'_A)_t \sin\theta + m_B v'_B$$
$$(1/32.2)(6.1994) = (1/32.2)(v_A)_n \cos 20° + (1/32.2)(2.120323) \sin 20° + (2/32.2)v'_B$$
$$(1/32.2)(v'_A)_n \cos 20° + (2/32.2)v'_B = 0.17001 \qquad (1)$$

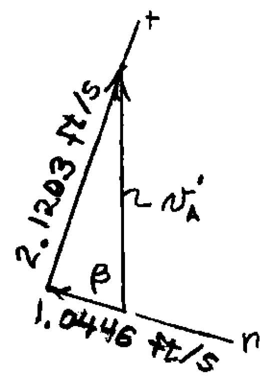
Coefficient of restitution:

$$(v'_B)_n - (v'_A)_n = e[(v_A)_n - (v_B)_n]$$
$$v'_B \cos\theta - (v'_A)_n = e[v_A \cos\theta - 0]$$
$$v'_B \cos 20° - (v'_A)_n = (0.7)(6.1994) \cos 20° \qquad (2)$$

Solving Eqs. (1) and (2) simultaneously for $(v'_A)_n$ and v'_B,

$$(v'_A)_n = -1.0446 \text{ ft/s}$$
$$v'_B = 3.2279 \text{ ft/s}$$

Resolve $\mathbf{v}_A$ into horizontal and vertical components.



$$\tan\beta = \frac{(v'_A)_t}{-(v'_A)_n} = \frac{2.1203}{1.0446}$$
$$\beta = 63.77° \qquad \beta + 20° = 83.8°$$
$$v'_A = \sqrt{(2.1203)^2 + (1.0446)^2} = 2.3637 \text{ ft/s}$$

(*a*) *Velocities immediately after impact.*

$$\mathbf{v}'_A = 2.36 \text{ ft/s} \searrow 83.8° \blacktriangleleft$$
$$\mathbf{v}'_B = 3.23 \text{ ft/s} \longrightarrow \blacktriangleleft$$

(*b*) *Maximum deflection of wedge B.*

Use conservation of energy: $T_{B1} + V_{B1} = T_{B2} + V_{B2}$

$$T_{B1} = \frac{1}{2} m_B v_B^2$$
$$V_{B1} = 0$$
$$T_{B2} = 0$$
$$V_{B2} = \frac{1}{2} k(\Delta x)^2$$

PROBLEM 13.188 (Continued)

The maximum deflection will occur when the block comes to rest (ie, no kinetic energy)

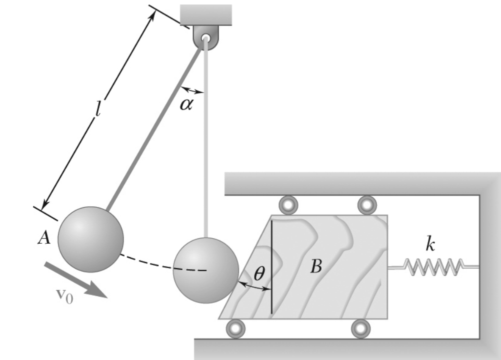
$$\frac{1}{2}m_B v_B^2 = \frac{1}{2}k(\Delta x)^2$$

$$(\Delta x)^2 = \frac{m_B v_B^2}{k} = \frac{\left(\frac{2 \text{ lb}}{32.2 \text{ ft/s}^2}\right)(3.2279 \text{ ft/s})^2}{2 \text{ lb/in (12 in/ft)}}$$

$$(\Delta x) = 0.1642118 \text{ ft}$$

$(\Delta x) = 1.971$ in. ◀

PROBLEM 13.189



When the rope is at an angle of $\alpha = 30°$ the 1-kg sphere A has a speed $v_0 = 0.6$ m/s. The coefficient of restitution between A and the 2-kg wedge B is 0.8 and the length of rope $l = 0.9$ m The spring constant has a value of 1500 N/m and $\theta = 20°$. Determine, (*a*) the velocities of A and B immediately after the impact (*b*) the maximum deflection of the spring assuming A does not strike B again before this point.

SOLUTION

Masses: $m_A = 1$ kg

$m_B = 2$ kg

<u>Analysis of sphere A as it swings down</u>:

Initial state: $\alpha = 30°, \quad h_0 = l(1-\cos\alpha) = (0.9)(1-\cos 30°) = 0.12058$ m

$$V_0 = m_A g h_0 = (1)(9.81)(0.12058) = 1.1829 \text{ N}\cdot\text{m}$$

$$T_0 = \frac{1}{2}mv_0^2 = \frac{1}{2}(1)(0.6)^2 = 0.180 \text{ N}\cdot\text{m}$$

Just before impact: $\alpha = 0, \quad h_1 = 0, \quad V_1 = 0$

$$T_1 = \frac{1}{2}m_A v_A^2 = \frac{1}{2}(1)v_A^2 = 0.5v_A^2$$

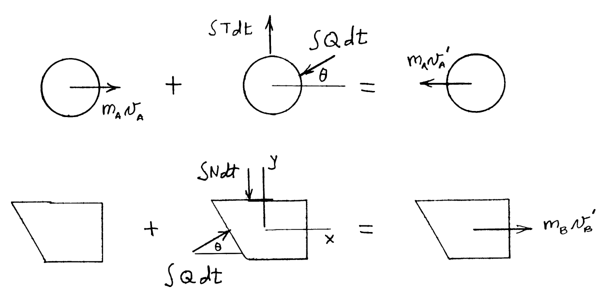
Conservation of energy: $T_0 + V_0 = T_1 + V_1$

$$0.180 + 1.1829 = 0.5v_A^2 + 0$$

$$v_A^2 = 2.7257 \text{ m}^2/\text{s}^2$$

$$\mathbf{v}_A = 1.6510 \text{ m/s} \longrightarrow$$

Analysis of the impact: Use conservation of momentum together with the coefficient of restitution. $e = 0.8$.



Note that the ball rebounds horizontally and that an impulse $\int Tdt$ is applied by the rope. Also, an impulse $\int Ndt$ is applied to B through its supports.

Both A and B:

Momentum in x-direction:

$$m_A(v_A)_x + 0 = m_A(v'_A)_x + m_B(v'_B)_x$$

$$(1)(1.6510) = (1)(v'_A)_x + (2)(v'_B)_x \qquad (1)$$

Coefficient of restitution: $(v_A)_n = (v_A)_x \cos\theta$

$$(v_B)_n = 0, \quad (v'_A)_n = (v'_A)_x \cos\theta, \quad (v'_B)_x \cos 30°$$

$$(v'_B)_n - (v'_A)_n = e[(v_A)_n - (v_B)_n]$$

$$(v'_B)_x \cos\theta - (v'_A)_x \cos\theta = e[(v_A)_x \cos\theta]$$

Dividing by $\cos\theta$ and applying $e = 0.8$ gives

$$(v'_B)_x - (v'_A)_x = (0.8)(1.6510) \qquad (2)$$

Solving Eqs. (1) and (2) simultaneously,

$$(v'_A)_x = -0.33020 \text{ m/s}$$

$$(v'_B)_x = 0.99059 \text{ m/s}$$

$\mathbf{v}'_A = 0.330 \text{ m/s} \longleftarrow$ ◀

(*a*) *Velocities immediately after impact.* $\mathbf{v}'_B = 0.991 \text{ m/s} \longrightarrow$ ◀

(*b*) Maximum deflection of wedge B.

Use conservation of energy: $T_{B1} + V_{B1} = T_{B2} + V_{B2}$

$$T_{B1} = \frac{1}{2} m_B v_B^2$$

$$V_{B1} = 0$$

$$T_{B2} = 0$$

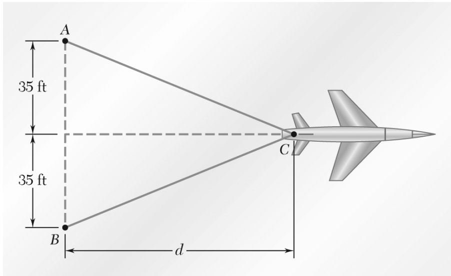
$$V_{B2} = \frac{1}{2} k(\Delta x)^2$$

The maximum deflection will occur when the block comes to rest (ie, no kinetic energy)

$$\frac{1}{2} m_B v_B^2 = \frac{1}{2} k(\Delta x)^2$$

$$(\Delta x)^2 = \frac{m_B v_B^2}{k} = \frac{(2)(0.99059 \text{ m/s})^2}{1500 \text{ N/m})}$$

$(\Delta x) = 0.0362$ m $\qquad \Delta x = 36.2$ mm ◀

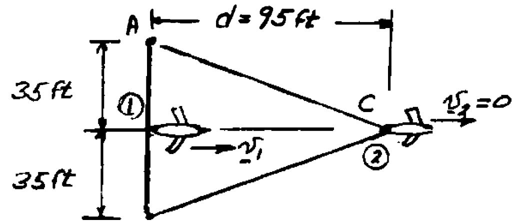


PROBLEM 13.190

A 32,000-lb airplane lands on an aircraft carrier and is caught by an arresting cable. The cable is inextensible and is paid out at A and B from mechanisms located below dock and consisting of pistons moving in long oil-filled cylinders. Knowing that the piston-cylinder system maintains a constant tension of 85 kips in the cable during the entire landing, determine the landing speed of the airplane if it travels a distance $d = 95$ ft after being caught by the cable.

SOLUTION

Mass of airplaine: $$m = \frac{W}{g} = \frac{32000 \text{ lb}}{32.2 \text{ ft/s}^2} = 993.79 \text{ lb} \cdot \text{s}^2/\text{ft}$$



Work of arresting cable force.

$$Q = 85 \text{ kips} = 85000 \text{ lb.}$$

As the cable is pulled out, the cable tension acts paralled to the cable at the airplane hook. For a small displacement

$$\Delta U = -Q(\Delta l_{AC}) - Q(\Delta l_{BC})$$

Since Q is constant,

$$U_{1\to 2} = -Q\left[\overline{AC} + \overline{BC} - \overline{AB}\right]$$

For $$d = 95 \text{ ft}, \qquad \overline{AC} = \overline{BC} = \sqrt{(35)^2 + (95)^2} = 101.24 \text{ ft}$$

$$U_{1\to 2} = -(85000)(101.24 + 101.24 - 70) = -11.261 \text{ ft} \cdot \text{lb}$$

Principle of work and energy: $$T_1 + U_{1\to 2} = T_2$$

$$\frac{1}{2}mv_1^2 + U_{1\to 2} = \frac{1}{2}mv_2^2$$

Since $v_2 = 0$, we get

$$v_1^2 = -\frac{2U_{1\to 2}}{m} = -\frac{(2)(-11.261)}{993.79} = 22.663 \times 10^3 \text{ ft}^2/\text{s}^2$$

Initial speed: $v_1 = 150.54$ ft/s $\qquad v_1 = 102.6$ mi/h ◀

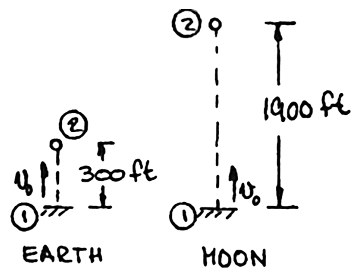
PROBLEM 13.191

A 2-oz pellet shot vertically from a spring-loaded pistol on the surface of the earth rises to a height of 300 ft. The same pellet shot from the same pistol on the surface of the moon rises to a height of 1900 ft. Determine the energy dissipated by aerodynamic drag when the pellet is shot on the surface of the earth. (The acceleration of gravity on the surface of the moon is 0.165 times that on the surface of the earth.)

SOLUTION

Since the pellet is shot from the same pistol the initial velocity v_0 is the same on the moon and on the earth.

Work and energy.



Earth: $$T_1 = \frac{1}{2}mv_0^2$$

$$U_{1-2} = -mg_E(300\text{ ft}) - E_L$$

$$(E_L = \text{ Loss of energy due to drag})$$

Moon: $$T_1 = \frac{1}{2}mv_0^2 \qquad T_2 = 0$$

$$U_{1-2} = -mg_M(1900) \qquad T_1 - 300mg_E - E_L = 0 \tag{1}$$

$$T_2 = 0$$

$$T_1 - 1900mg_M = 0 \tag{2}$$

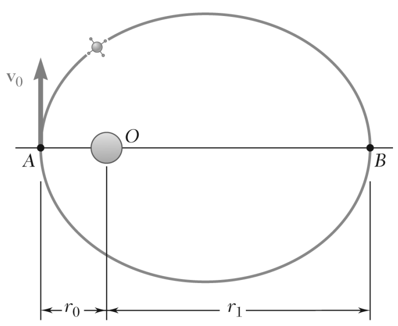
Subtracting (1) from (2) $$-1900mg_M + 300mg_E + E_L = 0$$

$$g_M = 0.165g_E$$

$$m = \frac{(2/16)}{g_E}$$

$$E_L = (1900)\frac{(2/16)}{g_E}(0.165g_E) - 300\frac{(2/16)}{g_E}g_E$$

$$E_L = 1.688\text{ ft}\cdot\text{lb} \blacktriangleleft$$



PROBLEM 13.192

A satellite describes an elliptic orbit about a planet of mass *M*. The minimum and maximum values of the distance *r* from the satellite to the center of the planet are, respectively, r_0 and r_1. Use the principles of conservation of energy and conservation of angular momentum to derive the relation

$$\frac{1}{r_0}+\frac{1}{r_1}=\frac{2GM}{h^2}$$

where *h* is the angular momentum per unit mass of the satellite and *G* is the constant of gravitation.

SOLUTION

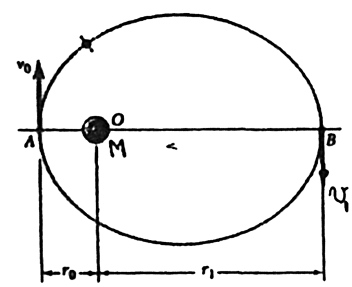
Angular momentum:

$$h = r_0, \quad v_0 = r_1 v_1$$

$$b = r_0 v_0 = r_1 v_1$$

$$v_0 = \frac{h}{r_0} \quad v_1 = \frac{h}{r_1} \qquad (1)$$

Conservation of energy:



$$T_A = \frac{1}{2}mv_0^2$$

$$V_A = -\frac{GMm}{r_0}$$

$$T_B = \frac{1}{2}mv_1^2$$

$$V_B = -\frac{GMm}{r_1}$$

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}mv_0^2 - \frac{GMm}{r_0} = \frac{1}{2}mv_1^2 - \frac{GMm}{r_1}$$

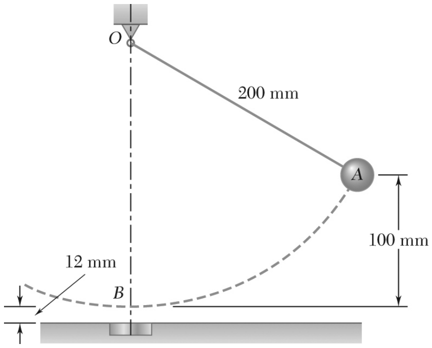
$$v_0^2 - v_1^2 = 2GM\left[\frac{1}{r_0}-\frac{1}{r_1}\right] = 2GM\left[\frac{r_1 - r_0}{r_1 r_0}\right]$$

Substituting for v_0 and v_1 from Eq. (1)

$$h^2\left[\frac{1}{r_0^2}-\frac{1}{r_1^2}\right] = 2GM\left[\frac{r_1 - r_0}{r_1 r_0}\right]$$

$$h^2\left[\frac{r_1^2 - r_0^2}{r_1^2 r_0^2}\right] = \frac{h^2}{r_1^2 r_0^2}(r_1 - r_0)(r_1 + r_0) = 2GM\left[\frac{r_1 - r_0}{r_1 r_0}\right]$$

$$h^2\left(\frac{1}{r_0}+\frac{1}{r_1}\right) = 2GM \qquad \left(\frac{1}{r_0}+\frac{1}{r_1}\right) = \frac{2GM}{h^2} \quad \text{Q.E.D.}$$



PROBLEM 13.193

A 60-g steel sphere attached to a 200-mm cord can swing about Point O in a vertical plane. It is subjected to its own weight and to a force **F** exerted by a small magnet embedded in the ground. The magnitude of that force expressed in newtons is $F = 3000/r^2$ where r is the distance from the magnet to the sphere expressed in millimeters. Knowing that the sphere is released from rest at A, determine its speed as it passes through Point B.

SOLUTION

Mass and weight: $m = 0.060$ kg

$$W = mg = (0.060)(9.81) = 0.5886 \text{ N}$$

Gravitational potential energy: $V_g = Wh$

where h is the elevation above level at B.

Potential energy of magnetic force:

$$F = \frac{3000}{r^2} = -\frac{dV}{dr} \quad (F\text{, in newtons, } r \text{ in mm})$$

$$V_m = -\int_{\infty}^{r} \frac{3000}{r^2} = \frac{3000}{r} \text{ N} \cdot \text{mm}$$

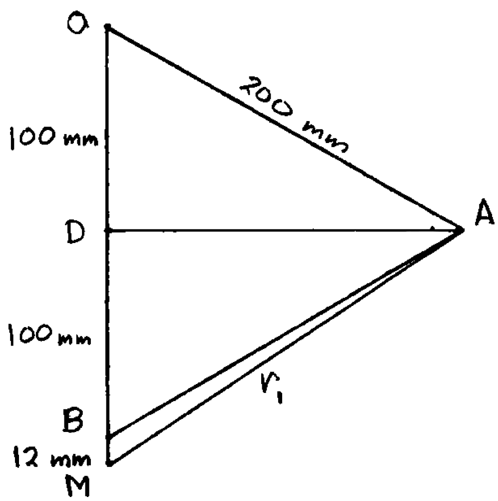
Use conservation of energy: $T_1 + V_1 = T_2 + V_2$

Position 1: (Rest at A.)

$$v_1 = 0 \qquad T_1 = 0$$

$$h_1 = 100 \text{ mm}$$

$$(V_g)_1 = (0.5886 \text{ N})(100 \text{ mm}) = 58.86 \text{ N} \cdot \text{mm}$$



From the figure, $\overline{AD}^2 = 200^2 - 100^2 \ (\text{mm}^2)$

$$\overline{MD} = 100 + 12 = 112 \text{ mm}$$

$$r_1^2 = \overline{AD}^2 + \overline{MD}^2$$

$$= 200^2 - 100^2 + 112^2$$

$$= 42544 \text{ mm}^2$$

$$r_1 = 206.26 \text{ mm}$$

$$(V_r)_1 = -\frac{3000}{r_1} = -14.545 \text{ N} \cdot \text{mm}$$

$$V_1 = 58.86 - 14.545 = 44.3015 \text{ N} \cdot \text{mm} = 44.315 \times 10^{-3} \text{ N} \cdot \text{m}$$

PROBLEM 13.193 (Continued)

Position 2. (Sphere at Point B.)

$$T_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}(0.060)v_2^2 = 0.030\ v_2^2$$

$$(V_g)_2 = 0 \qquad \text{(since } h_2 = 0\text{)}$$

$$r_2 = \overline{MB} = 12 \text{ mm} \qquad \text{(See figure.)}$$

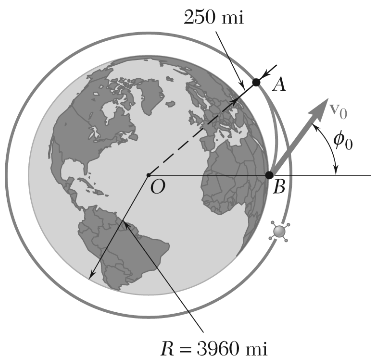
$$(V_m)_2 = -\frac{3000}{12} = -250 \text{ N} \cdot \text{mm} = -250 \times 10^{-3} \text{ N} \cdot \text{mm}$$

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 44.315 \times 10^{-3} = 0.030 v_2^2 - 250 \times 10^{-3}$$

$$v_2^2 = 9.8105 \text{ m}^2/\text{s}^2$$

$v_2 = 3.13$ m/s ◀

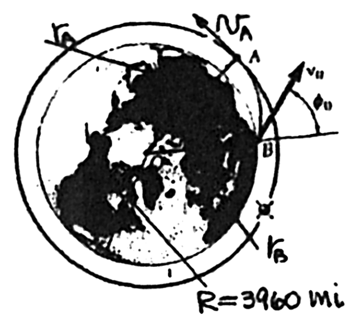


PROBLEM 13.194

A shuttle is to rendezvous with a space station which is in a circular orbit at an altitude of 250 mi above the surface of the earth. The shuttle has reached an altitude of 40 mi when its engine is turned off at Point *B*. Knowing that at that time the velocity $\mathbf{v}_0$ of the shuttle forms an angle $\phi_0 = 55°$ with the vertical, determine the required magnitude of $\mathbf{v}_0$ if the trajectory of the shuttle is to be tangent at *A* to the orbit of the space station.

SOLUTION

Conservation of energy:



$$T_B = \frac{1}{2}mv_0^2 \qquad V_B = -\frac{GMm}{r_B}$$

$$T_A = \frac{1}{2}mv_A^2 \qquad V_A = -\frac{GMm}{r_A}$$

$$GM = gR^2 \quad \text{(Eq. 12.30)}$$

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}\cancel{m}v_0^2 - \frac{gR^2}{r_B}\cancel{m} = \frac{1}{2}\cancel{m}v_A^2\frac{gR^2}{r_A}\cancel{m}$$

$$r_A = 3960 + 250 = 4210 \text{ mi}$$

$$v_A^2 = v_0^2 - \frac{2gR^2}{r_B}\left(1 - \frac{r_B}{r_A}\right)$$

$$r_B = 3960 + 40 = 4000 \text{ mi}$$

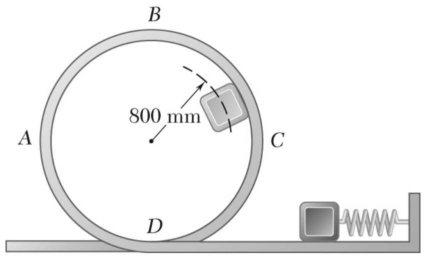
$$v_A^2 = v_0^2 - \frac{2(32.2)(3960 \times 528)^3}{(4000 \times 5280)}\left(1 - \frac{4000}{4210}\right)$$

$$v_A^2 = v_0^2 - 66.495 \times 10^6 \qquad (1)$$

Conservation of angular momomentum: $\quad r_A v_A = r_B v_B \sin\phi_0;$

$$v_A = (4000/4210)v_0 \sin 55° = 0.77829\, v_0 \qquad (2)$$

Eqs. (2) and (1) $\quad [1 - (0.77829)^2]\, v_0^2 = 66.495 \times 10^6 \qquad v_0 = 12{,}990 \text{ ft/s}$ ◀

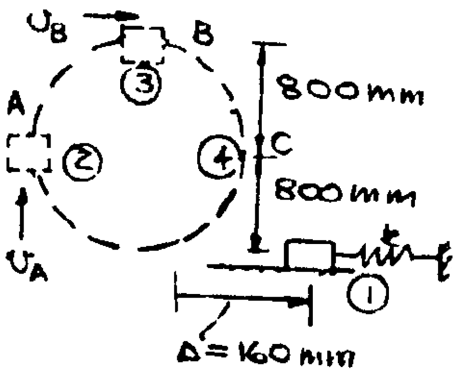


PROBLEM 13.195

A 300-g block is released from rest after a spring of constant $k = 600$ N/m has been compressed 160 mm. Determine the force exerted by the loop *ABCD* on the block as the block passes through (*a*) Point *A*, (*b*) Point *B*, (*c*) Point *C*. Assume no friction.

SOLUTION

Conservation of energy to determine speeds at locations *A*, *B*, and *C*.



Mass: $m = 0.300$ kg

Initial compression in spring: $x_1 = 0.160$ m

Place datum for gravitational potential energy at position 1.

Position 1: $v_1 = 0 \qquad T_1 = \frac{1}{2}mv_1^2 = 0$

$$V_1 = \frac{1}{2}kx_1^2 = \frac{1}{2}(600 \text{ N/m})(0.160 \text{ m})^2 = 7.68 \text{ J}$$

Position 2: $T_2 = \frac{1}{2}mv_A^2 = \frac{1}{2}(0.3)v_A^2 = 0.15v_A^2$

$$V_2 = mgh_2 = (0.3 \text{ kg})(9.81 \text{ m/s}^2)(0.800 \text{ m}) = 2.3544 \text{ J}$$

$$T_1 + V_1 = T_2 + V_2: \quad 0 + 7.68 = 0.15v_A^2 + 2.3544$$

$$v_A^2 = 35.504 \text{ m}^2/\text{s}^2$$

Position 3: $T_3 = \frac{1}{2}mv_B^2 = \frac{1}{2}(0.3)v_B^2 = 0.15v_B^2$

$$V_3 = mgh_3 = (0.3 \text{ kg})(9.81 \text{ m/s}^2)(1.600 \text{ m}) = 4.7088 \text{ J}$$

$$T_1 + V_1 = T_3 + V_3: \quad 0 + 7.68 = 0.15v_B^2 + 4.7088$$

$$v_B^2 = 19.808 \text{ m}^2/\text{s}^2$$

Position 4: $T_2 = \frac{1}{2}mv_C^2 = \frac{1}{2}(0.3)v_C^2 = 0.15v_C^2$

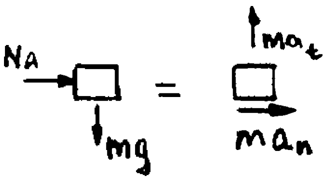
$$V_4 = mgh_4 = (0.3 \text{ kg})(9.81 \text{ m/s})(0.800 \text{ m}) = 2.3544 \text{ J}$$

$$T_1 + V_1 = T_4 + V_4: \quad 0 + 7.68 = 0.15v_C^2 + 2.3544$$

$$v_C^2 = 35.504 \text{ m}^2/\text{s}^2$$

PROBLEM 13.195 (Continued)

(*a*) Newton's second law at *A*:



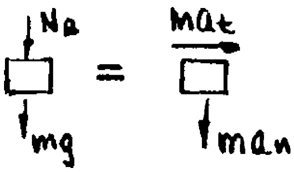
$$a_n = \frac{v_A^2}{\rho} = \frac{35.504 \text{ m}^2/\text{s}^2}{0.800 \text{ m}} = 44.38 \text{ m/s}^2$$

$\mathbf{a}_n = 44.38 \text{ m/s}^2 \longrightarrow$

$\xrightarrow{+} \Sigma F = ma_n: \quad N_A = ma_n$

$N_A = (0.3 \text{ kg})(44.38 \text{ m/s}^2)$ $\qquad \mathbf{N}_A = 13.31 \text{ N} \longrightarrow$ ◀

(*b*) Newton's second law at *B*:



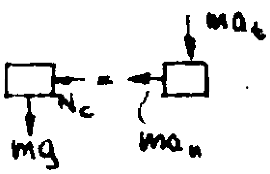
$$a_n = \frac{v_B^2}{\rho} = \frac{19.808 \text{ m}^2/\text{s}^2}{0.800 \text{ m}} = 24.76 \text{ m/s}^2$$

$\mathbf{a}_n = 24.76 \text{ m/s}^2 \downarrow$

$+\!\downarrow \Sigma F = ma_n: \quad N_B = mg = ma_n$

$N_B = m(a_n - g) = (0.3 \text{ kg})(24.76 \text{ m/s}^2 - 9.81 \text{ m/s}^2)$ $\qquad \mathbf{N}_B = 4.49 \text{ N} \downarrow$ ◀

(*c*) Newton's second law at *C*:

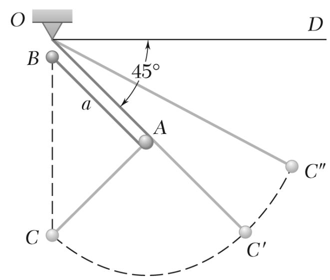


$$a_n = \frac{v_C^2}{\rho} = \frac{35.504 \text{ m}^2/\text{s}^2}{0.800 \text{ m}} = 44.38 \text{ m/s}^2$$

$\mathbf{a}_n = 44.38 \text{ m/s}^2 \longleftarrow$

$\xleftarrow{+} \Sigma F = ma_n: \quad N_C = ma_n$

$N_C = (0.3 \text{ kg})(44.38 \text{ m/s}^2)$ $\qquad \mathbf{N}_C = 13.31 \text{ N} \longleftarrow$ ◀



PROBLEM 13.196

A small sphere B of mass m is attached to an inextensible cord of length $2a$, which passes around the fixed peg A and is attached to a fixed support at O. The sphere is held close to the support at O and released with no initial velocity. It drops freely to Point C, where the cord becomes taut, and swings in a vertical plane, first about A and then about O. Determine the vertical distance from line OD to the highest Point C'' that the sphere will reach.

SOLUTION

Velocity at Point C (before the cord is taut).

Conservation of energy from B to C:

$$T_B = 0$$

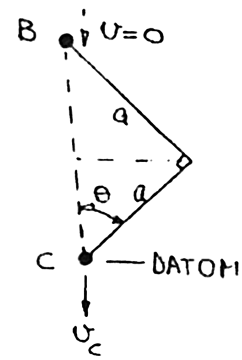
$$V_B = mg(2)\left(\frac{\sqrt{2}}{2}\right)a = mga\sqrt{2}$$

$$T_C = \frac{1}{2}mv_C^2 \qquad V_C = 0$$

$$T_B + V_B = T_C + V_C$$

$$0 + mga\sqrt{2} = \frac{1}{2}mv_C^2 + 0$$

$$v_C = \sqrt{2\sqrt{2}\,ga}$$



Velocity at C (after the cord becomes taut).

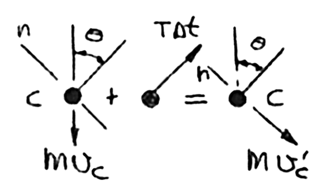
Linear momentum perpendicular to the cord is conserved:

$$\theta = 45°$$

$$\nwarrow + \quad -mv_C \sin\theta = mv'_C$$

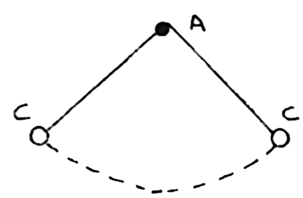
$$v'_C = \left(\sqrt{2\sqrt{2}}\right)\left(\frac{\sqrt{2}}{2}\right)\sqrt{ga}$$

$$v'_C = \sqrt{\sqrt{2}ga} = 2^{\frac{1}{4}}\sqrt{ga}$$



Note: The weight of the sphere is a non-impulsive force.

Velocity at C:



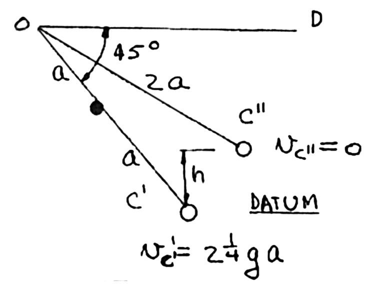
C to C' (conservation of energy): $\quad T_C = \frac{1}{2}m(v'_C)^2 \qquad V_C = 0$

$$T_{C'} = \frac{1}{2}m(v'_{C'})^2 \qquad V_{C'} = 0$$

Datum:
$$T_C + V_C = T_{C'} + V_{C'}$$
$$\frac{1}{2}m(v_C')^2 + 0 = \frac{1}{2}m(v_C')^2 + 0$$
$$v_C' = v_{C'}'$$

<u>C' to C''</u> (conservation of energy):

$$T_{C'} = \frac{1}{2}m(v_{C'}')^2$$
$$T_{C'} = \frac{1}{2}m\left(2^{1/4}\sqrt{ga}\right)^2$$
$$T_{C'} = \frac{\sqrt{2}}{2}mga$$

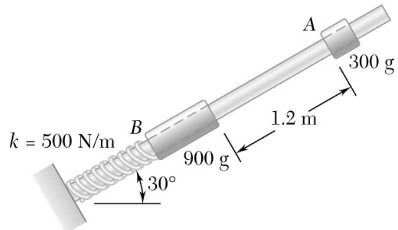


Datum:
$$T_{C'} + V_{C'} = T_{C''} + V_{C''}$$
$$V_{C'} = 0$$
$$T_{C''} = 0$$
$$V_{C''} = mgh$$

$$\frac{\sqrt{2}}{2}mga + 0 = 0 + mgh$$

$$h = \frac{\sqrt{2}}{2}a$$

$h = 0.707\ a$ ◀

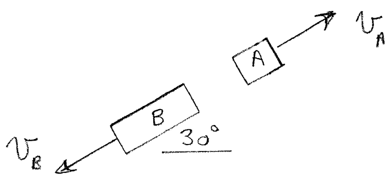


PROBLEM 13.197

A 300-g collar A is released from rest, slides down a frictionless rod, and strikes a 900-g collar B which is at rest and supported by a spring of constant 500 N/m. Knowing that the coefficient of restitution between the two collars is 0.9, determine (*a*) the maximum distance collar A moves up the rod after impact, (*b*) the maximum distance collar B moves down the rod after impact.

SOLUTION

After impact



Velocity of A just before impact, v_0

$$v_0 = \sqrt{2gh} = \sqrt{2(9.81\text{ m/s}^2)(1.2\text{ m})\sin 30°}$$

$$= \sqrt{2(9.81)(1.2)(0.5)} = 3.431\text{ m/s}$$

Conservation of momentum

$$\overset{+}{\swarrow}_{30°}\ m_A v_0 = m_B v_B - m_A v_A: \quad 0.3v_0 = 0.9v_B - 0.3v_A \quad (1)$$

Restitution

$$(v_A + v_B) = e(v_0 + 0) = 0.9v_0\,(2)$$

Substituting for v_B from (2) in (1)

$$0.3v_0 = 0.9(0.9v_0 - v_A) - 0.3v_A \qquad 1.2v_A = 0.51v_0$$

$$v_A = 1.4582\text{ m/s}, \ v_B = 1.6297\text{ m/s}$$

(*a*) A moves up the distance d where:

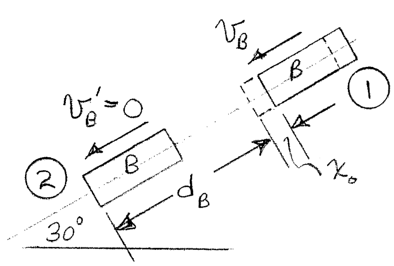
$$\frac{1}{2}\cancel{m_A}v_A^2 = \cancel{m_A}gd\sin 30°; \quad \frac{1}{2}(1.4582\text{ m/s})^2 = (9.81\text{ m/s}^2)d(0.5)$$

$$\nearrow_{30°}\ d_A = 0.21675\text{ m} = 217\text{ mm} \blacktriangleleft$$

(*b*) Static deflection = x_0, B moves down $\swarrow^{d_B}_{30°}$

Conservation of energy (1) to (2)

Position (1) – spring deflected, x_0



$$kx_0 = m_B g\sin 30°$$

$$T_1 + V_1 = T_2 + V_2: \quad T_1 = \frac{1}{2}m_B v_B^2, \ T_2 = 0$$

PROBLEM 13.197 (Continued)

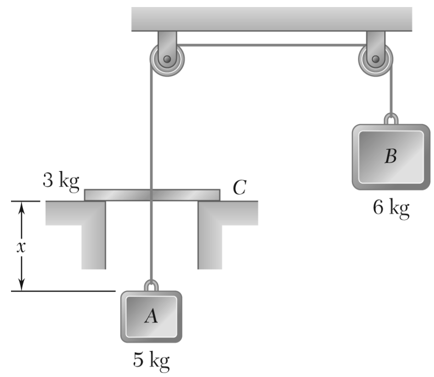
$$V_1 = V_e + V_g = \frac{1}{2}kx_0^2 + m_B g d_B \sin 30°$$

$$V_2 = V_e' + V_g' = \int_0^{x_0 + d_B} kx dx = \frac{1}{2}k\left(d_B^2 + 2d_B x_0 + x_0^2\right)$$

$$\frac{1}{2}kx_0^2 + mgd_B \sin 30° + \frac{1}{2}m_B v_B^2 = \frac{1}{2}k\left(d_B^2 + 2d_B x_0 + x_0^2\right) + 0 + 0$$

$$\therefore \quad kd_B^2 = m_B v_B^2; \quad 500d_B^2 = 0.9(1.6297)^2 \qquad d_B = 0.0691 \text{ m}$$

$d_B = 69.1 \text{ mm}$ ◀



PROBLEM 13.198

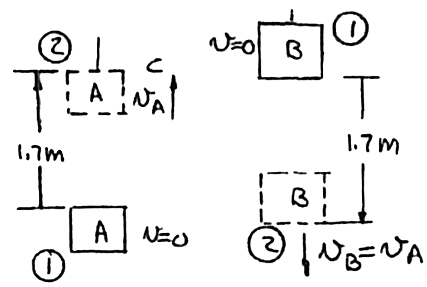
Blocks A and B are connected by a cord which passes over pulleys and through a collar C. The system is released from rest when $x = 1.7$ m. As block A rises, it strikes collar C with perfectly plastic impact ($e = 0$). After impact, the two blocks and the collar keep moving until they come to a stop and reverse their motion. As A and C move down, C hits the ledge and blocks A and B keep moving until they come to another stop. Determine (a) the velocity of the blocks and collar immediately after A hits C, (b) the distance the blocks and collar move after the impact before coming to a stop, (c) the value of x at the end of one compete cycle.

SOLUTION

(a) Velocity of A just before it hits C:

Conservations of energy:

Datum at ①:



Position ①:

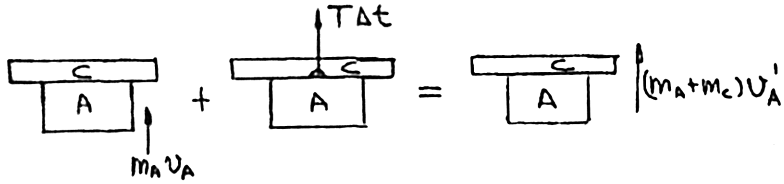
$$(v_A)_1 = (v_B)_1 = 0$$
$$T_1 = 0$$
$$v_1 = 0$$

Position ②:

$$T_2 = \frac{1}{2}m_A(v_A)^2 + \frac{1}{2}m_B v_B^2$$
$$v_A = v_B \quad \text{(kinematics)}$$
$$T_2 = \frac{1}{2}(5+6)v_A^2 = \frac{11}{2}v_A^2$$
$$V_2 = m_A g(1.7) - m_B g(1.7)$$
$$= (5-6)(g)(1.7)$$
$$V_2 = -1.7g$$

$$T_1 + V_1 = T_2 + V_2$$
$$0 + 0 = \frac{11}{2}v_A^2 - 1.7g$$
$$v_A^2 = \left(\frac{3.4}{11}\right)(9.81)$$
$$= 3.032 \text{ m}^2/\text{s}^2$$
$$v_A = 1.741 \text{ m/s}$$

Velocity of A and C after A hits C:



$v_A' = v_C'$ (plastic impact)

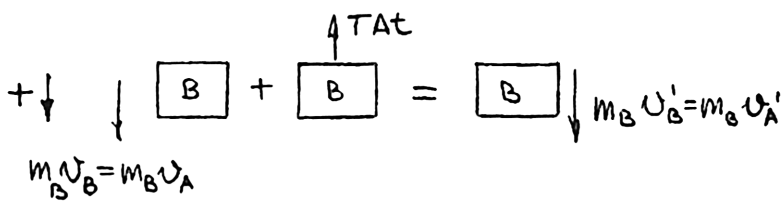
Impulse-momentum A and C:

$$+\uparrow \quad m_A v_A + T\Delta t = (m_A + m_C)v_A'$$

$$(5)(1.741) + T\Delta t = 8v_A' \tag{1}$$

$$v_B = v_A; \; v_B' = v_A' \text{ (cord remains taut)}$$

B alone:



$$m_B v_A - T\Delta t = m_B v_A'$$

$$(6)(1.741) - T\Delta t = 6v_A' \tag{2}$$

Adding Equations (1) and (2), $\quad 11(1.741) = 14v_A'$

$$v_A' = 1.3679 \text{ m/s}$$

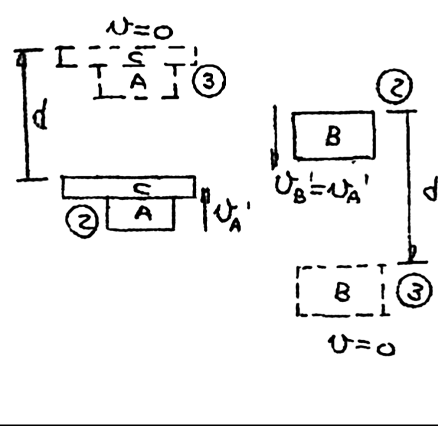
$$v_A' = v_B' = v_C' = 1.368 \text{ m/s} \blacktriangleleft$$

(*b*) Distance A and C move before stopping:

Conservations of energy:

Datum at ②:

Position ②:



$$T_2 = \frac{1}{2}(m_A + m_B + m_C)(v_A')$$

$$T_2 = \left(\frac{14}{2}\right)(1.3681)^2$$

$$T_2 = 13.103 \text{ J}$$

$$V_2 = 0$$

Position ③:

$T_3 = 0$

$$V_3 = (m_A + m_C)gd - m_B gd$$
$$V_3 = (8-6)gd = 2gd$$

$T_2 + V_2 = T_3 + V_3$

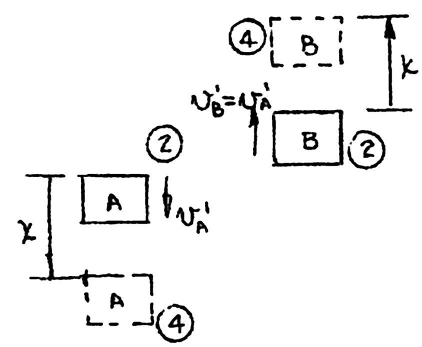
$$13.103 + 0 = 0 + 2gd$$

$$d = (13.103)/(2)(9.81) = 0.6679 \text{ m} \qquad d = 0.668 \text{ m} \blacktriangleleft$$

(*c*) As the system returns to position ② after stopping in position ③, energy is conserved, and the velocities of *A*, *B*, and *C* before the collar at *C* is removed are the same as they were in Part (*a*) above with the directions reversed. Thus, $v'_A = v'_C = v'_B = 1.3679$ m/s. After the collar *C* is removed, the velocities of *A* and *B* remain the same since there is no impulsive force acting on either.

Conversation of energy:

Datum at ②:



$$T_2 = \frac{1}{2}(m_A + m_B)(v'_A)^2$$
$$T_2 = \frac{1}{2}(5+6)(1.3679)^2$$
$$T_2 = 10.291 \text{ J}$$

$V_2 = 0$

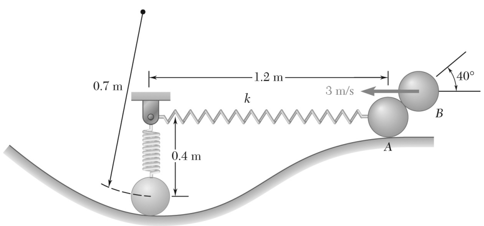
$$T_4 = 0 \quad V_4 = m_B gx - m_A gx$$
$$V_4 = (6-5)gx$$

$T_2 + V_2 = T_4 + V_4$

$$10.291 + 0 = (1)(9.81)x$$

$$x = 1.049 \text{ m} \blacktriangleleft$$

PROBLEM 13.199

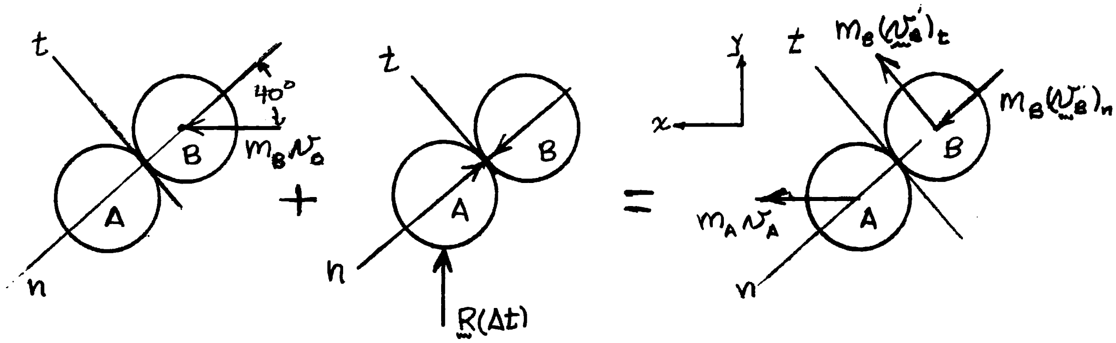


A 2-kg ball B is traveling horizontally at 10 m/s when it strikes 2-kg ball A. Ball A is initially at rest and is attached to a spring with constant 100 N/m and an unstretched length of 1.2 m. Knowing the coefficient of restitution between A and B is 0.8 and friction between all surfaces is negligible, determine the normal force between A and the ground when it is at the bottom of the hill.

SOLUTION

Ball B impacts on ball A. Use the principle of impulse and momentum.

$$\Sigma m\mathbf{v}_1 + \Sigma \mathbf{Imp}_{1\to 2} = \Sigma m\mathbf{v}_2$$



Velocity components: $v_0 = 10$ m/s

$$(v_0)_x = v_0 \quad (v_0)_n = v_0 \cos 40° \quad (v_0)_t = v_0 \sin 40°$$
$$(v_A)_x = v_A \quad (v_A)_n = v_A \cos 40°$$
$$(v_B)_x = (v_B)_n \cos 40° + (v_B)_t \sin 40°$$

Impulse-momentum for ball B alone.

t-direction:

$$m_B(v_0)_t = m_B(v_B)_t$$
$$(v_B)_t = (v_0)_t = 10\sin 40° = 6.4279 \text{ m/s} \qquad (1)$$

Impulse-momentum for balls A and B.

x-direction ←

$$m_B v_0 + 0 = m_A v_A + m_B(v_B)_x + m_B(v_B)_t$$
$$(2)(10) + 0 = 2v_A + 2[(v_B)_n \cos 40° + 6.4279 \sin 40°]$$
$$2v_A + 2(v_B)_n \cos 40° = 11.7365 \qquad (1)$$

Coefficient of restitution. $(e = 0.8)$

$$(v_B)_n = (v_A)_n = e[0 - (v_0)_n]$$

$$(v_B)_n - v_A \cos 40° = -(0.8)(10)\cos 40° \qquad (2)$$

Solving Eqs. (1) and (2) simultaneously,

$$v_A = 6.6566 \text{ m/s} \qquad (v_B)_n = -1.0291 \text{ m/s}$$

As ball A moves from the impact location to the lowest point on the path, the spring compresses and the elevation decreases. Since friction is negligible, energy is conserved.

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}m_A v_A^2 + (V_e)_1 + (V_g)_1 = \frac{1}{2}m_A v_2^2 + (V_e)_2 + (V_g)_2$$

Position 1: (Just after impact.)

$$T_1 = \frac{1}{2}m_A v_A^2 = \frac{1}{2}(2)(6.6566)^2 = 44.3101 \text{ J}$$

$$(V_e)_1 = 0 \quad \text{(The spring is unstretched.)}$$

$$(V_g)_1 = 0 \quad \text{(Datum)}$$

Position 2: (Lowest point on path.)

$$T_2 = \frac{1}{2}m_A v_2^2 = \frac{1}{2}(2)v_2^2 = v_2^2$$

For the spring, $x_2 = l_2 - l_0 = 0.4 \text{ m} - 1.2 \text{ m} = 0.8 \text{ m}$

$$F_e = kx_2 = (100)(0.8) = 80 \text{ N}$$

$$(V_2)_e = \frac{1}{2}kx_2^2 = \frac{1}{2}(100)(0.8)^2 = 32 \text{ J}$$

Elevation above datum: $h_2 = -0.4 \text{ m}$

$$(V_2)_g = m_A g\, h_2 = (2)(9.81)(-0.4) = -7.848$$

Conservation of energy:

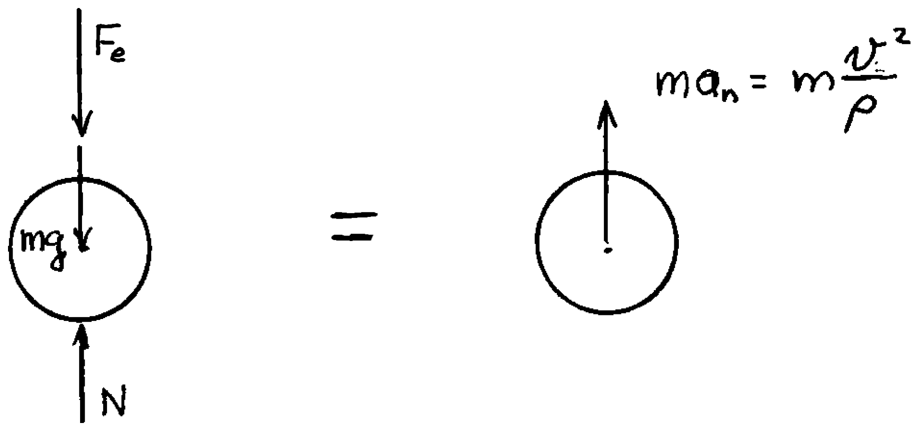
$$44.310 + 0 + 0 = v_2^2 + 32 - 7.848$$

$$v_2^2 = 20.158 \text{ m}^2/\text{s}^2 \qquad v_2 = 4.489 \text{ m/s}$$

Normal acceleration at lowest point on path:

$$a_n = \frac{v_2^2}{\rho} = \frac{20.158}{0.7} = 28.798 \text{ m/s}^2 \qquad \mathbf{a}_n = 28.8 \text{ m/s}^2 \uparrow$$

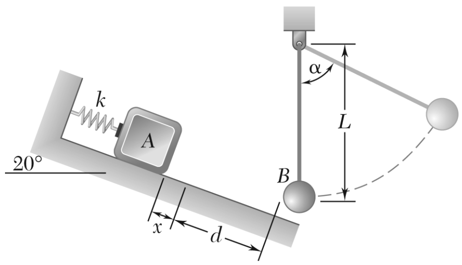
Apply Newton's second law to the ball.



$$+\uparrow \Sigma F = ma_n: \; N - mg - F_e = ma_n$$

$$N = mg + F_e + ma_n$$

$$= (2)(9.81) + 80 + (2)(28.798) \qquad N = 157.2 \text{ N} \blacktriangleleft$$



PROBLEM 13.200

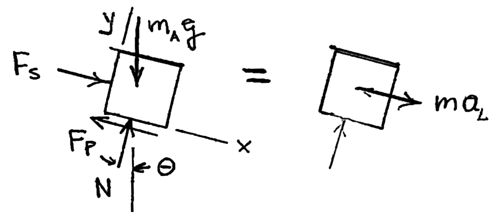
A 2-kg block A is pushed up against a spring compressing it a distance $x = 0.1$ m. The block is then released from rest and slides down the $20°$ incline until it strikes a 1-kg sphere B which is suspended from a 1 m inextensible rope. The spring constant $k = 800$ N/m, the coefficient of friction between A and the ground is 0.2, the distance A slides from the unstretched length of the spring $d = 1.5$ m and the coefficient of restitution between A and B is 0.8. When $\alpha = 40°$, determine (a) the speed of B (b) the tension in the rope.

SOLUTION

Data: $m_A = 2$ kg, $m_B = 1$ kg, $k = 800$ N/m, $x = 0.1$ m, $d = 1.5$ m

$\mu_k = 0.2$, $e = 0.8$, $\theta = 20°$, $\alpha = 40°$, $l = 1.0$ m

Block slides down the incline:



$$+\nearrow \; \Sigma F_y = 0$$

$$N - m_A g \cos\theta = 0$$

$$N = m_A g \cos\theta$$
$$= (2)(9.81)\cos 20°$$
$$= 18.4368 \text{ N}$$
$$F_f = \mu_k N = (0.2)(18.4368)$$
$$= 3.6874 \text{ N}$$

Use work and energy. Datum for V_g is the impact point near B.

$$T_1 = 0, \quad (V_1)_e = \frac{1}{2}kx_1^2 = \frac{1}{2}(800)(0.1)^2 = 4.00 \text{ J}$$

$$(V_1)_g = m_A g h_1 = m_A g(x+d)\sin\theta = (2)(9.81)(1.6)\sin 20° = 10.7367 \text{ J}$$

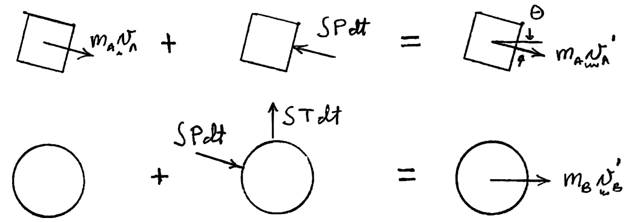
$$U_{1\to 2} = -F_f(x+d) = -(3.6874)(1.6) = -5.8998 \text{ J}$$

$$T_2 = \frac{1}{2}m_A v_A^2 = \frac{1}{2}(1)(v_A^2) = 1.000\, v_A^2 \qquad V_2 = 0$$

$$T_1 + V_1 + U_{1\to 2} = T_2 + V_2: \quad 0 + 4.00 + 10.7367 - 5.8998 = 1.000\, v_A^2 + 0$$

$$v_A^2 = 8.8369 \text{ m}^2/\text{s}^2$$
$$\mathbf{v}_A = 2.9727 \text{ m/s} \searrow 20°$$

Impact: Conservation of momentum.



Both A and B, horizontal components $\xrightarrow{+}$:

$$m_A v_A \cos\theta + 0 = m_A v_A' \cos\theta + m_B v_B$$

$$(2)(2.9727)\cos 20° = 2v_A' \cos 20° + (1.00)v_B \tag{1}$$

Relative velocities:

$$(v_B')_n - (v_A')_n = e[(v_B)_n - (v_A)_n]$$

$$v_B' \cos\theta - v_A' = e[v_A - 0]$$

$$v_B' \cos 20° - v_A' = (0.8)(2.9727) \tag{2}$$

Solving Eqs. (1) and (2) simultaneously,

$$v_A' = 1.0382 \text{ m/s}$$

$$v_B' = 3.6356 \text{ m/s}$$

Sphere B rises: Use conservation of energy.

$$T_1 = \frac{1}{2} m_B (v_B')^2 \quad V_1 = 0$$

$$T_2 = \frac{1}{2} m_B v_2^2 \qquad V_2 = m_B g h_2 = m_B g l(1-\cos\alpha)$$

$$T_1 + V_1 = T_2 + V_2: \quad \frac{1}{2} m_B (v_B')^2 + 0 = \frac{1}{2} m_B v_2^2 + m_B g(1-\cos)$$

$$v_2^2 = (v_B')^2 - 2gl(1-\cos\alpha)$$

$$= (3.6356)^2 - (2)(9.81)(1-\cos 40°)$$

$$= 8.6274 \text{ m}^2/\text{s}^2$$

(*a*) Speed of B: $v_2 = 2.94$ m/s ◀

(*b*) Tension in the rope: $\rho = 1.00$ m

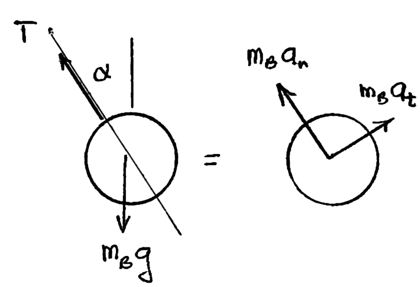
$$a_n = \frac{v_2^2}{\rho} = \frac{8.6274}{1.00} = 8.6274 \text{ m/s}^2$$

$$+\nwarrow \Sigma F_n = m_B a_n:$$

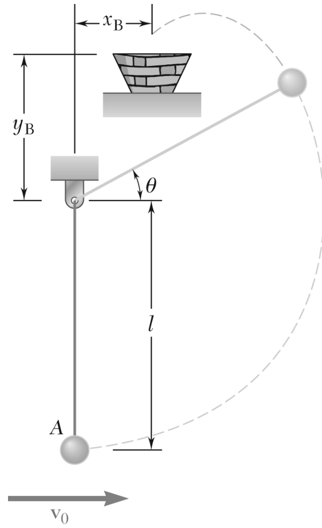
$$T - m_B g \cos\alpha = m_B a_n$$

$$T = m_B (a_n + g\cos\alpha)$$

$$= (1.0)(8.6274 + 9.81\cos 40°)$$



$T = 16.14$ N ◀



PROBLEM 13.201*

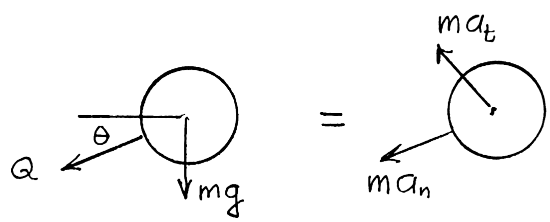
The 2-lb ball at A is suspended by an inextensible cord and given an initial horizontal velocity of $\mathbf{v}_0$. If $l = 2$ ft, $x_B = 0.3$ ft and $y_B = 0.4$ ft determine the initial velocity $\mathbf{v}$ so that the ball will enter in the basket. *Hint*: use a computer to solve the resulting set of equations.

SOLUTION

Let position 1 be at A. $\quad v_1 = v_0$

Let position 2 be the point described by the angle θ where the path of the ball changes from circular to parabolic. At position 2 the tension Q in the cord is zero.

Relationship between v_2 and θ based on $Q = 0$. Draw the free body diagram.



$$+\swarrow \Sigma F = 0: \quad Q + mg\sin\theta = ma_n = \frac{mv_2^2}{\ell}$$

With $Q = 0$, $\quad v_2^2 = g\ell\sin\theta \quad \text{or} \quad v_2 = \sqrt{g\ell\sin\theta}$ (1)

Relationship among $v_0, v_2,$ and θ based on conservation of energy.

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}mv_0^2 - mg\ell = \frac{1}{2}mv_2^2 + mg\ell\sin\theta$$

$$v_0^2 = v_2^2 + 2g\ell(1+\sin\theta) \tag{2}$$

<u>x and y coordinates at position 2</u>:

$$x_2 = \ell \cos\theta \tag{3}$$

$$y_2 = \ell \sin\theta \tag{4}$$

Let t_2 be the time when the ball is in position 2.

<u>Motion on the parabolic path</u>. The horizontal motion is

$$\dot{x} = -v_2 \sin\theta$$

$$x = x_2 - (v_2 \sin\theta)(t - t_2) \tag{5}$$

At Point B, $\quad x = x_B \quad \text{and} \quad t = t_B. \qquad$ From Eq. (5),

$$(t_B - t_2) = \frac{\ell \cos\theta - x_B}{v_\theta \sin\theta} \tag{6}$$

Vertical motion: $\quad \dot{y} = v_2 \cos\theta - g(t - t_2)$

$$y = y_2 + (v_2 \cos\theta)(t - t_2) - \frac{1}{2} g(t - t_2)^2$$

At Point B,

$$y_B = \ell \sin\theta + (v_2 \cos\theta)(t_B - t_2) - \frac{1}{2} g(t_B - t_2)^2 \tag{7}$$

<u>Data</u>: $\quad \ell = 2\text{ ft}, \quad x_B = 0.3\text{ ft}, \quad y_B = 0.4\text{ ft}, \quad g = 32.2\text{ ft/s}^2$

With the numerical data,

Eq. (1) becomes $\quad v_2 = \sqrt{64.4 \sin\theta} \qquad (1)'$

Eq. (6) becomes $\quad t_B - t_2 = \dfrac{2\cos\theta - 0.3}{v_2 \sin\theta} \qquad (6)'$

Eq. (7) becomes $\quad y_B = 2\sin\theta + (v_2 \cos\theta)(t_B - t_2) - 16.1(t_B - t_2)^2 \qquad (7)'$

<u>Method of solution</u>. From a trial value of θ, calculate v_2 from Eq. (1)′, $t_B - t_2$ from Eq. (6)′, and y_B from Eq. (7)′. Repeat until $y_B = 0.4$ ft as required.

Try $\theta = 30°$. $\quad v_2 = \sqrt{64.4 \sin 30°} = 5.6745\text{ ft/s}$

$$t_B - t_2 = \frac{2\cos 30° - 0.3}{5.6745 \sin 30°} = 0.50473\text{ s}$$

$$y_B = 2\sin 30° + (5.6745 \cos 30°)(0.50473) - (16.1)(0.50473)^2$$
$$= -0.62116\text{ ft}$$

Try $\theta = 45°$.

$$v_2 = \sqrt{64.4 \sin 45°} = 6.7482$$

$$t_B - t_2 = \frac{2\cos 45° - 0.3}{6.7482 \sin 45°} = 0.23351 \text{ s}$$

$$\begin{aligned} y_B &= 2\sin 45° + (6.7482\cos 45°)(0.23351) - (16.1)(0.23351)^2 \\ &= 1.65060 \text{ ft} \end{aligned}$$

Try $\theta = 37.5°$.

$$v_2 = \sqrt{64.4 \sin 37.5°} = 6.2613 \text{ ft/s}$$

$$t_B - t_2 = \frac{2\cos 37.5° - 0.3}{6.2613 \sin 37.5°} = 0.33757 \text{ s}$$

$$\begin{aligned} y_B &= 2\sin 37.5° + (6.2613\cos 37.5°)(0.33757) - (16.1)(0.33757)^2 \\ &= 1.05972 \text{ ft} \end{aligned}$$

Let $u = \theta - 30°$. The following sets of data points have be determined:

$$(u,\ y_B) = (0°, -0.62114 \text{ ft}), (7.5°, 1.05972 \text{ ft}), (15°, 1.65060 \text{ ft})$$

The quadratic curve fit of this data gives

$$y_B = -0.62114 + 0.29678\, u - 0.009688711\, u^2$$

Setting $y_B = 0.4$ ft gives the quadratic equation

$$-0.009688711\, u^2 + 0.29678\, u - 1.02114 = 0$$

Solving for u, $\quad u = 3.95° \quad \text{and} \quad 26.68°$

Rejecting the second value gives $\quad \theta = 30° + u = 33.95°$.

Try $\theta = 33.95°$.

$$v_2 = \sqrt{64.4 \sin 33.95°} = 5.997 \text{ ft/s}$$

$$t_B - t_2 = \frac{2\cos 33.95° - 0.3}{5.9971 \sin 33.95°} = 0.40578 \text{ s}$$

$$\begin{aligned} y_B &= 2\sin 33.95° + (5.997\cos 33.95°)(0.40578) - (16.1)(0.40578)^2 \\ &= 0.48462 \text{ ft} \end{aligned}$$

The new quadratic curve-fit is based on the data points

$$(u,\ y_B) = (0°, -0.62114 \text{ ft}), (3.95°, 0.48462 \text{ ft}), (7.5°, 1.05972 \text{ ft}).$$

The quadratic curve fit of this data is

$$y_B = -0.62114 + 0.342053907\, u - 0.015725232\, u^2$$

Setting $y_B = 0.4$ ft gives

$$-0.015725232\, u^2 + 0.342053907\, u - 1.02114 = 0$$

Solving for u,

$$u = 3.572° \qquad \theta = 30° + 3.572° = 33.572°$$

Try $\theta = 33.572°$.

$$v_2 = \sqrt{64.4 \sin 33.572°} = 5.9676 \text{ ft/s}$$

$$t_B - t_2 = \frac{2 \cos 33.572° - 0.3}{5.9676 \sin 33.572°} = 0.41406 \text{ s}$$

$$\begin{aligned} y_B &= 2 \sin 33.572° + (5.9676 \cos 33.572°)(0.41406) - (16.1)(0.41406)^2 \\ &= 0.40445 \text{ ft} \end{aligned}$$

which is close enough to 0.4 ft.

Substituting $\theta = 33.572°$ and $v_2 = 5.9676$ ft/s into Eq. (2) along with other data gives

$$v_0^2 = (5.9676)^2 + (2)(32.2)(2)(1 + \sin 33.572°) = 235.64 \text{ ft}^2/\text{s}^2$$

$$\mathbf{v}_0 = 15.35 \text{ ft/s} \longrightarrow \blacktriangleleft$$